AN

ELEMENTARY COURSE

OF

CIVIL ENGINEERING,

FOR THE USE OF

THE CADETS

OF THE

UNITED STATES' MILITARY ACADEMY.

Dennis Hart

BY

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TO

THOMAS NEWTON, Esq.,

OF NORFOLK, VIRGINIA.

Sir,

As one of the earliest pioneers in the march of Internal Improvement in our own State, and among its most able and successful advocates in our National Councils, during the unusually long period in which you occupied the station of a Representative in Congress, there is no one to whom this little volume could with more propriety be dedicated than to yourself. But strong as are these claims on the attention of its author, a more pleasing duty falls to his lot, in laying it before you as an humble offering to an early and kind friend,—one to whom he is indebted for his first important step in life, his admission as a Cadet into our only National School; he therefore, Sir, requests your acceptance of its dedication to you as a tribute of grateful remembrance; and, in offering it, begs to renew his sincere wishes for your health and prosperity.

D. H. MAHAN.

United States' Military Academy,
West Point, June 1, 1837.

a.

PREFACE.

The best apology which the author of this work can offer, for ushering a new book into the scientific world, is the absolute want of some treatise of an elementary character, arranged as a text book for instruction, on the subject of which it treats. So far as the author's information extends, there is, he believes, no work of a similar character either in English or in French, the two languages which are within the comprehension of the majority of persons engaged in the profession of civil engineers, except the Programma of the Course of Civil Constructions, as taught, many years back, at the Polytechnic School, by M. Sganzin, an eminent French engineer, and which has been translated into English and published in our own country. This work, it is unnecessary to say to those into whose hands it may have fallen, is almost a dead letter to persons who are entirely unacquainted with the subject of constructions, and to those who are, it leaves much to be desired; being, as its title shows, nothing more than a very meagre outline of the Course taught by the able Professor, whose name it bears, and was designed by him only to refresh the memories of the pupils who had profited by his instruction.

Charged with teaching in the Military Academy a course of civil constructions, conjointly with the one of military

engineering, the author was forced to gather his means from every source within his reach, and to embody them in the most convenient manner, for the benefit of his pupils. The information thus collected, assumed by degrees the form of the work now laid before the public. It is but a part of the course taught, at present, to the Cadets of the Military Academy; being what the author deemed would be most suited to the wants of other seminaries of learning, and to young men who are preparing themselves as civil engineers.

The author claims no merit for the contents of the work; the ideas and precepts, which it embodies, are but a part of the common stock belonging to the profession in general; he has only endeavored to exercise a discriminating judgment in selecting from the various sources of information, both written and oral, to which he had access, what seemed to him most worthy of confidence. The faults of the work, of which no one can be more sensible than the author is, are peculiarly his own; its merits must be shared among many abler authorities both among the dead and living.

With respect to the matter selected and its arrangement, the author has only to say, that he has chiefly aimed at being sound and clear; he has given the work as popular a form as he conscientiously could; omitting nothing, however, which the cause of sound instruction seemed to demand. Could he have prevailed upon himself, in obedience to what appears to be the pervading taste of the moment, to have pursued a different course, he should have held himself not only as recreant to that cause, but as having contributed to weaken the powers of the human mind, by relieving it of the necessity for patient investigation and close reasoning which alone can give it that vigor which achieves excellence and commands success in every department of life.

United States' Military Academy,
June 1, 1837.

NOTE.

As the author frequently receives letters from persons, about commencing the study of civil engineering, asking his counsel, both as to the best course to be

pursued, and the best works to be studied, he would, in this place, respectfully offer the following remarks:

A thorough acquaintance with the mathematics, the author considers as indispensable to a successful pursuit of this profession; without this essential ground work, he confesses, that he is entirely at a loss to conceive, how any sound acquirements, other than a few mechanical rules, acquired in the routine of practice, can be made in it. Without wishing to prejudice the works of others, the author would call attention to the very complete Course of Mathematics of *Professor* Davies, late Professor of Mathematics in the Military Academy, as the best that has fallen under his observation in the English language. The great success of this gentleman as a teacher, is alone a sufficient guarantee of the excellence of his works, in the arrangement of which, moreover, he has followed the best of mathematical schools, the French. As more immediately connected with the mathematics, and as an indispensable branch of knowledge to the engineer, the author would also mention a Treatise on Topography, by *Lieutenant* Eastman of the U. S. Army, now acting as *Assistant Teacher* of *Drawing* in the Military Academy, which is now in the hands of the publisher, and may be shortly expected from the press. The merits of this gentleman as a finished draughtsman may lead us to expect an excellent work on this branch.

Connected with the Art of Constructions and applied mathematics, the author would mention the name of M. Navier. The European reputation of this eminent *savan* and engineer, would render eulogium from the author more, if possible, than supererogatory. His name is connected, either as author or editor, with the ablest works on the subject under consideration, that have appeared in France within the last twenty years; and the best counsel that the author could give to every young engineer, is to place in his library every work of science to which M. Navier's name is in any way attached.

Of treatises on special branches of civil engineering, the author would mention,

Smeaton's works generally.

The Articles *Bridge*, *Canals*, and *Carpentry*, in the Edinburgh Encyclopedia, and the Supplements both to it and to the Encyclopedia Britannica.

Tredgold on *Carpentry*.

Tredgold on *Cast Iron*.

Transactions of the Society of Civil Engineers.

De Pambour on *Locomotives*.

Wood on *Rail-Roads*.

Parnell on *Common Roads*.

Storrow on the *Conveyance of Water*.

De Gerstner (traduit par M. Girard) sur les *Chemins à Ornières*.

Treussart sur les *Mortiers*.

The works containing descriptions of particular constructions, are too numerous to be mentioned here. The author would recommend every young engineer to procure all of this character, from authentic sources, that his means will admit of; for he will find in them those practical details, which cannot enter in

an elementary treatise. Among works of this character, the author would, particularly, call attention to the able reports made by the engineers of our country, within the last twenty years, on the various works of internal improvements undertaken within this period. The young engineer will find in these reports information, which he would seek for in vain at any other source.

The author is led to hope, that the profession will receive an invaluable addition to the common stock, on that very imperfectly understood subject, among our builders, the composition of mortars, from *Colonel* Totten of the *United States Corps of Engineers.* This distinguished officer has made this subject a particular study, for many years back; and has instituted a wide range of experiments, at the works under his charge, on the different varieties of lime found in our country. It is to this gentleman, that the author is indebted for the confirmation of many of the facts, laid down in this Course, under the head of Mortars.

The author is indebted for the drawings of most of the plates, accompanying this work, to one of his pupils, Lieutenant *J. Carle Woodruff, of the* U. S. Army, *at present* Assistant Prof. of Engineering *in the Military Academy.*

CONTENTS.

MATERIALS.

	Page.
Classification of Materials,	1
Natural Stones or Rocks,	1
Siliceous Stones,	2
Granite and Gneiss,	2
Sienite,	4
Sand Stone,	5
Mill-Stone, Soap-Stone, and Mica Slate,	6
Argillaceous Stones,	6
Gray-wacke and Gray-wacke Slate,	6
Roofing Slate,	7
Calcareous Stones,	7
Lime Stone, Granular and Compact,	8
Marble, variegated, varieties of	8
Gypsum, or Plaster of Paris,	9
General Observations on Building Stone,	10
Artificial Stone and Cements,	13
Quick Lime,	13
Sand,	17
Cements,	17
Puzzolano and Trass,	18
Mortar,	19
Beton, Grub-stone Mortar or Concrete,	24
General Observations on Mortar,	24
Brick,	27
Fire-brick,	30
Tiles,	31
Wood,	31
Felling of Trees for Timber,	32
Seasoning of Timber,	33

	Page.
Defects and Diseases of Timber, - - - - -	34
Oak, varieties of - - - - - - - - -	36
Pine, varieties of - - - - - -	37
Locust, Chesnut, Cedar, &c. - - - - - -	38
Strength of Timber, - - - - - - -	39
Metals, - - - - - - - - -	39
Cast Iron, - - - - - - -	40
Forged Iron, - - - - - - - -	42
General Results of the Strength of Materials in general, -	44
Practical Deductions on the Strength of Materials, - -	48
MASONRY, - - - -	53
Classifications of Masonry, - - - - - -	53
Cut Stone, - - - - - - - -	53
Rubble Masonry, - - - - - - -	58
Brick Masonry, - - - - - - -	59
Foundations, - - - - - - - -	59
Construction of Masonry, - - - - - -	75
Masonry of Arches, - - - - - - -	80
Influence of Season on Masonry, - - - -	84
CARPENTRY, - - - - -	85
Frames, - - - - - - - - - - -	85
Algebraical Formula relating to the Strength of Straight Timber,	86
Wooden Arches, - - - - - - - - -	91
Frames of Straight Timber, - - - - - -	97
General Remarks on Frames, - - - - - -	103
Joints, - - - - - - - -	104
Built Beams, - - - - - - - -	105
Scarf Joints, - - - - - - - -	106
Mortise and Tennon Joint, - - - - - -	107
Dove-tail Joint, - - - - - - -	108
Bolts and Straps, - - - - - - -	108
ROADS, - - - -	109
Roconnaissance of Lines for Roads, - - - - -	109
Survey, - - - - - - -	112
Map and Memoir, - - - - - - -	112
Survey of a Common Road, - - - - -	113
Equalizing Excavations and Embankments, - - -	117
Grading, - - - - - - -	120

	Page.
Drainage, - - - - - - - -	125
Road Coverings, - - - - - - -	128
Cross Dimensions of Roads, - - - - -	136
Repairs and Materials, - - - - - -	137
BRIDGES, - - - - -	138
Classification of Bridges, - - - - - -	138
Definitions relating to a Bridge, - - - - -	138
Survey for a Bridge, - - - - - - -	138
Basis of Plan of a Bridge, - - - - - -	139
Locality, - - - - - - - - -	140
Water Way, - - - - - - - -	140
Form and Dimensions of Bays, - - - - - -	142
Dimensions of Arches, - - - - - -	145
Dimensions and Form of Piers, - - - -	145
Arrangement of the Facing, - - - - - -	148
Superstructure, - - - - - - - -	148
Style of Architecture, - - - - - -	149
Masonry of a Bridge, - - - - - - -	150
Approaches, - - - - - - - -	152
Water Wings, - - - - - - - -	154
Enlargment of Water Way, - - - - - -	155
Centres for Arches, - - - - - - -	155
Wooden Bridges, - - - - - - -	159
RAIL-ROADS, - - -	168
Form and Materials of Rails, - - - - -	169
Supports of Rails, - - - - - -	172
Arrangement of Rail Tracks, - - - - -	174
Inclined Planes on Rail Roads, - - - - - -	176
Sidings and Crossings, - - - - - -	177
CANALS, - - - - -	178
Classification of Navigable Canals, - - - - -	179
Location, - - - - - - - - -	182
Cross Section, - - - - - - -	182
Summit Level, - - - - - - - -	183
Deep Cuts and Tunnels, - - - - - -	184
Supply of Water, - - - - - - -	188
Feeders and Reservoirs, - - - - - -	193
Lift of Locks, - - - - - - - -	196
Reaches, - - - - - - - -	198

Page.
Locks, - - - - - - - - - 198
Lock-Gates, - - - - - - - - 204
Culverts, - - - - - - - - - 206
Aqueducts, - - - - - - - - 207
Canal Bridges, - - - - - - - - 208
Waste Wiers, - - - - - - - 208
Reach Dams, - - - - - - - - 209
Guard or Tide Lock, - - - - - - 209

RIVERS, - - - - - 209

SEA-COAST IMPROVEMENTS, - - 216

Roadsteads, - - - - - - - - 216
Harbors, - - - - - - - - 220
Dikes and Sea Walls, - - - - - - - 225

SUPPLEMENT, - - - - 227

Angle of Friction. *Note I.* - - - - - - 229
Methods of Laying out Circular Arcs on the Ground. *Note II.* 231
Excavations and Embankments. *Note III.* - - - 233
On Oval Curves. *Note IV.* - - - - - - 250
Equilibrium of Arches. *Note V.* - - - - - 255
Equilibrium of Sustaining Walls. *Note VI.* - - 278
Pressure of Voussoirs on Centres. *Note VII.* - - - 296
Pressure on Frames and Supports of Wooden Bridges. *Note VIII.* 299
Elevation of Exterior Rail of a Rail Track. *Note IX.* - 301
Gauging of Water Courses. *Note X.* - - - - - 305
Length of Tail Wall of Locks. *Note XI.* - - - 308
Strain on Lock Gates. *Note XII.* - - - - - 309

EXPLANATIONS OF THE PLATES.

Plate 1.

Fig. 1. *Vertical section of a Spheroidal Lime Kiln to be heated with wood.*

A. Fire-place.

B. Inner Lining of the kiln of fire brick.

C. General mass of the kiln.

D. Shed to cover the top of the kiln.

E. Spherical dome, formed of the larger blocks of lime stone just above the flues of the fire-place, to give the flame a freer passage through the body of the stone.

F. Body of lime stone.

Fig. 2. *Plan and Section of a Conical Lime Kiln for pit or charcoal.*

A,A′. Annular vault into which the lime is received after being burnt.

G,G′. Entrances to the annular vault.

B. General mass of the kiln.

C. Small spherical vault with flues at top to supply air to the combustible.

D. Conical nucleus of stone to facilitate the passage of the burnt stone into the openings made in the annular arch.

E. Small parapet wall around the top of the kiln.

F. Body of lime stone and combustible material.

Fig. 3. *Plan and Section of a Kiln for burning Clay for Cement.*

A, A. Fire-places.

B, B′. Oven in which the clay is burnt.

Fig. 4. *Elevation of a Mortar Mill.*

A. Circular trough for the reception of the materials.

B. Wheel running in the trough to mix the materials.

C. Axle of the wheel to which a horse is attached.

D. Vertical axle which receives the axle of the wheel.

Fig. 5. *Section of an Ordinary Mortar Mill.*

A. Vertical axle which receives the rakes.

B. Horizontal arm to which the horse is attached at the hoop C.

FIG. 6. *Elevation of a Cut Stone wall with a great batter.*
a, a Exterior stone with elbow joints.
b Bottom stone termed a *die* or *newell.*
FIG. 7. *Vertical Section of a Wall with Cut Stone facing* A, *and Rubble backing* B.
FIG. 8. A *Elevation*, B *Plan*, and C *Section of a Wall, formed of headers and stretchers.*
a. Stretcher.
b. Header.
FIG. 9. A *Elevation*, and B *Plan* of a Wall formed with dove-tailed or *joggled* blocks, connected with iron cramps and bolts.
C. Perspective view of a single block.
D. Perspective view of two blocks joined.
FIG. 10. *Cross Section of a Grillage and Platform on Piles.*
a. Cupping.
b. Planks of Platform.
c. String pieces.
d. Cross pieces.
e. Notches into which the string pieces are fitted.
h, h. Piles.
A. Enrockment of broken stone.

PLATE 2.

FIG. 11. *Cross Section of a Wall laid on a bed of Sand.*
A. Sand.
B. Masonry.
FIG. 12. *Cross Section of a Wall laid on Sand Piles.*
A, A. Pile holes filled with sand.
B. Masonry of wall.
FIG. 13. *Cross Section of a Coffer Dam.*
a, a. Piles.
b, b. Exterior string pieces connecting the piles.
c. Cross pieces notched on the string pieces.
e. Interior ribband pieces against which the sheeting-piles d are driven.
A. Puddling.
FIG. 14. *Cross Section of a floating Caisson.*
a. Bottom of scantling and plank to receive the masonry A.
b. Uprights against which the plank of the sides are nailed.
c. Cross pieces notched on the uprights and connecting the sides.
d. Iron bars to connect the bottom and sides.
e. String pieces notched on the Cross pieces to confine them.

FIG. 15. *Cross Section and Plan of a Guide Grating for Piles.*
a, a'. Cross pieces of the grating.
b, b'. String pieces notched in pairs on cross pieces.
c, c, c'. Foundation piles.
d, d'. Scaffolding piles.
e, e'. Guide piles for settling the grating.

FIG. 16. *Plan and Cross Section of a fixed Caisson.*
a,a,a',a'. Principal uprights.
b,b' &c. String pieces notched in pairs on the uprights.
c,c' &c. Heavy sheeting piles driven between the string pieces.
d,d'. Iron bars connecting the sides of the caisson.

FIG. 17. *Cross Section of a Coffer Dam, à la Treussart.*
a, a. Exterior and interior sheeting to confine the puddling B.
b. String pieces to connect the sheeting piles.
c. Cross pieces notched on the string pieces.
A. Bed of beton to receive the masonry C.

FIG. 18. *Cross Section of a Foundation.*
A. Batter of foundation.
B. Offsets of foundation.
C. Masonry of superstructure.

FIG. 19. *Cross Section of Foundation with Reversed Arches.*
a. Reversed arches.
b. Askew backs.
B. Superstructure.

FIG. 20. *Longitudinal Section of a Stone Arch.*
A. Arch stones or voussoirs.
B. Key-stone.
C. Mass of abutment.
D. Backing of arch.
E. Askew back stone.
a. Springing line.
b. Crown of arch or intrados.

PLATE 3.

FIG. 21. *Elevation of a heavy Brick Arch.*
A and B. Shells.
C. Blocks.

FIG. 22. *Cross Section of a Brick Arch at the Crown.*
a. First course at intrados.
b and c. Intermediate courses.
d. Course at intrados.

FIG. 23. *Horizontal Beam attached to a wall at* A, *loaded with a weight* W *at* D, *and with the equal weights* w *at the intermediate points as* B *and* C.

FIG. 24. *Horizontal Beam supported on two props and loaded with a weight* W *at any point* C.

FIG. 25. *Horizontal Beam attached to the wall at* C *and resting on a prop at* B *and loaded with a weight* W *at the middle point* C.

FIG. 26. *Horizontal Beam resting on three props and loaded with the weights* W *and* W' *between the props.*

FIG. 27. *Horizontal Beam resting on a fixed vertical support and having a weight* W *applied at* C.

FIG. 28. *Inclined Beam with a weight* W *applied at* B.

FIG. 29. *Inclined Beam with a weight* W *applied at* C.

FIG. 30. *Bent Beam.*

FIG. 31. *Built beam without vertical joints confined by iron straps* a.

FIG. 32. *Built beam with vertical joints confined by iron straps.*

FIG. 33. *Built beam with keys* b *and straps.*

FIG. 34. *Built beam with indented courses and straps.*

FIG. 35. *Built beam strengthened by braces and uprights.*

A. Upper beam.

B. Lower beam.

a, a. Uprights connecting the beams A and B.

b and c. Diagonal braces.

FIG. 36. *Wooden Arch.*

a, a. Uprights at equal intervals which transmit the weight diffused over C to the arch.

FIG. 37. *Inclined beams confined at* A *and* B *with a weight suspended from their point of junction* C.

FIG. 38. *Horizontal beam resting on two fixed inclined supports with a weight* W *suspended at* C.

FIG. 39. *Horizontal beam supported by a brace with a weight* W *applied at* C.

FIG. 40. *Horizontal beam sustained by a fixed upright and inclined brace with a weight* W *suspended from* C.

FIG. 41. *Same as* FIG. 40, *except the upright is braced.*

FIG. 42. *Horizontal beam resting on two props and sustained by braces.*

FIG. 43. *Horizontal beam on two fixed uprights and strengthened with braces.*

PLATE 4.

FIG. 44. *Iron Strap arranged for screwing up.*
a. Strap.
b. Cross piece with eyes to receive the ends of the strap.
c. Screw nuts to confine the piece b.

FIG. 45. *Double Wedged Key.*
a and b. Wedges.
c. Piece with a square notch to receive the wedges.

FIG. 46. *Built indented Beam with an iron King Bolt a.*

FIG. 47. *Fished Beam.*
a and b. Pieces abutting end to end and confined by the lateral pieces c and d, and bolts.

FIG. 48. *Fished Beam confined by lateral pieces arranged with indents, bolts and keys.*

FIG. 49. *Scarfed Beam confined by straps bolts and keys.*

FIG. 50. *Scarfed Beam for cross strains, secured by straps, keys and a bottom piece* c.

FIG. 51. *Scarfed Beam for a cross and longitudinal strain, secured by a double-wedge key* c, *a bottom strap* d, *and bolts.*

FIG. 52. *Mortise and Tenon joint.*
a. Tenon.
b. Mortise.
c. Pin hole.

FIG. 53. *Mortice Joint for inclined pieces.*
a b c. Notch made in the piece B to receive the end of A.
d. Form of the tenon of the piece A.

FIG. 54. *Dove-tail joint.*

FIG. 55. *Iron Strap for securing a joint near a wall.*
a, a, a. Lateral pieces of the strap.
c, c. Cross pieces to secure the lateral pieces and the joint.

FIG. 56. *Iron Strap for securing a joint between two beams.*

FIG. 57. *Cross Section of a Road in Cutting.*
A. Road surface.
B. Side slopes.
C. Top surface drain.

FIG. 58. *Cross Section of a Road in Cutting on Slate with side slopes cut into steps and covered with earth.*

FIG. 59. *Cross Section of Road in Filling.*

FIG. 60. *Cross Section of Road in Side Cutting.*

FIG. 61. *Cross Section of a Road in steep Side Cutting.*
A. Filling.
B. Sustaining wall of filling.

C. Breast wall of cutting.
D. Parapet wall of foot-path.

PLATE 5.

FIG. 62. *Level for forming Road Surface.*
A. Level.
b c. Horizontal line through the middle point of the road-way.
a, a. Sliding gauges to mark the distances of the points of the road surface below b c.

FIG. 64. *Cross Section of a covered Drain.*
A. Drain.
a, a. Side walls.
b. Top stones.
c. Bottom stones.
d. Broken stone or large gravel laid over brush.

FIG. 63. *Cross Section of Broken Stone Road Covering.*
A. Road surface.
B. Side channels.
C. Foot-path.
D. Covered drains, or culverts, leading from side channels to the side drains E.

FIG. 65. *Paved Road Covering.*
A. Pavement.
C. Curb stone.
D. Flagging of side walk.

FIG. 66. *Cross Section of Road through a Cross Drain.*
A. Inlet to cross drain for water from the side channel.
B. Cross drain.
C. Side drain.

FIG. 67. *Elevation of the Head of a Bridge.* (*Full Centre Arch.*)
A. Starlings.
B. Hood or cap of starling.
C. Voussoirs.
D. Spandrel courses.
E. Parapet of bridge.

FIG. 68. *Elevation of Head of a Bridge.* (*Segment Arch.*)
A. Starling.
B. Hood.
C. Voissoirs.
D. Spandrel courses.
E. Parapet.

FIG. 69. *Elevation as in* FIGS. 67, 68. (*Oval Arch of three Centres.*)

Plate 6.

Fig. 70. *Elevation same as preceding.* (*Oval of eleven Centres.*)
F. Enlargement of water-way, see Fig. 83.
Fig. 71. *Plan of a Starling with Plane Faces.*
A. Starling.
B. Pier.
Fig. 72. *Plan of a Starling with Curved Faces.*
Fig. 73. *Plan of semi-circular Starling.*
Fig. 74. *Plan of semi-elliptical Starling.*
Fig. 75. *Cross Section of the Parapet and Cornice of a Bridge.*
a. Cornice.
b. Consoles or modillions.
c. Parapet.
d. Foot-path.
Fig. 76. *Plan of the Foundation of an Abutment.*
d. Face of the abutment.
c. Wing walls.
a. End counterforts or buttresses.
b. Intermediate buttress.
Fig. 77. *Longitudinal Section of the Waterloo Bridge and its Centre.*
C. Voussoirs of arch.
E. Balustrade parapet.
O. Reversed arch over inner spandrel courses.
P. Longitudinal walls of brick to sustain the flagging Q, on which the road material rests.
a, a. Top and bottom striking plates.
b. Wedge block for easing the centre.
c, c, &c. Struts of the centre.
d, d, &c. Stirrup pieces bolted in pairs on the struts.
e, e. Cast-iron sockets to receive the ends of the struts.
f, f. Curved back of the rib.
g, g. Chock blocks to strengthen the joints of the pieces f.
h, h, &c. Main props or shores of the centre.
Fig. 78. *Cross Section of a Bridge over the Pier, showing the two Systems of sustaining the road material, either by flagging, or by groined arches.*
a, a. Cross section of walls which sustain the flagging c.
b, b. Piers of the groined arches.
d. Section through the crown of the groined arches.
e. Exterior facing walls of the bridge.

Plate 7.

Fig. 79. *Elevation and Plan, showing the manner of arranging the Embankments of the Approaches, when the Head Walls of the Bridge are simply prolonged.*

a, a′. Side slope of embankment.

b, b′. Dry stone facing of the embankment where its end is rounded off, forming a quarter of a cone finish.

f, f′. Flight of steps for foot passengers to ascend the embankment.

c, c′. Embankment arranged as above, but simply sodded.

d, d′. Facing of dry stone for the side slopes of the banks.

e, e′. Facing of the bottom of the stream.

Fig. 80. *Elevation and Plan of Arched-wing Wall.*

A, A′. Arch of the wing wall.

B, B′. Side slope of the embankment sustained by the wall A, A′.

Fig. 81. *Plan and Elevation of an ordinary Wing Wall for a Culvert or Bridge.*

a, a′. Face of wing wall.

b, b′. Side slope of embankment.

c, c′. Top of wing wall.

o, o′. Fender or guard stones on top of the bridge.

Fig. 82. *Elevation and Plan of a Return Wall and Wing Wall.*

A, A′. Starling.

B, B′. Return wall for the widened approach to the bridge.

C, C′. Face of wing wall.

D, D′. Top of surface of wing wall.

E, E′. Parapet.

F, F′. Newel.

Fig. 83. *Cross Section of* Fig. 70 *of the Bridge of Neuilly.*

A. Starling.

B. Hood of starling.

F. Enlargement of water-way.

Fig. 84. *Rib of a Plank Centre.*

a, a. Curved back pieces.

b, b. Bolsters.

Fig. 85. *Rib of Heavy Centre with intermediate points of support.*

a. Curved back pieces.

b, b. Radial struts.

c, c. Horizontal beam resting on intermediate supports d on which the struts rests.

e, e. Braces bolted in pairs on the struts.

f. Bolsters.

[Plate 8.]

FIG. 86. *Rib of a Cocked Centre without intermediate points of support.*

a. Curved back pieces.

b, b. Struts.

c, c. Straining beams.

d, d. Main shores.

e. Striking plates.

f. Bolsters.

FIG. 87. *Rib of a Cocked Centre on the same principle as* FIG. 86.

b. Main struts.

c. Main straining beam, which, with the main struts, form the main strength of the rib.

FIG. 88. *Rib of a Cocked Centre with details.* (Tredgold's Carpentry.)

a, a. Main props.

b, b. Caps of the props to receive the striking plates.

c, c. Sills of do.

A. Bottom plate.

B. Top plate.

C. Wedge block.

D. Main strut resting in a cast iron socket on B.

E. Main straining beam.

F, F. Auxiliary struts.

G, G. Stirrup pieces bolted in pairs over the struts, &c.

H, H. Curved back pieces.

I, I. Longitudinal ties bolted on the ribs in pairs.

d, d. Chock pieces.

FIG. 89. *Wooden Arch Bridge.*

A. Arch formed of several courses of bent beams confined by straps.

B. Stone abutment.

a, a. Vertical uprights bolted in pairs on the arch and principal beams of the superstructure to transmit the weight of the latter to the former.

b, b. String pieces of the road-way.

c, c. Cross pieces or joists.

d, d. Plank to receive the road materials.

e, e. Hand railing.

f, f. Diagonal struts which assist to relieve the arch from a part of the weight of the superstructure.

FIG. 90. *Elevation of a Wooden Pier.*

a, a. Piles of substructure.

b, b. Capping of piles arranged to receive the ends of the uprights c, c, which supports the string pieces i, i.

d. Upper fender beam.

e. Lower fender beam.

f. Horizontal ties bolted in pairs on the uprights.

g, g. Diagonal braces bolted in pairs on the uprights.

h. Capping of the uprights placed under the string pieces.

A. Road-way.

B. Parapet.

Fig. 91. *Plan, Elevation, and Cross Section, showing the arrangement of the Capping of the Foundation Piles with the Uprights.*

a. Piles.

b. Capping of four beams bolted together.

c. Uprights.

Fig. 92. *Elevation of the arrangement of a Wide Foundation for a Wooden Pier.*

a. Upright.

b, b. Piles of the foundation.

c, c. Capping of the piles.

d, d. Struts to strengthen the uprights.

e, e. Clamping pieces bolted in pairs on the uprights.

Fig. 93. *Elevation and Plan of a Simple Ice Breaker.*

a, a. Foundation piles.

b, b. Capping of piles.

c, c. Uprights.

d. Inclined fender beam shod with iron.

Fig. 94. *Elevation and Plan of the Frame of an Ice Breaker to be filled in with Broken Stone.*

Fig. 95. *Elevation of simplest form of Wooden Bridge.*

a. Uprights.

b. Capping of uprights.

c. String pieces.

Fig. 96. *Elevation of Wooden Bridge with Corbels.*

d. Corbel pieces.

Plate 9.

Fig. 97. *Elevation of Wooden Bridge with Struts, &c.*

e. Struts.

Fig. 98. *Elevation of Wooden Bridge with Straining Beam, &c.*

f. Straining beam.

Fig. 99. *Same Principle as* Fig. 98.

FIG. 100. *Elevation of Wooden Bridge with King Post and Struts.*
c. String piece to receive the flooring, &c.
e. Struts.
g. King post.

FIG. 101. *Elevation of Wooden Bridge with Queen Posts, &c.*
g. Queen posts bolted in pairs on the other pieces.
h. Straining beam.

FIG. 102. *Same principle as* FIG. 101.

FIG. 103. *Elevation of Wooden Bridge in which the string pieces are sustained by a Bent Beam* b.

FIG. 104. *Wooden Arch with road-way resting on it.*

FIG. 105. *Wooden Arch with Road-way suspended from it.*
A. Arch.
B. Roof covering of bridge.
C. String pieces suspended from the arch by the stirrup pieces g bolted in pairs to both.
i. Cross joists for the flooring of the road-way.
m. Flooring plank.
o, o. Diagonal braces to stiffen the system.

FIG. 106. *Cross Section of the Road-way of a Bridge with details.*
C, C. String pieces.
D. Cross pieces at intervals of a few feet notched on all the string pieces.
E. Intermediate cross pieces of smaller scantling forming with the pieces D the flooring for the road materials.
F. Curb beam for confining the road material.
G. Cross joists on which the longitudinal pieces I, I are laid to receive the plank H of the foot-path.
a, a. Cast iron plates placed between the string pieces and cross joists.
b. Iron grating for parapet.
c. Iron stancheons to brace the parapet.

FIG. 107. *Side elevation of Wrought Iron Fish-bellied Rail.*

FIG. 108. *Cross Section of Rail and Wrought Iron Chair.*
a. Rail.
b. Chair.
c. Clamping pieces confining the chair by means of the bolts d.
A. Wooden sleeper to which the chair is fastened.

FIG. 109. *Cross Section of Cast Iron Chair and Rail.*
a. Rail.
b. Chair.
c. Iron wedge to confine the rail to the chair.

FIG. 110. *Plan and Cross Section of the system of Wood and Iron Rail Roads.*

A, A. Cross sleepers.

B, B'. String pieces on which the iron plates are fastened.

a'. Iron plates.

b. Arrangement of the joints between the plates.

c, c'. Wooden wedges to confine the string pieces to the sleepers.

D, D'. Broken stone supports.

PLATE 10.

FIG. 111. *Plan of a Siding.*

A, A. Main track.

B, B. Siding.

a. Fixed switch.

b. Movable switch.

FIG. 112. *Plan of a Crossing.*

A. Main track.

B. Track of Crossing.

a, a. Turn outs.

FIG. 113. *Cross Section of a Canal in Level Cutting.*

A. Water-way.

B. Tow-paths.

C. Berms.

D. Side drains.

E. Puddling of clay or sand.

FIG. 114. *Cross Section of a Canal in Side Cutting lined with masonry.*

A. Water-way.

B. Tow-paths.

D. Embankment.

a. Masonry lining.

FIG. 115. *Cross Section of a Canal in Thorough Cutting.*

E. Side slopes of cutting.

FIG. 116. *Cross Section of an Earthen Dam through its Culvert.*

A. Mass of dam towards the reservoir faced with stone.

B. Exterior mass of the dam.

C. Puddling.

D. Culvert for drawing off the water.

E. Valves for closing the tunnel.

F. Brick well with spiral stairs to reach E.

G. Reservoir.

Fig. 117. *Cross Section of the Arch and Centre of a Tunnel.*

A. Excavation of the arch.
B. Working shaft.
C. Arch.
D. Reversed arch of the bottom.
a. Back curved pieces of the centre.
b. Radial pieces connected by screws, by which the centre can be taken apart.
c. Struts.

Plate 11.

Fig. 118. *Plan and Longitudinal Section of a Canal Lock.*

A, A′. Chamber or side walls.
B, B′. Lift walls.
C, C′. Bottom of the chamber reversed arch.
D, D′. Wing or return walls of the head-bay.
E, E′. Wing or return walls of the fore-bay.
F. Bottom of fore-bay.
G. Bottom of tail-bay.
a, a′. Upper mitre sill.
b, b′. Lower mitre sill.
c, c′. Gate chambers or recesses.
d, d′. Hollow quoins.
e. Flat bottom for the free play of the gates.
o, o′. Grooves for stop-plank.

Fig. 119. *Cross Section of the Lock through the Chamber.*

A″. Side walls.
B″. Lift wall.
C′. Reversed arch of the chamber.
a″. Mitre sill.
m″. Coping of side wall.

Fig. 120. *Leaf of a Lock-Gate.*

a. Quoin post.
b. Mitre post.
c, c′. Bottom cross piece.
d, d′. Top cross piece.
e. Diagonal brace.
f. Balance beam.
A, A′. Valve.
m′. Plank foot way for crossing the lock.

D

FIG. 121. *Cross Section of a Stone Aqueduct.*
A. Water way.
B. Mass of beton of sides.
C. Mass of beton of the bottom.
D. Elevation of the soffit of the arch.
E. Elevation of the pier.
a. Tow paths.
b. Parapet.

FIG. 122. *Cross Section and Plan of the Waste Wier and Culvert emptying into a side drain.*
B, B′. Side walls of waste wier.
C. Bottom of waste wier.
D. Lift of waste wier.
E, E′. Side drains.
a, a. Sliding gates of waste wier.
b, b′. Bridge across waste wier.
c, c′. Groves for stop-plank.

FIG. 123. *Cross Section of the Cherbourg Break Water.*
a b. Inner face.
b c. Top or walk of break water.
c d. Parapet.
d e f g. Exterior face.

FIG. 124. *Cross Section of a Wooden Jetty.*
a. Foundation piles.
b. Inclined side pieces.
c. Middle upright.
d. Cross pieces bolted in pairs.
e. Struts.
m. Longitudinal pieces bolted in pairs.
o. Parapet.

See plates 12 — 14.

CIVIL ENGINEERING.

ELEMENTARY COURSE

OF

CIVIL ENGINEERING.

MATERIALS.

THE materials used in building are arranged in four classes: they are, 1. *Natural Stones*; 2. *Artificial Stone* and *Cements*; 3. *Wood*; 4. *The Metals*.

An accurate knowledge of the physical and chemical properties of materials is of essential importance to the engineer, to enable him to form a correct estimate of the advantages to be derived from their proper application to the purposes of constructions, so as to satisfy the conditions of judicious economy, and skilful workmanship. He should, therefore, be acquainted with their absolute and relative strength, the resistance which they offer to friction and shocks, the changes which they undergo from exposure to the atmosphere, and to the more ordinary chemical agents, as fire, salt water, &c., and, finally, the time and labor required in preparing them for the purposes of building.

NATURAL STONES OR ROCKS.

Natural Stones, or *Rocks*, are composed of an aggregation of several simple mineral substances. They are variously classified by naturalists, either according to their chemical

constituents, or from their external appearances and physical properties. These classifications, although essential for scientific arrangement, are of minor importance to the engineer; as the principal points requiring his attention, are those which render stone a suitable material for building.

The most essential properties of stone, as a building material, are *strength*, or the resistance which is offered to rupture, caused either by compression, extension, or a cross strain; *hardness*, or the capability of resisting shocks, and attrition; and *durability*, or the unchangeable character of the stone, when exposed to the extremes of temperature, to the atmosphere, and to chemical agents. These properties can be readily ascertained by a few simple experiments, which will be noticed under their proper heads.

As the stones commonly used for building, may be arranged in three classes, it is usual, for greater convenience, to adopt this classification, which is: 1. The *Siliceous Class*, or the one, of which Silex is the base, or principal constituent element; 2. The *Argillaceous*, of which Argile is the base; and 3. The *Calcareous*, of which Lime is the base.

SILICEOUS STONES.

Granite, Gneiss, and *Sienite*, are commonly known to builders by the general appellation of granite, owing to the great resemblance of their external characters, and of their physical properties.

Granite and gneiss differ in fact, rather in the aggregation of their constituent elements, than in any other essential particular. In the former the aggregation of the particles is mostly homogeneous, giving the stone a uniform appearance and the property of splitting readily in all directions;—whereas, in gneiss, the particles are disposed in layers, which give the stone a laminated appearance, and cause it to yield more readily to the chisel and wedge in the direction of the layers, than in any other.

The constituent elements of these stones are *Quartz*, *Feld-spar*, and *Mica*. These elements are easily distinguishable on an examination of a sample of the stone. The quartz presents a transparent or semi-transparent appearance, somewhat approaching to that of glass of a milky hue; the feldspar is usually either whitish or reddish, presenting more of an opaque appearance than the quartz, and where the surface has been for some time exposed to the action of the weather, having a dull white character; the mica is found in scales, of greater or less size, of a dark color, when seen in the mass, but transparent and resembling the small scales of a fish, when detached from the block. When uniformly distributed throughout the mass, these constituents give the stone a uniform color, generally some shade of gray; but occasionally a slight reddish hue, owing mostly to the color of the feldspar.

The quality of the stone depends on the aggregation of the particles, their size, and the proportion of each. The best is usually that in which the particles are fine, and uniformly disseminated throughout the mass.

If the quartz predominates, and particularly if its grains are large, the stone will be hard and brittle, and, therefore, will present great difficulties to being wrought and dressed to a uniform surface. The feldspar decomposes when exposed to the atmosphere for a long period, and, if in excess, will be injurious to the quality. The mica is also subject to decomposition, and, when in excess, gives the stone a character of weakness, which causes it to be known by the appellation of *tender* or *soft* granite.

Besides their peculiar elements, foreign minerals are nearly always to be met with in these two stones. The most deleterious are schorl and iron and its ores, particularly the sulphurets. The iron becoming oxidized, when the stone is exposed to the air, destroys it very rapidly, particularly if in large quantities. The sulphurets, by decomposition, yield sulphuric acid, which soon destroys the texture of the stone,

by acting on the mica and feldspar. And schorl, when in abundance, gives the stone a character of great brittleness, which, in some cases, renders it entirely unfit for a building material.

Sienite, though frequently mistaken for granite, is composed principally of particles of *hornblende* and feldspar. The general appearance of the mass is either gray or of a reddish tint. The particles of the hornblende are readily distinguishable, by their greenish tint, when the stone is moistened. Quartz and mica are likewise generally found in sienite, and give it more the appearance of granite.

The structure of this stone usually resembles that of granite; but it is sometimes found in layers.

Granite, gneiss, and sienite, for strength, hardness and durability, occupy the first rank as building materials.—Neither of them resist very high temperatures, although gneiss, when the mica in it is very abundant, has, in some cases, been used with success, as a facing for fire-places and furnaces, subjected to a strong heat. Granite and sienite are the most suitable for the purposes of cut or dressed stone, particularly in cases where great solidity is indispensable, owing to the large blocks in which they can be procured from the quarry, and the perfect accuracy with which the surfaces can be wrought. Gneiss seldom splits evenly, and is, therefore, more suitable for rubble and hammered stone. It is also an excellent material for flagging stone, for which it is very extensively used in many of our cities. All three of these stones are in very common use with us, for structures requiring great solidity and permanency; as the revetment walls of fortifications, quay-walls, sea-walls, light-houses, &c.

There are very extensive quarries of granite, gneiss, and sienite, in the United States—lying either upon, or contiguous to, several of our principal rivers, as those on the Potomac, the Hudson, and the Thames. The quarries at Chelmsford and Quincy, Massachusetts, which afford sienite chiefly, have furnished the materials for the principal public works on the

seaboard. There are, also, extensive quarries of the best samples, in Maine and New-Hampshire.

Sand Stone. This stone, of which there are two principal varieties, the *red* and the *gray*, is generally known to builders, under the name of *free-stone.* It is composed of small particles of quartz, united by an argillaceous or calcareous cement.

Both varieties are very extensively used in building.—They are generally strong and durable, and though they yield readily to the chisel, and other tools, are sufficiently hard to resist the wear and tear to which any part of an edifice is ordinarily exposed.

Sand stone is frequently so porous, as to absorb a large quantity of moisture, which, when acted upon by the frost, causes the surface of the stone to disintegrate, or to split off in scales. The gray sand stone is more liable to this defect than the red, and requires a thin coating of mortar, paint, or a white-wash of hydraulic lime, to protect it from the action of the atmosphere.

Sand stone, of both varieties, has been used with us in the construction of our public works: in some cases, as the principal material, but, mostly, for the cut stone of the angles, for the coping, for the water tables, &c. Its inferiority to granite, and its liability to disintegrate, render it more suitable to ordinary structures; and its use is now mostly confined to edifices built principally of brick, or of rubble work. It should, moreover, only be used as ashlar or cut stone, because it adheres very badly to mortar, and is, therefore, not suitable for rubble work, the principal strength of which depends on the adhesion between the stones and mortar.

There are many large quarries of both varieties of this stone in the United States. An extensive bed of it, from fifteen to twenty miles broad, and nearly four hundred miles long, stretches from the Connecticut river to the Rappahannock. It is found, also, abundantly in the Eastern States.

All the stones belonging to the siliceous class, of which

there is a great variety, are eminently suitable for the purposes of building, either as cut stone or rubble. Among those less in use than those already described, are the *Buhr* or *Mill-stone*, which, from its great hardness, durability, and porosity, forms an excellent rubble stone; the *Soap-stone*, which is principally used as a fire-stone, for the facing of fire-places and furnaces, and *Mica Slate*, which is also a good fire stone, and forms a good material, both for rubble work, and for flagging.

ARGILLACEOUS STONES.

Nearly all the stones known to builders as *Slate Stone*, belong to this class. The most remarkable varieties, are those denominated the *Trap Rocks* by mineralogists, which consist either of *Basalt*, or *Green Stone*. Basalt, it is said, does not occur as a distinct stone in the United States. It is very remarkable for its great strength and hardness, though it is less durable than many varieties of the siliceous class.

Green stone, so called from the greenish tinge it exhibits when wet, is found very abundantly with us. One of the most remarkable localities is on the Hudson river; that part of its banks known as the Palisade Rocks, being composed almost entirely of it.

Green stone is a good building material, when it does not contain any large quantity of iron, as this metal, by becoming oxidized, very soon entirely destroys the texture of the stone, causing it to break up into small fragments or scales. It is only suitable for rubble work, owing to its being found chiefly in small tabular prismatic masses. From the facility with which it is quarried, and its unchangableness in salt water, it has been used with us for break-water stone.

Gray Wacke, and *Gray Wacke Slate*, properly belong to the sand stones. They are composed of the fragments of several other minerals in a granular state, united by an argillaceous cement. Both of these stones make a good building

material for rubble work; the gray wacke slate is in very common use as a flagging and coping stone. They are found, abundantly, in many parts of our country. Several quarries are worked in Connecticut, Massachusetts, and New-York, from which an excellent flagging and coping stone is obtained.

Common Roof Slate requires no particular description. There are many varieties of this stone which are very suitable for rubble work. The best for roof covering, is that which splits into thin even layers, is free from the ores of iron, particularly the sulphurets, which are most deleterious to it, and absorbs but little moisture.

Good roof slate is quarried extensively in many parts of the United States, chiefly in Maine, at Hoosack, in New-York, and nearer the Susquehanna river, in the counties of York and Lancaster, Pennsylvania.

CALCAREOUS STONES.

This very abundant class, composed of innumerable varieties, is the most useful building material known to the engineer and architect, both for common and ornamental purposes; arising from the strength, hardness, durability, and beauty both of color and polish, which it is known to possess. It also furnishes the principal ingredient in the composition of every variety of cement used for uniting stones artificially.

Calcareous stones, distinguished by the more common appellation of *Lime Stone* and *Marble*, are composed principally of *Carbonate of Lime* combined with the metallic oxides, and several other foreign minerals. They seldom occur in a pure state; when found so, the color of the stone is a pure white, and it is shown, by analysis, to be composed of lime, carbonic acid, and a small quantity of water.

The general properties of this class of stone, both physical and chemical, are so well known, as hardly to require any description. Its effervescence with acids, and the effects of

heat on it, both of which disengage the carbonic acid, are facts, with which almost every person must be acquainted.

Mineralogists distinguished two general divisions of this class: 1. *Granular* Lime Stone. 2. *Compact* Lime Stone.

The granular presents the distinct appearance of an aggregation of grains of variable size, from very fine to coarse, apparently the result of an irregular crystallization. The compact has a fine uniform texture, without any appearance of grains, some of the varieties being quite loose and earthy in their texture.

As a building material, the calcareous stones are classed in two divisions: 1. The Common Lime Stone; 2. The Marbles.

Each of these divisions furnish an equally good stone for building, but the marbles are mostly reserved for ornamental purposes, owing to the fine polish which the stones, from which they are procured, are susceptible of receiving. The term marble, is frequently applied by builders to all stones which receive a high polish, and it is the proper signification of the term; but it is now usually applied only to those varieties of lime stone which are polished. The compact lime stone furnishes a great variety of *variegated* marbles, but, generally, they are not so highly estimated as those furnished by the granular, owing to their inferiority in hardness and polish.

The colors of variegated marble, are owing to the metallic oxides, and the names of the different varieties are taken from some peculiarity, either in the appearance or color of the stone. Some of the best known are the *Veined;* the *Bird's-eye;* the *Conglomerate*, of which there are two kinds; the *Breccia*, composed of broken angular fragments united by a natural calcareous cement, and the *Pudding Stone*, composed of round pebbles similarly united; the *Lumachella*, which exhibits a variety of shells united by a calcareous cement; the *Florence Stone* or *Ruin Marble*, so called from some fancied resemblance to ruins, exhibited by the figures on the polished surface; the *Verd Antique*, which is of a green color, and is

named after a stone much esteemed by the Ancients, &c. &c.

The localities of lime stone in our country are numerous and extensive; the quarries furnishing both an ordinary building stone, and variegated marbles, which are, in every respect, equal to any in the world. Most of the quarries lie in a range of granular lime stone, which is about seven hundred miles long, and varies from two to twenty miles in breadth, stretching from N. E. to S. W., nearly parallel to the seacoast.

A beautiful breccia is obtained on the Potomac; the verd antique comes from near New-Haven, in Connecticut; and every variety of variegated marble is found in the various quarries of compact lime stone.

Quick lime, both for ordinary building and for hydraulic constructions, is made in large quantities at most of these localities, from which it is sent to every part of the Union.— The most celebrated kilns are at Thomaston, in Maine, which furnish common lime; and those in New-York, which furnish a hydraulic lime, known as *Rosendale cement.*

Plaster of Paris, or *Gypsum*, is a Sulphate of Lime; its principal use is for stucco work in the interior of edifices, for which purpose it is prepared by calcination, over a strong fire. It is too soft for a building stone, and although forming, when properly prepared, a cement which hardens very rapidly, it is not suitable for the purposes of mortar, because, having a strong affinity for water, it absorbs it from the atmosphere, and increases so much in volume, as to occasion cracks in walls built with it; and, in exposed situations, it very soon commences to exfoliate. This stone is found abundantly in our country, principally near salt springs.

General Observations on the properties of Stone, as a Building Material.

Strength, hardness, and durability, are the essential points to be considered in stone for building. Stone which has one of these properties, in most cases, possesses also the other two.

It is not always easy to judge of the quality of a building stone from its external appearances, but it has in general been found, that when the texture is uniform and compact, the grain fine, the color dark, and the specific gravity great, the stone will possess all the essential qualities.

There are also certain defects that can be ascertained by the eye, which should cause stone to be rejected as a building material, though belonging to a good class; these are cracks, cavities, and foreign minerals, particularly the various forms under which iron is found. The effect of these is to render the stone weak, brittle, and liable to disintegration, from the decomposition of the metallic ores. Great hardness is likewise objectionable, when the stone is to be prepared by the chisel, owing to the labor required to work it; and as the stones of this character generally wear smooth, and become polished by attrition, they are unsuitable for stairs, pavements, &c., where accidents might happen from slipping. Brittleness is a defect which frequently accompanies hardness, particularly in coarse grained stones; it prevents the stone from being wrought to a true surface, and from receiving a smooth edge at the angles. The coarse grained stones are, moreover, more liable to rapid disintegration than those of a fine texture.

Experiments on an extensive scale have been made in France and England, to ascertain the comparative and absolute strength of the most common building stones.—Small cubical blocks, the sides of which measured about four superficial inches, were selected for the experiments, and the following general results were obtained:

1st. The strongest and hardest stones were found to be

those having a dark color, a compact uniform texture, and the greatest specific gravity. These last qualities generally accompanied each other, and the strength and hardness increased in proportion as they predominated, without, however, exhibiting any uniform law of variation.

2d. That, for the same stone, the strength varied nearly in the proportion of the area of the base. For equal bases, the circle gave the most favorable results for strength, and after it those polygons, which most nearly approximated to it. For the same volumes, those were strongest, in which the altitudes were equal to the diameters of the bases, and the strength decreased, either as the altitude or the area of the base was increased—the volume remaining the same.

3d. In the blocks submitted to a compressive force, it was observed, that a slight yielding took place under a weight equal to about one half of that which was necessary to crush the stone; and when rupture ensued, the stones of a crystalline texture generally split into needle form fragments, parallel to the direction of the force; whilst the amorphous stones broke into small pyramidal fragments, having a common vertex near the centre of the block.

4th. From a comparison of the weight borne by the stone in some of the boldest structures in Europe, with the results of these experiments, it appears that it would not be safe to submit any stone to a permanent strain, which would be greater than one-tenth of the weight required to crush a small block of it, of the size of those used for the experiments.

5th. That the following order of strength and hardness, was observed in the more common building stones—Basalt, Granite, Lime Stone, and Sand Stone.

The terms hard and soft, as applied to stones, are not very definite. Workmen term those stones hard, which can be sawed into slabs only by the agency of the grit saw; and soft, those which can be separated by a common saw.

A knowledge of these qualities is essential to the engineer, to enable him to fix a price on the preparation of the mate-

rial; and, also, to select such as are most suitable to resist attrition and shocks.

The durability of stone is best ascertained, by examining samples in edifices which have stood for a long period. When this cannot be done, the stone must be submitted to direct experiments. The best for ascertaining the effects of the atmosphere, is to expose a sample during two years, in some very exposed position. With regard to the ordinary chemical agents, as fire and sea water, direct experiments must likewise be made.

Frost is one of the most powerful destructive agents in its action on stone; its effects are seen mostly in very porous stones, and in those of a slaty or fibrous texture;—in the former it causes a disintegration, or exfoliation at the surface:—in the latter, the block splits into several fragments. The action of frost may be ascertained, by placing a sample of the stone in a saturated solution of some crystallizing salt, and after it has remained long enough to have thoroughly imbibed the liquid, by allowing it to dry, the action on the stone of the crystals of the salt, whilst in the act of forming, will be similar to that of water, whilst freezing.

The durability of stone, quarried from cliffs or ledges, may be more readily ascertained than that taken from underground quarries, owing to the certain indications which the exposed position of the cliffs must present.

Within the ordinary ranges of temperature, stone is too slightly affected by contraction or expansion, to cause any perceptible changes in a large mass of masonry; but it has been found, upon careful observations made with us,* that the contraction and expansion of the blocks of a long line of coping are sufficiently great to crush mortar between the blocks which is soon washed out of the joints, to the great detriment of the masonry beneath the coping.

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* The experiments were made by Lieutenant Bartlett, of the Engineer Corps, (now Professor of Natural and Experimental Philosophy, at the Military Academy,) on the works at Fort Adams, Newport, R. I.

ARTIFICIAL STONES AND CEMENT.

The term *Artificial Stone*, is applied to any composition to which, by an artificial process, the general properties of natural stone are imparted; such, for example, are mortar and brick, both of which, when properly prepared, possess, in an eminent degree, the qualities of good stone.

The term *Cement*, is applied to certain mineral substances, found either in a natural state, or else prepared artificially, which being mixed with common lime, impart to it the property of hardening under water.

The ingredients that usually enter into the composition of mortar are *Slaked Lime* and *Sand*, to which sufficient water is added, to bring the mixture to a proper consistence or *temper* before using it for the purposes to which it is to be applied; when the mortar is to be used for hydraulic works, a certain proportion of cement is to be added to the other ingredients.

Lime. When lime stone is submitted to a high temperature for some length of time, the water, and nearly all the carbonic acid which enter into its composition, are driven off, and the result obtained, is known by the name of *Quick Lime*. The stone in this new state, shows a strong avidity for water, which it absorbs even from the atmosphere; and when water is poured over the stone, it swells and cracks, evolving a very considerable degree of heat, and finally, falls into a fine white powder, in which state it is denominated *Slaked Lime*, and belongs to that class of chemical substances, denominated hydrates.

If the lime stone is a perfectly pure carbonate, it will absorb about three and a half times its bulk in the process of slaking, and the hydrate will be found increased about the same quantity.

Owing to the great consumption of lime, its preparation has become a distinct branch of the Useful Arts, and the material is a common commercial article. But as the engineer is sometimes thrown into situations, where it may

be necessary to prepare the material himself, the following outline of the process may prove of some service:

The *Lime Kilns*, or furnaces, in which the stone is calcined, are either of a spheroidal form, (Figs. 1 and 2,) or that of an inverted truncated cone. The former is most suitable for a wood fire; the latter, when the combustible used is either pit or charcoal. The interior of the kiln should be lined with some material, which, like soap stone or fire brick, is capable of resisting a very powerful heat; and a position should be chosen for the kiln, where it will not be exposed to the wind; and a shed roof should, moreover, be placed over the mouth, to shelter it from rain.

If the stone has been newly quarried, it will, generally, be moist enough to facilitate the escape of the carbonic acid; otherwise, water should be poured over it. The stone is broken up into lumps of about half a cubic foot, when the fire is of wood, but into much smaller pieces, when either pit or charcoal is used. The stone is then placed in the kiln;—and, in the case of pit or charcoal being used, in alternate layers with the fuel; the fire, in both cases, being ignited at the bottom of the kiln.

The burning is commenced, when wood is used, by keeping up a slow fire of uniform temperature, from twelve to forty-eight hours in duration. This gentle heat drives off the moisture from the stone, which appears on the surface like a perspiration, when the stone is said to sweat; and, when it has been nearly driven off, which, in most cases, will be in about ten or twelve hours, the stone will begin to blacken from the smoke; at this stage, the heat should be slightly increased, and be kept at a uniform degree, until the smoke is consumed, when it should be raised to its greatest intensity, and be kept so until the calcination is complete, which may be ascertained by the following indications:—The color of the flame above the mouth of the kiln, will appear either of a pale yellow, or white color, being indistinctly visible; the stone in the kiln will have decreased

about one sixth of its original bulk, and the whole mass will present either a glowing red heat, or a whitish rosy hue.

The management of the fire is a subject, in all cases, for careful experiment, as its effects are very different, varying with the quality of the stone. The lime from the pure carbonates, is never injured by the most powerful heat of ordinary kilns; but the impure carbonates may be rendered almost useless, if over-burned. The effects of intense heat on the latter, being to cause the stone to slake very slowly, and to fall into small lumps, instead of giving a fine impalpable powder; and, if it be used when it is only partially slaked, the mortar after a series of years, will be found to have increased considerably in bulk; disfiguring the surface of walls, if it has been employed for plastering, or injuring the masonry by causing cracks, if used for stone work—all of which effects arise from the gradual slaking of the small lumps, by slowly absorbing moisture from the atmosphere.

As a building material, lime is divided by engineers into two classes: 1. *Common Lime*; 2. *Hydraulic* or *Water Lime*. Common lime is, also, sometimes termed *Fat* Lime, from the appearance and feeling of the paste made from it with water, whilst hydraulic lime, with the same quantity of water, yielding a thin paste, is denominated *Meagre* Lime. This difference of appearance in the paste of the two, it must, however, be observed, does not sèrve in all cases to distinguish the two classes: for some varieties are very meagre, without possessing the slightest hydraulic properties.

The distinction between the two classes consists in the uses to which they are applicable. The mortar of common lime will never harden under water, or in very moist places, as the foundations of edifices, or the interior of very thick walls, and, therefore, is only suitable for dry positions and thin walls; whereas, hydraulic lime yields a mortar which sets readily, and soon becomes nearly as hard as stone in all moist situations.

To ascertain the properties of a lime stone, direct experi-

ment should always be resorted to; for the external appearance of the stone does not present any indications which can be relied on with certainty. The simplest method consists in calcining a small portion of the stone to be tried, over a common fire on a plate of iron, slaking it and kneading it into a thick paste; this paste being placed at the bottom of a glass vessel, and carefully covered with water, will, in a short time, give evidence of the quality of the stone. If, after several days, it is found not to have set, the stone may be pronounced as affording common lime; but if it has become firmer or hard, it may be safely classed among the hydraulic varieties, and its excellence will be shown by the quickness with which it hardens.

It is only within a few years back, that scientific men have come to any certain conclusions, as regards the causes of this peculiar property of hydraulic lime. For a long time, it was ascribed to the presence of metallic oxides; then, to the manner of slaking the lime and mixing the ingredients of mortar; but careful analysis and experience have finally settled the question; and it is now fully ascertained, that this property is owing to the presence of *Argile* or *common Clay* in the stone, which, after the stone is calcined, forms a compound possessing this highly important quality. It still, however, remains to be determined, whether the presence of both the elements of argile, which are *Silex* and *Alumine*, is necessary to impart this property. Alumine alone, it is known, does not; and an hydraulic lime has been found in France, in which it is stated that silex is alone present.

Whatever may be the solidifying principle, a most important fact to engineers is put beyond a doubt, that an artificial hydraulic lime can be made equal in quality to the best natural varieties, by mixing common lime in a slaked state, with any mineral substance, of which argile is the predominant constituent, by simply exposing it to a suitable degree of heat, and afterwards converting it into a fine powder, before mixing it with the lime.

Sand. The term sand, is applied to any mineral substance in a granular state, where the grain is of an appreciable size and insoluble in water.

Sand is classified, either from its principal constituent element, as *Siliceous Sand*, *Argillaceous*, &c., or from the size of the grain, as *Coarse*, *Fine*, and *Middling Sand;* the latter classification being chiefly in use among builders.

As this material is either procured from pits, or from the shores of rivers or the sea, it is denominated also *Pit Sand* or *River Sand*, from the locality where it is obtained.

Pit sand is superior to river sand for all purposes where mortar is required to possess great strength. Its grain is more angular and porous than that of river sand ; the latter becoming smooth and polished from the constant attrition between the grains, this roughness gives the lime a better hold on the grains ; besides, it is generally freer from the impurities, as salts, &c., which are found in the sand taken from the shores of the sea and tide water rivers.

River sand, owing to its superior whiteness, and the small size of its grain, is well suited for plastering for the interior of edifices.

Fine pit sand should not leave any stain on the fingers, when rubbed between them. If it is found to contain earthy impurities, or salts, it must be passed through several waters in flat vats ; the water being renewed and poured off, until it no longer appears turbid.

The siliceous class is superior to every other, owing to the great strength and hardness of its grains.

Cements. These substances are obtained sometimes in a natural state, or they can be prepared artificially, by calcining pure argile, and most of the argillaceous stones. Their uses, as has been already stated, are to form an artificial hydraulic lime, by mixing them with common lime.

All the substances which serve as cements, contain nearly the same constituent elements, in about the same proportions. They are argile, in which the alumine varies between one

fourth and one half of the silex, with a small portion of the oxides of iron and manganese, and the carbonate of lime, potash, and soda.

The argile, or clay, constitutes the essential ingredient of cements. The metallic oxides seem to play a neutral part when not in excess; the oxide of iron, for example, when very abundant in clay, it appears, from some experiments, acts injuriously on the quality of the cement. The carbonates of soda and potash, and the muriate of soda, or common salt, produce, it is said, a favorable effect, when the heat is by accident carried beyond the degree for suitable calcination. The other foreign ingredients found in clay, as the carbonate of magnesia, &c., do not appear to affect the quality of the cement.

Puzzolano and *Trass*, or *Terras*, are the most celebrated natural cements. They are both of volcanic origin, the former being found in a pulverulent state, near Mount Vesuvius; the latter near Andernach on the Rhine, where it occurs in fragments, and is ground fine and exported for hydraulic works.

The constituent elements of both these natural products, are nearly the same, and as follows, in one hundred parts:

Silex, ...	55 to 60 parts.
Alumina, ...	20 " 19 "
Iron, ...	20 " 15 "
Lime, ...	5 " 6 "

Common potter's clay, such as is suitable for making pottery and tiles, furnishes the best artificial cement, when properly calcined. The best proportions of the constituent elements, are when the alumina is nearly one third of the silex, with an addition of about five hundredths of lime. With these proportions, the clay requires to be brought to a state of calcination somewhat inferior to that of brick, denominated cherry red. When the lime is in greater quantity than this, the calcination must not be carried so far, or the cement will be injured; but when the clay is entirely free from lime, it

must be submitted to a high temperature, but not so great as to produce vitrifaction.

If a current of air be passed over the clay whilst calcining, (Fig. 3,) it is found that the mortar made of the cement, hardens much sooner, and is, moreover, stronger than that made of clay burned in a close kiln.

As the quality of the cement, essentially depends on the degree of calcination, and as this must be mainly regulated by the quantity of lime that enters into the composition of the clay, it is very important to ascertain, in the first place, this element, before burning the clay. This may be done by taking a small quantity of the clay, which should be thoroughly dried by a gentle heat and be then converted into a fine powder, and carefully weighed. This powder should then be formed into a very thin paste with water, and muriatic acid be added to this paste as long as any effervescence takes place ; so soon as this ceases, the whole should be filtered, and the residue left on the filtering paper, be carefully washed with distilled water, so as to free it entirely of the muriates that may have been formed ; it is then carefully dried by a gentle heat and weighed : the loss will be nearly the quantity of lime contained in the clay.

As this analysis cannot always be practised, direct experiment should be resorted to in its stead. Three samples of the clay to be tried, should be submitted to a temperature that will convert one of the samples into pale brick, the second into cherry brick, and the third into hard brick, without, however, producing vitrifaction. A mortar is then made from the cement of each of these samples, by taking equal parts of common lime, sand, and the cement. These three specimens of mortar are then placed under water, and the quality of the cement is inferred from the promptness of induration, and the strength of the specimens. If there are brick yards where the clay to be tried is used for making brick, the samples for trial may be taken from the kilns.

Mortar. This material is divided, like lime, into two

classes, *Common Mortar*, and *Hydraulic Mortar*, according to the uses to which it is to be applied.

Besides these divisions, there is *Grout*, which is a very thin tempered mortar, used in a liquid state, and *Beton*, *Concrete* or *Grub-stone Mortar*, formed by mixing up small chippings of stone with hydraulic mortar.

Before mixing the lime with the other ingredients in mortar, it should be carefully and thoroughly slaked. Previous to the knowledge now possessed with respect to mortar, great importance was attached to the methods of slaking the lime, but the inefficacy of any of these methods for the improvement of mortar, has been long established, and engineers at present pay no regard to them; confining their attention to effecting a thorough slaking, most suitable to the different varieties of lime.

As hydraulic lime would harden, if made into a paste, and suffered to stand any length of time before using it, no more water should be used in slaking it, than is just sufficient to convert it into a fine dry powder; and from about one fourth to one third the bulk of the lime is sufficient for this purpose, if carefully poured over.

Common lime may be slaked with any quantity of water without injury; and it is a common practice for builders to slake a large quantity at a time, and allow it to stand in a state of paste until wanted for use; but this method is said to yield a mortar inferior to that made from lime recently slaked, or from lime, which is simply converted into a dry powder, either by the common process, or by spontaneous slaking, from absorbing moisture from the atmosphere. And lime, which is allowed to stand over some months, yields a superior mortar to that recently slaked.

With regard to hydraulic lime, it is stated that, it should never be allowed to remain longer than two or three days in a slaked state, before using it; and even in that case, it should be carefully covered over with sand. Lime, which had been allowed to stand for some time, lost most of its

hydraulic properties, which was supposed to be owing to its absorbing oxygen from the atmosphere, as a considerable quantity of this gaz was found in it.

This fact is of great importance to the engineer, for many of the varieties of hydraulic lime, commonly termed cements, which are ordinary articles of commerce, may prove, upon examination, to have lost their original properties by remaining a long time slaked, before being used for mortar.

The best proportions of the ingredients for mortar, can be ascertained alone by direct experiment, as some varieties of lime will require more sand than others. The practice of builders in regulating these proportions, is usually bad, as they add too much sand, which gives a weak mortar, which generally adheres but slightly to the stone, and is apt to become pulverulent. Regarding the lime, simply as a cement, uniting the particles of the sand, the proper proportions of the two would appear to be those, in which the lime is sufficient to fill up the voids between the particles of sand; which quantity may be readily ascertained by filling any measure, first with the sand, and then pouring into it as much water, as will fill up the voids; by which means, the bulk of lime, required for the same purpose, will be known. Mortar made with the proportions thus determined, has been found to give very satisfactory results.

From a great number of experiments very carefully made with common lime and sand, it appears, that two parts of sand and one of lime, with sufficient water to convert the whole into a ductile paste, form the best mortar; and that when a greater disproportion, than this, exists between the lime and sand, the quality of the mortar is inferior, and becomes so much the more so, as the disproportion is greater. The proportions, however, in most common use with builders, ers, from four to six parts of sand to one of lime, and about two thirds of a part of water. As a general rule, it is found that the quality of the mortar is not affected by the quantity of water used in tempering it, unless it is brought to a tem-

per so soft, that the particles of sand sink to the bottom of the mass. Soft tempered mortar has the advantage over hard tempered, of being more plastic under the trowel, and of not drying so rapidly as to impair the strength of the mortar, after it has set, which frequently takes place in the hard tempered in warm weather, if the precaution be not taken, of keeping the stone or brick in a moist state some time before laying it.

Hydraulic mortar, made of natural hydraulic lime, will take about two parts and a half of sand. And when an artificial hydraulic mortar is made, the common lime, sand, and cement, should be in equal proportions.

It is not usual to add a cement to mortar made of lime naturally hydraulic, unless it is so only in an inferior degree, but experiments have shown, that a certain dose of cement improves the quality of the mortar, made of the best hydraulic lime, the quantity of the cement being smaller, as the quality of the lime is stronger; and, in all cases, when a cement is added, it has been found that the mortar will require a smaller dose of lime, than when sand and lime are alone used.

Mortar, made of fine sand, is superior in quality to that made from any other, and is, moreover, the only kind suitable for cut stone masonry, or any kind of work, where close joints, and accurate finishing, are indispensable. For rubble work, a mixture of coarse and middling sand, will answer all the purposes of good workmanship, and is, besides, more economical than mortar of fine sand.

In mixing the ingredients, the main point to be observed, is to obtain as homogeneous a mass as possible; any labor bestowed beyond this point, will be thrown away. To effect this in the most thorough manner, the lime and cement must be in a state of the most minute division, before they are mixed together, and the sand added. If the lime stone has been properly calcined, it will be converted by the ordinary process of slaking into an impalpable powder, but most gene-

rally, either from being over, or under calcined, it breaks up simply into very small lumps when water is poured over it, and will remain in this state a long time before the slaking is thoroughly effected. Whenever the lime is found in this state, it requires to be ground down. in a mortar mill, before the other ingredients are added to it. The mill used for this purpose, (Fig. 4,) is ordinarily a large circular trough, from twenty to thirty feet in diameter, from out to out; the section of the trough is generally trapezoidal, being about eighteen inches wide at top, and about twelve inches in depth; the sides of the trough are built of hard brick or stone, which is laid on a very solid foundation. The lime and cement are ground down by the action of a heavy wheel of wood, from six to eight feet in diameter, or by a large stone roller of the form of a frustrum of a cone; the width of each being about ten inches, to allow them to play freely in the trough when set in motion, which is effected either by horse or some other power. Sufficient water should be added to the lime and cement, to bring them to the consistence of a very thin paste. Sand is usually added until the mortar will not adhere to the sides of the wheel or roller. A mill of a different construction is used to mix the ingredients when the lime and cement do not require to be ground or have been ground by themselves, without any addition of sand, in the mill just described. This mill (Fig. 5) consists of a large firkin, similar in shape, to a churn, about six feet high, and three or four in width at top. A vertical axle, into which several horizontal arms, fashioned like a common rake, are set, is so arranged, that it can be easily put in motion by a horse. The ingredients are slightly mixed in a wet state, and, being thrown into the mill, are thoroughly mixed by the action of the arms of the axle. An opening is left in the side of the mill, near the bottom, through which the mortar flows out into a trough or some other receptacle; from which it is taken, and is worked over with a hoe, with the addition of sufficient water to bring it to a soft temper.

In preparing beton, the followiug proportions have been found to succeed perfectly in some recent structures.

Hydraulic Lime (unslaked,) - - - -	0,30	parts.
Sand (middling,) - - - - - -	0,30	"
Cement (common clay,) - - - -	0,30	"
Gravel (coarse,) . - - - - -	0,20	"
Chippings of stone, - - - - -	0,40	"

The lime, sand, and cement, are, in the first place, thoroughly worked up into a homogeneous mass of a hard temper; this mass is suffered to rest in a heap about twelve hours, it is then spread out into a layer about six inches thick, and the gravel and stone are evenly spread over it, and the whole well mixed up. The mass before it is used is suffered to remain until it has partially set, which will require from twelve to thirty-six hours, according to the quality of the mortar. This delay is found to improve the quality of the beton.

This material depends on the quality of the mortar for its excellence. It is not stronger than simple hydraulic mortar, but it is far more economical. The gravel, which enters into its composition, is used to fill up the voids between the fragments of stone, which would, otherwise, be filled by the mortar alone.

Broken brick may replace the fragments of stone, when the latter cannot be had; or gravel alone may be used.

There is no subject, connected with the art of the engineer, upon which more ingenuity has been uselessly expended, than upon that of mortar. Misled by erroneous, or forced interpretations of some passages of the Ancients, particularly of Vitruvius, various hypotheses have been formed, to explain the superior properties of the mortar found in the remains of ancient edifices, over that of a modern date; and almost universal failure, for a long period, attended all the experiments made in conformity with these hypotheses, as they were not conducted according to the only sure method of investigation in such cases, a careful analysis. The fal-

lacy, both of the hypotheses adopted, and the results obtained, led scientific engineers to treat the subject in a more rational manner, and with a success, which has fully repaid the care bestowed on it. The true nature both of lime and mortar, thanks to the labors of Vicat, Raucourt and Treussart, men who stand at the head of the professions of civil and military engineers in France, is now perfectly understood, and the results, owing to the light that they have thrown on the subject, may, with certainty, be predicted. It is now placed beyond a doubt, that neither the methods followed in slaking the lime, mixing the ingredients, nor the age, are the causes of the great strength and hardness of some kinds of mortar, although they doubtless exercise some influence; but that these qualities are attributable, almost solely, to the nature of the lime. That, with common lime and sand, a mortar is obtained, which is suitable only for dry exposures; and that no age, nor preparation, will cause it to harden in moist situations, such as foundations, the interior of heavy walls, and constructions under water. That there are natural varieties of lime stone, which possess this peculiar property of hardening under water, and in moist situations, and are, therefore, alone suitable for hydraulic mortar, and that wherever this natural hydraulic lime cannot be procured, an artificial mortar can be prepared, fully equal to that made of the natural lime, by adding some natural or artificial cement to common lime and sand.

With regard to the action of the lime on the sand, the most careful analysis, thus far, has not been able to detect any appearance of a chemical combination between the two; and it is the received opinion, that the union between them is simply of a mechanical character;—the lime entering the pores of the sand, and thus connecting the particles much in the same way, as the particles of granular stones are connected by a natural cement. The sand itself serves the important purposes of causing the mass to shrink uniformly, whilst the hardening or *setting* of the mortar is still in progress, and

thus prevents any cracking, which must always be the result of irregularity in the shrinking; it promotes the rapid dessication of the mass, and is conducive both to solidity and economy, from its superior strength, hardness, and cheapness, to lime.

No perfectly satisfactory solution has yet been given for the hardening of either common or hydraulic mortar. That the former acquires strength and hardness with age, experience has very conclusively shown; and it was, for some time, supposed that this arose from a gradual conversion of the lime into a carbonate, by the slow absorption of carbonic gas from the air; but, from experiments conducted with great care, it seems that only a very thin coating on the surface undergoes this change, and that no more gas can be detected in the interior of the mass, than is usually retained by lime, which has been submitted to the greatest heat of ordinary kilns. As to the action which takes place in hydraulic lime, it is accounted for on the supposition, that a chemical combination takes place between the lime and argile, when mixed in a moist state, being a compound formed with new properties, distinct from those of the constituent elements; this combination requiring a longer or shorter time to become complete, depending on the energy of the ingredients.

From the above views of the nature of mortar, it appears that its good qualities, as a building material, essentially depend, 1, on the kind of lime; 2, on the strength and hardness of the sand; 3, on the adhesion between the lime and sand, which will depend on the roughness and porosity of the particles of sand, and the care bestowed in thoroughly incorporating the ingredients.

The experiments, made on the strength of mortar, have led to no satisfactory conclusions, except so far as to institute a comparison of the effects produced by using various proportions of the same or different ingredients, and from the more or less care taken in mixing them. As to the absolute strength, no definite results, of course, can be arrived at,

owing to the variable proportions of the ingredients, until some greater uniformity shall be adopted in the practice of engineers generally, for determining the proportions, and the method of mixing them.

The most interesting experiments on the strength of mortar, are those detailed by General Treussart in his work on this subject. He chose for his experiments small rectangular parallellopipeds, six inches long, and squaring two inches by two inches. These were placed on props at each end, leaving a bearing of four inches between the props. A common weighing scale was attached by a hook, or stirup, to the middle point of the parallellopiped, and weights added until it broke. He found that mortar, formed of equal parts of common lime, sand, and cement, bore in this way, before rupture took place, from 220 to 440 pounds; and he recommends, that for heavy masonry exposed to the air, the mortar used should bear from 220 to 330 pounds, when submitted to this test, and that for hydraulic works, from 330 to 440 pounds. With regard to mortar of common lime and sand, its strength was found, by his experiments, to be very inferior to that in which cement entered. The best samples were those made with one part of lime and two parts of sand; some of these bore at the moment of rupture, from 60 to 100 pounds.*

Brick. This material is properly an artificial stone, formed by submitting common clay, which has undergone suitable preparation, to a temperature sufficient to convert it into a semi-vitrified state.

Brick may be used for nearly all the purposes to which stone is applicable, for when carefully made, its strength, hardness, and durability, are but little inferior to the more ordinary kinds of building stone. It remains unchanged under the extremes of temperature; resists the action of water; sets

* Experiments on this very interesting subject to engineers have been made by Colonel Totten, of the United States' Corps of Engineers at Fort Adams. So far as they have been prosecuted, they agree with the results given in the preceding remarks.

firmly and promptly with mortar; and being both cheaper and lighter than stone, is preferable to it for many kinds of structures, as arches, the walls of houses, &c.

The art of brick-making is a distinct branch of the useful arts, and does not properly belong to that of the engineer. But as the engineer is frequently obliged to prepare this material himself, the following outline of the process may prove of service.

The best brick earth is composed of a mixture of pure clay and sand, deprived of pebbles of every kind, but particularly of those which contain lime, and pyritous, or other metallic substances; as these substances, when in large quantities, and in the form of pebbles, act as fluxes, and destroy the shape of the brick, and weaken it by causing cavities and cracks; but in small quantities, and equally diffused throughout the earth, they assist the vitrification, and give it a more uniform character.

Good brick earth is frequently found in a natural state, and requires no other preparation for the purposes of the brick-maker. When he is obliged to prepare the earth by mixing the pure clay and sand, direct experiments should, in all cases, be made, to ascertain the proper proportions of the two. If the clay is in excess, the temperature required to semi-vitrify it, will cause it to warp, shrink and crack; and, if there is an excess of sand, complete vitrification will ensue, under similar circumstances.

The quality of the brick depends as much on the care bestowed on its manufacture, as on the quality of the earth. The first stage of the process, is to free the earth from pebbles, which is most effectually done by digging it out early in the autumn, and exposing it in small heaps to the weather during the winter. In the spring, the heaps are carefully riddled, if necessary, and the earth is then in a proper state to be kneaded or tempered. The quantity of water required in tempering, will depend on the quality of the earth; no more should be used, than will be sufficient to make the earth so

plastic, as to admit of its being easily moulded by the workman. About half a cubic foot of water to one of the earth, is, in most cases, a good proportion. If too much water be used, the brick will not only be very slow in drying, but it will, in most cases, crack, owing to the surface becoming completely dry, before the moisture of the interior has had time to escape; the consequence of which will be, that the brick, when burnt, will be either entirely unfit for use, or very weak.

Machinery is now coming into very general use in moulding brick; it is superior to manual labor, not only from the labor saved, but from its yielding a better quality of brick, by giving it great density, which adds to its strength.

Great attention is requisite in drying the brick before it is burned. It should be placed, for this purpose, in a dry exposure, and be sheltered from the direct action of the wind and sun, in order that the moisture may be carried off slowly and uniformly from the entire surface. When this precaution is not taken, the brick will generally crack from the unequal shrinking, arising from one part drying more rapidly than the rest.

The burning and cooling, should be done with equal care. A very moderate fire should be applied under the arches of the kiln for about twenty-four hours, to expel any remaining moisture from the raw brick; this is known to be completely effected, when the smoke from the kiln is no longer black. The fire is then increased until the bricks of the arches attain a white heat; it is then allowed to abate in some degree, in order to prevent complete vitrification, and it is alternately raised and lowered in this way, until the burning is complete; which may be ascertained by examining the bricks at the top of the kiln. The cooling should be slowly effected; otherwise the bricks will not withstand the effects of the weather. It is done by closing the mouths of the arches, and the top and sides of the kiln in the most effectual manner, with moist clay and burnt brick, and allowing the kiln to remain in this state, until the warmth has perfectly subsided.

Brick of a good quality, exhibits a fine, compact, uniform texture when broken across; gives a clear ringing sound, when struck; and is of a cherry red, or brownish color. Three varieties are found in the kiln; those which form the arches, denominated *arch brick*, are always vitrified in part, and present a grayish glassy appearance at one end; they are very hard but brittle, and of inferior strength, and set badly with mortar; those from the interior of the kiln, usually denominated *body*, *hard*, or *cherry-brick*, are of the best quality; those from near the top and sides, are generally under burnt, and are denominated *soft*, *pale*, or *salmon brick*, they have neither sufficient strength, nor durability, for heavy masonry, nor the outside courses of walls, which are exposed to the weather.

The quality of good brick may be improved by soaking it for some days in water, and reburning it. This process increases both the strength and durability, and renders the brick more suitable for hydraulic constructions, as it is found not to imbibe water so readily after having undergone it.

The size and form of bricks present but trifling variations. They are generally rectangular parallellopipeds, from eight to nine inches long, from four to four-and-a half wide, and from two to two-and-a-quarter thick. Thin brick is generally of a better quality than thick, because it can be dried and burned more uniformly.

Brick presents great diversity in its strength, arising, principally, from its greater or less density; the densest made of the same earth, being uniformly the strongest. It was found on experiment, that good brick having the specific gravity of 2,168, required 1200 pounds on a square inch, to crush it.

Fire-brick. This material is used for the facing of furnaces, fire-places, &c., where a very high degree of temperature is to be sustained. It is composed of a very refractory species of clay, that will remain unimpaired by a degree of heat, which would vitrify and completely destroy ordinary brick. A very remarkable brick of this nature, has been

made of Agaric Mineral; it remains unchanged by the highest temperature, is one of the worst conductors of heat known, and is so light, as to float on water.

Tiles. As a roof covering, tiles are, in many cases, superior to slate and metallic coverings, both for strength and durability. They are, therefore, very suitable for the roofing of arches, as their great weight is not so objectionable, as in the case of common roofs of frame work.

Tiles are made of the best potters' clay, and in order to make them both thin and strong, are moulded with great care to give them the greatest density. They are of very variable form and size, the worst being the flat square form, which, owing to the warping of the clay in burning, seldom makes a perfectly water tight covering.

WOOD.

This material holds the next rank to stone, owing to its durability and strength, and the very general use made of it in constructions. To suit it to the purposes of the engineer, the tree is felled after having attained its mature growth, and the trunk, the larger branches that spring from the trunk, and the main parts of the root, are cut into suitable dimensions, and seasoned; in which state, the term *timber* is applied to it. The crooked, or *compass* timber of the branches and roots, is mostly applied to the purposes of ship-building, for the knees and other parts of the frame work of vessels, requiring crooked timber. The trunk furnishes all the straight timber.

The trunk of a full grown tree, presents three distinct parts: the *bark*, which forms the exterior coating, the *sap wood*, which is next to the bark, the *heart*, or inner part, which is easily distinguishable from the sap wood by its greater firmness and darker color.

The heart forms the essential part of the trunk, as a building material. The sap wood possesses but little strength, and is subject to rapid decay, owing to the great quantity of fermentable matter contained in it; and the bark is not only

without strength, but, if suffered to remain on the tree after it is felled, it hastens the decay of the sap wood and heart.

Trees should not be felled for timber until they have attained their mature growth, nor after they exhibit symptoms of decline; otherwise, the timber will be less strong, and far less durable. Most forest trees arrive at maturity between fifty and one hundred years, and commence to decline after one hundred and fifty, or two hundred years. The age of the tree can, in all cases, be ascertained by cutting into the centre of the trunk, and counting the rings, or layers of the sap and heart, as a new ring is formed each year in the process of vegetation. When the tree commences to decline, the extremities of the old branches, and particularly the top, exhibit signs of decay.

Trees should not be felled whilst the sap is in circulation; for this substance is of a peculiarly fermentable nature, and, therefore, very productive of destruction to the wood. The proper seasons for felling are in winter, during the months of December, January, and February, and in midsummer, during July. All other seasons are bad; but the spring is peculiarly so, for the tree then contains the greatest quantity of sap. As the sap wood, in most trees, forms a large portion of the trunk, experiments have been made, for the purpose of improving its strength and durability. These experiments have been mostly directed towards the manner of preparing the tree, before felling it. One method consists in *girdling*, or making an incision with an axe around the trunk completely through the sap wood, and suffering the tree to stand in this state, until it is dead; the other consists in *barking*, or stripping the entire trunk of its bark, without wounding the sap wood, early in the spring, and allowing the tree to stand until the new leaves have put forth and fallen, before it is felled. The sap wood of trees, treated by both of these methods, was found very much improved in hardness, strength, and durability; the results from girdling were, however, inferior to those from barking.

The seasoning of timber is of the greatest importance, not only to its durability, but to the solidity of the structure, for which it may be used; as a very slight shrinking of some of the pieces, arising from the seasoning of the wood, if used in a green state, might, in many cases, cause material injury, if not complete destruction to the structure. Timber is considered as sufficiently seasoned, for the purposes of frame work, when it has lost about one-fifth of the weight, which it has in a green state. Several methods are in use for seasoning timber; they consist either in an exposure to the air for a certain period in a sheltered position, which is termed *natural seasoning*, in immersion in water, termed *water seasoning*, or in boiling or *steaming*.

For natural seasoning, the trunk, so soon as the tree is felled, should be deprived of its bark, and be removed to some dry position, until it can be sawed into suitable scantling. It should then be piled in a perfectly dry situation, and be exposed to a free circulation of the air, but sheltered from the direct action of the wind, rain, and sun. By taking these precautions, an equable evaporation of the moisture will take place over the entire surface, which will prevent either warping or splitting, which necessarily ensues, when one part dries more rapidly than another. It is farther recommended, instead of piling the pieces on each other in a horizontal position, that they be laid on cast iron supports properly prepared, and with a sufficient inclination to facilitate the dripping of the sap from one end; and that heavy round timber be bored through the centre, to expose a greater surface to the air, as it has been found that it cracks more in seasoning than square timber.

Natural seasoning is preferable to any other, as timber seasoned in this way, is both stronger and more durable than when prepared by any artificial process. Most timber will require, on an average, about two years to become fully seasoned in the natural way.

Water seasoning may be resorted to when despatch is ne-

cessary; the trunk is immersed in water about a fortnight, and then taken out and dried in a sheltered position before using it. The sap wood is rendered less liable to decay by this process, as a large proportion of the fermentable matter is dissolved by the water; but the general strength of the timber is impaired by this loss. Fresh running water is considered the best for timber, which is to be used in the frame work of houses, as the salt which is taken up by timber, immersed in salt water, keeps it always in a moist state, by attracting moisture from the atmosphere.

Steaming is mostly in use for ship building, where it is necessary to soften the fibres, for the purpose of bending large pieces of timber. It impairs the strength of the timber, but renders it less subject to decay, and to warp and crack. About four hours is said to be sufficient for steaming the largest sized pieces.

When timber is used for posts partly imbedded in the ground, it is usual to char the part thus imbedded, to preserve it from decay. This method is only serviceable when the timber has been previously well seasoned, as it then acts as a preventive, both of worms and the rot; but for green timber, it is highly injurious, as by closing the pores, it prevents the evaporation from the surface, and thus causes fermentation and rapid decay within.

The most durable timber is procured from trees of a close compact texture, which, on analysis, yield the largest quantity of carbon. And those which grow in moist and shady localities furnish timber which is weaker and less durable than that from trees growing in a dry open exposure.

Timber is subject to defects, arising either from some peculiarity in the growth of the tree, or from the effects of the weather. Straight-grained timber, free from knots, is superior in strength and quality, as a building material, to that which is the reverse.

The action of high winds, and severe frosts, injures the tree whilst standing: the former separating the layers from

each other, forming what is denominated *rolled timber;* the latter cracking the timber in several places, from the surface to the centre. These defects, as well as those arising from worms or age, are easily seen by examining a cross section of the trunk.

The *wet* and *dry rot* are the most serious causes of the decay of timber; as all the remedies thus far proposed to prevent them, are too expensive to admit of a very general application. Both of these causes have the same origin, fermentation, and consequent putrefaction. The wet rot takes place in wood exposed, alternately, to moisture and dryness; and the dry rot is occasioned by want of a free circulation of air, as in confined warm localities, like cellars, and the more confined parts of vessels.

Trees of rapid growth, which contain a large portion of sap wood, and timber of every description, when used green, where there is a want of a free circulation of air, decay very rapidly with the rot.

Of the various remedies, proposed to prevent the rot, the application of salt around the timber, is said to succeed in ship building; and boiling the timber for some hours in a solution of copperas, or in one of corrosive sublimate, is said to answer the purpose for house carpentry. The best means is to use only well seasoned timber, and to procure a free circulation of fresh air around it, to prevent the accumulation of moisture, or the action of a warm, damp, stagnant, atmosphere.

The durability of timber varies greatly under different circumstances of exposure. If placed in a sheltered position, and exposed to a free circulation of fresh air, it will last for centuries, without any very sensible changes in its physical properties. The same is known to take place, when it is entirely immersed in water, or imbedded in the ground, or in thick walls, so as not to be affected by the atmosphere. In salt water, however, particularly in warm climates, timber is rapidly destroyed by several kinds of worms, which soon

reduce it to a honeycomb state. But if the wood is imbedded in mud, the worm will not attack it, nor does it usually eat down farther than about a foot from the bottom.

The best seasoned timber will not withstand the effects of exposure to the weather for a much greater period than twenty-five years, unless it is protected by a coating of paint or pitch, or of oil laid on hot, when the timber is partly charred over a light blaze. These substances themselves, being of a perishable nature, require to be renewed from time to time, and will, therefore, be serviceable only in situations which admit of their renewal. They are, moreover, more hurtful than serviceable to unseasoned timber, as by stopping up the pores of the exterior surface, they prevent the moisture from escaping from within, and, therefore, promote one of the chief causes of decay.

The forests of our own country produce a great variety of the best timber for every purpose, and supply abundantly, both our own and foreign markets. The following genera are in most common use.

Oak. About forty-four species of this tree are enumerated by botanists, as found in our forests, and those of Mexico. The most of them afford a good building material, except the varieties of red oak, the timber of which is weak, and decays rapidly.

The White Oak. (*Quercus Alba.*) So named from the color of its bark, is among the most valuable of the species, and is in very general use, but is mostly reserved for naval constructions; its trunk, which is large, serving for heavy frame work, and the roots and larger branches affording the best compass timber. The wood is very strong and durable, and of a slightly reddish tinge; it is not suitable for boards, as it shrinks about $\frac{1}{32}$ in seasoning, and is very subject to warp and crack.

This tree is found most abundantly in the Middle States. It is seldom seen in comparison with other forest trees, in the Eastern and Southern States, or in the rich valleys of the Western States.

Post Oak (*Quercus Obtusiloba.*) This tree seldom attains a greater diameter than about fifteen inches, and, on this account, is mostly used for posts, from which use it takes its name. The wood has a yellowish hue, and close grain, and is said to exceed white oak in strength and durability, and is, therefore, an excellent building material for the lighter kinds of frame work. This tree is found most abundantly in the forests of Maryland and Virginia, and is there frequently called *Box White Oak*, and *Iron Oak*. It also grows in the forests of the Southern and Western States, but is rarely seen farther north than the mouth of the Hudson River.

Chesnut White Oak (*Quercus Prinus Palustris.*) The timber of this tree is strong and durable, but inferior to the two preceding species. The tree is abundant from North Carolina to Florida.

Rock Chesnut Oak (*Quercus Prinus Monticola.*) The timber of this tree is mostly in use for naval constructions, for which it is esteemed inferior only to the white oak. The tree is found in the Middle States, and as far north as Vermont.

Live Oak (*Quercus Virens.*) The wood of this tree is of a yellowish tinge; it is heavy, compact, and of a fine grain, it is stronger and more durable than any other species, and, on this account, it is considered invaluable for the purposes of ship building, for which it is exclusively reserved.

The live oak is not found farther north than the neighborhood of Norfolk, Virginia, nor farther inland, than from fifteen to twenty miles from the seacoast. It is found in abundance along the coast south, and in the adjacent islands as far as the mouth of the Mississippi.

Pine. This very interesting genus is considered inferior only to the oak, from the excellent timber afforded by nearly all of its species. It is regarded as a most valuable building material, owing to its strength and durability, the straightness of its fibre, the ease with which it is wrought, and its applicability to all the purposes of constructions in wood.

Yellow Pine (*Pinus Mitis.*) The heart wood of this tree is fine grained, moderately resinous, strong, and durable; but the sap wood is very inferior, decaying very rapidly on exposure to the weather. The timber is in very general use for frame work, &c.

This tree is found throughout our country, but in the greatest abundance in the Middle States. In the Southern States, it is known as *Spruce Pine*, and *Short-leaved Pine.*

Long-leaved Pine, or Southern Pine, (*Pinus Australis.*) This tree has but very little sap wood; and the resinous matter is very uniformly distributed throughout the heart wood, which presents a fine compact grain, having more hardness, strength, and durability, than any other species of the Pine, owing to which qualities, the timber is in very great demand.

The tree is first met with near Norfolk, Virginia, and from this point south, it is abundantly found.

White Pine, or Northern Pine, (*Pinus Strobus.*) This tree takes its name from the color of its wood, which is white, soft, light, straight-grained, and durable. It is inferior in strength to the species just described, and has, moreover, the defect of swelling in damp weather, and of not giving a firm hold to nails. Its timber is, however, in great demand as a a good building material, being almost the only kind in use in the Eastern and Northern States, for the frame work and joinery of houses, &c.

The finest specimens of this tree grow in the forests of Maine. It is found in great abundance, between the 43 and 47 parallels, N. L.

Among the forest trees, in less general use than the oak and pine, the *Locust*, the *Chesnut*, the *Red Cedar*, and the *Larch*, hold the first place for hardness, strength, and durability. They are chiefly used for the frame work of vessels. The chesnut, the locust, and the cedar, are preferred to all other trees for posts.

The Black, or Double Spruce, (*Abies Nigra*) also affords

an excellent building material, its timber being strong, durable, and light.

The *Juniper* or *White Cedar*, and the *Cypress*, are very celebrated for affording a material, which is very light, and of great durability, when exposed to the weather; owing to these qualities, it is almost exclusively used for shingles and other exterior coverings. These two trees are found, in great abundance, in the swamps of the Southen States.

Strength of Timber. From a variety of experiments, made to ascertain the strength of different kinds of wood, it appears that there is but little difference in the strength of those varieties, which are in most common use as building materials.

The resistance which timber of oak and pine offers to a force of extension, acting parallel to the direction of the fibres, is very nearly the same in each; and on an average, may be stated, according to the results of experiments, at 10,000 pounds on the square inch before rupture ensues. The resistance to rupture by compression, is about one half the resistance to rupture by a force of extension, and may be taken on an average for the two kinds of timber, at 4000 pounds on the square inch. In practice, timber should not be exposed to a permanent strain, greater than one fifth of that which will cause rupture, when the force acts parallel to the direction of the fibres. To ascertain the limits of the resistance to a force acting in a perpendicular direction to the fibres, it will be necessary to examine the analytical expressions given by writers on the strength of materials; and as these expressions are equally applicable to all materials, they will be given with their applications in another place.

METALS.

Iron. This metal is very extensively used for the purposes of the engineer and architect, both in the state of *cast iron*, and *forged* or *wrought iron.*

Cast iron is one of the most valuable building materials, owing to its great strength, hardness and durability, and the ease with which it can be cast, or moulded, into the best forms, for the purposes to which it is to be applied.

Cast iron is divided into two principal varieties, the *gray cast iron*, and *white cast iron*. There exists a very marked difference between the properties of these two varieties. There are, besides, many intermediate varieties, which partake more or less of the properties of these two, as they approach, in their external appearances, nearer to the one or the other.

Gray cast iron, when of a good quality, is slightly malleable in a cold state, and will yield readily to the action of the file, when the hard outside coating is removed. This variety is also sometimes termed soft gray cast iron; it is softer and tougher than the white iron. When broken, the surface of the fracture presents a granular structure; the color is gray, and the lustre is what is termed metallic, resembling small brilliant particles of lead strewed over the surface.

White cast iron is very hard and brittle; when recently broken, the surface of the fracture presents a distinctly marked crystalline structure, the color is white, and lustre vitreous, or bearing a resemblance to the reflected light from an aggregation of small crystals.

The gray iron is most suitable where strength is required; and the white, where hardness is the principal requisite.

The color and lustre, presented by the surface of a recent fracture, are the best indications of the quality of iron. A uniform dark gray color, and high metallic lustre, are indications of the best and strongest. With the same color, but less lustre, the iron will be found to be softer and weaker, and to crumble readily. Iron without lustre, of a dark and mottled color, is the softest and weakest of the gray varieties.

Iron of a light gray color and high metallic lustre, is usually very hard and tenacious. As the color approaches to

white, and the metallic lustre changes to vitreous, hardness and brittleness become more marked, until the extremes of a dull or grayish white color, and a very high vitreous lustre, are attained, which are the indications of the hardest and most brittle of the white variety.

The quality of cast iron may also be tested, by striking a smart stroke with a hammer on the edge of a casting. If the blow produces a slight indentation, without any appearance of fracture, it shows that the iron is slightly malleable, and, therefore, of a good quality; if, on the contrary, the edge is broken, it indicates brittleness in the material, and a consequent want of strength.

The strength of cast iron will depend not only on the quality of the melted metal, but also upon its temperature at the moment it is thrown into the mould; the position of the mould itself; and the manner in which the cooling is performed. All of these circumstances render it very difficult to judge of the quality of a casting from a bare inspection of its external characters: but, in general, if the exterior presents a uniform appearance, without any inequalities on the surface, it will be an indication of uniform strength throughout.

Gray cast iron offers a greater resistance to a force of extension, than the white cast, in a ratio of nearly eight to five; but the white cast offers the greatest resistance to a compressive force.

The strength of the gray cast iron is very variable, depending on the quantity of carbon that is combined with it. Its resistance to rupture by a force of extension, in the best varieties, does not exceed 20,000 pounds on the square inch. It is found, moreover, that the strength of bars, cast in vertical moulds, is superior to those which are cast horizontally, and that large bars are stronger than small ones, in a ratio, which is greater than the areas of their sections.

The resistance of cast iron to compression is very great; from experiments, it appears that, it will bear a weight varying

between 90,000 and 140,000 pounds on the square inch, before rupture takes place by compression.

Forged Iron. The color, lustre, and texture of a recent fracture, present, also, the most certain indications of the quality of forged iron. The fracture submitted to examination, should be of bars at least one inch square, or, if of flat bars, they should be at least half an inch thick; otherwise, the texture will be so greatly changed, arising from the greater elongation of the fibres, in bars of smaller dimensions, as to present none of those distinctive differences observable in the fracture of large bars.

The surface of a recent fracture of good iron, presents a clear gray color, and high metallic lustre; the texture is granular, and the grains have an elongated shape, and are pointed and slightly crooked at their ends, giving the idea of a powerful force having been employed to produce the fracture. When a bar, presenting these appearances, is hammered or drawn out into small bars, the surface of fracture of these bars, will have a very marked fibrous appearance, the filaments being of a white color and very elongated.

When the texture is either laminated or crystalline, it is an indication of some defect in the metal, arising either from the mixture of foreign ingredients, or else, from some neglect in the process of forging.

Burnt iron is of a clear gray color with a slight shade of blue, and of a slaty texture. It is soft and brittle.

Cold short iron, or iron that cannot be hammered when cold without breaking, presents nearly the same appearance as burnt iron, but its color inclines to white. It is very hard and brittle.

Hot short iron, or that which breaks under the hammer when heated, is of a dark color without lustre.

The fibrous texture, which is only developed in small bars by hammering, is an inherent quality of good iron; those varieties, which are not susceptible of receiving this peculiar texture, are of an inferior quality, and should never be used

for purposes requiring great strength: the filaments of these varieties are short, and the fracture is of a deep color, between lead gray and dark gray.

The best forged iron presents two varieties; the *hard* and *soft*. The hard variety is very strong and ductile, but does not yield to the hammer so readily as the soft. It preserves its granular texture a long time under the action of the hammer, and only developes the fibrous texture, when beaten or drawn out into small rods: its filaments then present a silver white appearance.

The soft variety is weaker than the hard; it yields easily to the hammer, and it commences to exhibit, under its action, the fibrous texture in tolerably large bars. The color of the fibres, is between a silver white and lead gray.

Iron may be naturally of a good quality, and, still, from being badly refined, not present the appearances, which are regarded as sure indications of its excellence. Generally, however, if the surface of fracture presents a texture partly crystalline and partly fibrous, or a fine granular texture, in which some of the grains seem pointed and crooked at the points, together with a light gray color without lustre, it will indicate natural good qualities, which require only careful refining to be fully developed.

The strength of forged iron is very variable, as it depends not only on the natural qualities of the metal, but also upon the care bestowed in forging, and the greater or less compression of its fibres, when drawn or hammered into bars of different sizes.

The resistance offered by the best kinds of forged iron to rupture by a force of extension, may be stated, on an average, at 60,000 pounds on the square inch for bars whose cross section is greater than one square inch. It has been found that in comparing the relative strength of bars of different sizes, small bars are the strongest. Bars having a cross section of half a square inch, will require a force of extension, equivalent to 70,000 pounds on the square inch, to produce rupture;

and bars having a cross section of a quarter of a square inch, will bear from 80,000 to 95,000 pounds on the square inch, before rupture ensues. With equal areas, flat bars are stronger than square ones, and round bars are stronger than either the flat or square bars.

There are no satisfactory experiments on the resistance of forged iron to rupture from a compressive force. A knowledge of this resistance would be of little practical use, as forged iron is never used for vertical supports; cast iron and wood being much superior to it for such purposes.

STRENGTH OF MATERIALS IN GENERAL.

The following algebraical expressions are given by writers on the subject of the strength of materials, to determine the effect of a given weight to produce either rupture, or a certain flexure, when the direction of the force is perpendicular to the fibres of the solid, producing what is termed a *transversal* or *cross strain.*

Representing by

W. the weight applied,

w. the weight of the solid,

f. the depression caused by the applied weight,

l. the length of the solid between the points of support,

b. the breadth, or horizontal thickness of the solid,

d. the depth, or vertical thickness.

The following expression represents the relation between the applied weight and the depression; or what is termed the stiffness of the solid,

$$W + \tfrac{5}{8}\,w = E.\ \frac{4bd^3f}{l^3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

from which expression, there results,

$$E = (W + \tfrac{5}{8}\,w)\frac{l^3}{4bd^3f}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

or, if the weight of the solid be neglected,

$$E = W\,\frac{l^3}{4bd^3f}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

The quantity E, in the expressions (2) and (3), is constant for the same material, and is termed the *coefficient of elasticity*. It represents that weight, which, applied on a unit of surface of a solid of a given length, would cause an extension equal to the length of the solid, supposing it to be perfectly elastic. The relations, therefore, shown by the expressions (2) and (3), are true only, when the depressions, caused by the applied weight, are so small as not to injure the elasticity of the material.

When the applied weight is sufficiently great to produce rupture, the following expressions, in which the same notation is followed, as in the foregoing expressions, are found to obtain,

$$W + \frac{w}{2} = R \frac{2bd^2}{3l}, \quad \ldots \ldots \ldots \quad (4.)$$

from which,

$$R = \left(W + \frac{w}{2}\right)\frac{3l}{2bd^2}, \quad \ldots \ldots \ldots \quad (5.)$$

The quantity R, in the expressions (4) and (5), is constant for the same material, and is denominated the *coefficient of tenacity*, or *rupture*. It represents the weight, which applied on a unit of surface of a given material, will cause rupture. The value of W and R, obtained from expressions (4) and (5), are true only in cases, where the depressions at the instant of rupture, are so small, that they may be neglected in calculation; and as this is generally the case in all practical applications, these expressions may be used in all such cases.

To determine the value of the constants E and R, direct experiments must be made on each kind of material. The experiments are made in the following manner: A sample of the material, whose cross section is rectangular, is placed horizontally on two props, the distance between which is represented by the quantity *l*; weights are applied to the material at the middle point between the props, and the depressions of the middle point below the points resting on the props, caused by each weight are noted until rupture takes place. To de-

termine the quantity E, the weight W, which produces a very slight flexure of the materials, with its corresponding depression f are substituted in expressions (2) and (3); and to obtain R, that value of W, which in the experiment, causes rupture, is substituted in expression (5). Having, in this way, obtained the value of E and R, the value of W in the expressions (1) and (4), can be calculated in all other cases; or any one of the quantities be found in terms of the rest.

When the weight, instead of being applied at the middle of the solid, expression (1), is equally distributed throughout its length, it is shown, both by calculation and experiment, that the depression will be less, in the ratio of 5 to 8, or, in other words, that $\frac{5}{8}$ W placed at the middle of the solid, will produce the same depression as W, equally distributed over the whole length between the props.

When the solid, instead of resting on two props, is fixed at one extremity, and a weight is applied at the other, it is found that the depression, arising from this weight, is to that caused by the same weight uniformly distributed throughout the solid, in the ratio of 8 to 3.

With regard to the expression (4), it is found that, when a weight is uniformly distributed over the solid, the same effect will be produced, as if half that weight were applied at the middle point.

When one end of the solid is fixed, and a weight is applied at the other end, the expression (4) becomes

$$W + \frac{w}{2} = R\frac{bd^2}{6l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

When a solid is maintained in a vertical position, and a sufficient weight is applied at the top to produce flexure, the following expressions will give the least weight, under which the solid will commence to bend,

$$W = 0.823\,E.\frac{bd^3}{l^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

the cross section of the solid being rectangular, and the same notation being adopted as in the preceding expressions.

If the cross section of the solid is a circle, the radius of the circle being represented by r, the expression for the least weight, becomes

$$W. = E.\frac{31\ r^4}{4\ l^3}, \quad . \quad . \quad . \quad . \quad . \qquad (8.)$$

The preceding expressions show the relations between the weights, which produce either rupture, or a certain flexure, and the dimensions of solids of the forms which are most usually met with in practice. There now remains to be found the values of the constants E and R, and to assign such limits to them, as are in accordance with safety in practice.

In the following table, will be found the results of experiments made upon timber and iron, with the values of E and R, deduced by substituting the data which they furnish, in the expressions (2) and (5), without taking into consideration the values of w, which enter into those expressions, as they were generally so small, in the cases considered, as not to affect sensibly the result. The inch, and pound avoirdupois are taken as the units of measure, and weight.

Table of the values of E *determined from experiments.*

	Material.	Value of l.	Value of b.	Value of d.	Value of W.	Value of f.	Value of $E = W\frac{l^3}{4bd^3f}$.	Authors of Experiments.
		inches.	inches.	inches.	lbs.	inches.	lbs.	
1	Oak (*English*)	84	2	2	200	1.280	1 450 000	Barlow.
2	Oak (*Canadian*)	84	2	2	225	1.080	1 930 000	"
3	Pine (*American*)	84	2	2	150	0.931	1 492 000	"
4	Oak (*English*)	30	1	1	137	0.5	1 850 000	Tredgold.
5	White Spruce (*Canadian*)	24	1	1	180	0.5	1 244 000	"
6	White Pine -	85.2	2.75	5.55	777	0.177	1 444 000	Lt. Brown
7	Black Spruce	85.2	2.75	5.55	892	0.177	1 658 000	"
8	Southern Pine	85.2	2.75	5.55	1175	0.177	2 183 000	"
9	Cast Iron (*soft Gray*) -	78	1.5	3	440	0.075	17 190 000	Tredgold.
10	Same (*hard and brittle*)	36	0.9	0.9	100	0.10	17 780 000	"
11	Forged Iron (*French*)	78.7	1.18	0.43	22	0.944	30 090 000	Duleau.
12	Same (*Swedish*)	36	1	1	560	0.30	21 780 000	Barlow.
13	Same (*English*)	36	1	1	560	0.25	26 120 000	"

From these values, it appears that, the following may be

taken as the mean value of E: for Oak, 1 700 000, for Pine, 1 400 000, for Cast Iron, 17 500 000, for Forged Iron, 26 000 000.

Experiments 6, 7, and 8, were made on our own timber at *Fort Adams*, Newport, R. I., by *Lieutenant* Brown, of the *U. S. Corps of Engineers.*

Table of the values of R. *determined from experiments.*

	Material.	Specific Gravity.	Value of l.	Value of b.	Value of d.	Value of w.	Value of $R = w.\frac{l}{2bd^2}$
			inches.	inches	inches.	lbs.	
1	Oak (*English*) -	,934	84	2	2	637	10 030
2	Oak (*Canadian*)	,872	84	2	2	673	10 600
3	White Spruce (*American*) - -	,465	24	1	1	285	10 260
4	White Pine -	,455	85.2	2.75	5.55	5 189	7 829
5	Black Spruce - -	,490	85.2	2.75	5.55	5 646	8 518
6	Southern Pine -	,872	85.2	2.75	5.55	9 237	13 940
7	Cast Iron - -	7,207	36	1	1	897	48 400
8	Same - -	7,207	36	1	1	800	43 200

From this table, it appears that the following mean values may be assigned to R: Oak, 10 000, Spruce and Pine, 8 800, Cast Iron, 46 000.

Practical deductions respecting the strength of materials.

From the foregoing expressions, and from the tables deduced from them, the laws which regulate the resistance of materials to flexure and rupture, caused by a cross strain, may be easily ascertained; but the values obtained from these expressions require to be modified in practice, for the materials used in structures of every character, must not only be strong enough to resist any effort, whose action for a short time would cause rupture, but they must also be secure from the effects of permanent or frequent repetition of such action, and the alterations to which they may be subjected from accidents, and the effects of time. To ascertain the effects produced by these several causes of change, resort must be had to data, furnished by experiments; and none can be relied upon with more security, than those which are ob-

tained by an examination of structures which have already withstood the test of time.

The main point to be determined in all cases, is the effort to which a material may be exposed, without injury to its elasticity; for if this be impaired in ever so slight a degree, the physical constitution of the material will be virtually destroyed, and it can no longer oppose that reaction on which security depends.

The following results, drawn from experience, will serve to regulate this limit.

Resistance of Stone to Rupture,

The principal results, drawn from the experiments of this material, have already been given. The following table, showing the weight on a superficial foot in some of the most remarkable structures in the world, for boldness of design, will furnish some additional facts of importance to builders.

Pillars of the dome of St. Peter's (*Rome*)	33 330 lbs.
Pillars of the dome of St. Paul's (*London*)	39 450 "
Pillars of the dome of St. Geneviève (*Paris*)	60 000 "
Pillars of the church of Toussaint (*Angers*)	90 000 "
Lower courses of the piers of the *Bridge of Neuilly*	3 600 "

From experiments made on small cubes of the stone, of which these structures are built, the base of the cube being nearly four superficial inches, the following results were obtained.

The stone used in St. Peter's is a calcareous Tufa, called *Travertino;* it is crushed by about 536 000 pounds on the square foot.

Saint Paul's is built of a lime stone, known to mineralogists as *Oolite*, and to builders as *Portland stone;* it crushes under a weight of 537 000 pounds on the square foot.

Saint Geneviève is built of a lime stone, which crushes under a weight of 456 000 pounds on the square foot.

The church of Toussaint is built of a very hard shell lime

stone of a reddish color, which crushes under a weight of 900 000 pounds on the square foot.

The bridge of Neuilly is built of a lime stone, which crushes under 570 000 pounds on the square foot.

Resistance of timber to rupture by extension or compression.

The general results of the strength of wood have already been given; from existing structures, it appears, that security will be attained by limiting the weight borne to $\frac{1}{5}$ of that which would cause rupture by compression. Although the elasticity of timber is said not to be affected by a force, which is between one fourth and one third of that which causes rupture, still, it is prudent not to submit it to so great a permanent strain.

To determine the limits of the cross strain, to which timber can be submitted with safety, it should be borne in mind, that the degree of flexure, caused by the strain, must not impair the elasticity of the fibres, so that when the strain is taken off, the piece may regain its natural form. There are no special experiments, from which this limit can be ascertained, but, from an examination of existing structures, it seems, that timber may be exposed, with entire safety, to a cross strain equal to $\frac{1}{10}$ of that which would cause rupture. In applying, therefore, the expressions (4) and (6) to given cases, a value R′ must be substituted for R, equal to one tenth of that which is contained in the tables; this value for oak, for example, would be 1000.

As the value of E for oak is 1 700 000 pounds, the elongation or compression of the fibres, arising from a value of R 1000 pounds, will be $\frac{1}{1700}$, or nearly 0.0006, which may, therefore, be taken as the greatest that will admit of perfect security.

When a vertical beam is pressed by a force at top, it has been ascertained by experiments, that if the length is greater than 8 or 10 times the thickness, rupture will take place by the bending of the beam, and that, when the length is less than 8 times the thickness, the beam will yield by crushing.

To determine the limits of the strain in this case, the expressions (7) and (8), which show the least weight that will cause flexure under these circumstances, must be used, but from a comparison of the values furnished by these expressions, and those determined by experiments, it appears, that the latter will be greater than the former, in all cases, where the length is less than 20 times the thickness: therefore, in all cases, where the length is less than 20 times the thickness, the weight to be borne, will be estimated from that by which rupture is caused by crushing the fibres. But as any slight lateral strain in addition to this, would cause the beam to give way, this weight must be farther reduced, depending on the length of the beam.

The experiments, made to ascertain this reduction, indicate that, for wood, the weight borne should be reduced to the $\frac{5}{6}$, when the length is equal to 12 times the thickness; and to $\frac{1}{2}$, when it is 24 times the thickness.

For cast iron, the weight should be reduced to the $\frac{2}{3}$, when the length is 4 times the thickness; to about the $\frac{1}{2}$, when it is 8 times the thickness; and to the $\frac{1}{15}$, when it is 36 times the thickness.

For forged iron, the weight should be reduced to the $\frac{5}{8}$, when the length is 12 times the thickness; and to $\frac{1}{2}$, when it is 24 times the thickness.

The following is about the crushing weights for a square inch of these materials, when the length is once or twice the thickness.

Oak and Pine - - - - -	4 000 pounds
Forged Iron - - - -	60 000 "
Cast Iron - - - - -	140 000 "

Having thus ascertained the reduction of the crushing weight required by the length, or of that given by the expressions (7) and (8), the total permanent weight borne, should be only about $\frac{1}{10}$ of this for wood, and between $\frac{1}{4}$ and $\frac{1}{8}$ for forged, or for cast iron.

Resistance of forged iron to a longitudinal extension, and to a cross strain.

This material may be exposed to a permanent longitudinal strain, between $\frac{1}{4}$ and $\frac{1}{6}$ of that which would cause rupture.

When submitted to a cross strain, a value for R, equal to $\frac{1}{4}$ of that determined from experiments, or 15 000 pounds, must be substituted in the expression (4) and (6), comparing this with the value of E furnished by the table, it will appear, that forged iron may be submitted with safety to a force, which would produce an elongation of the fibre, equal to $\frac{15}{6200}$ or 0.0005.

Resistance of cast iron to a cross strain.

The value of R, contained in the table, for this material, is 46 000 pounds, and that for E is 17 500 000.

Cast iron may be submitted with safety, to a strain equal to $\frac{1}{4}$ of that which causes rupture. It will, therefore, be necessary to substitute 11 500 for R in the expressions (4) and (6.) Comparing this reduced value with that of E, it appears, that the fibres can bear with safety an elongation equal to 0.0065.

With regard to the expression (1), it is usually applied to find the weight which will cause a given deflection, when the dimensions of the solid are given. If, for example, it be required to find the weight, which will cause a deflection of $\frac{1}{40}$ of an inch for every foot in length of the solid, which would be equivalent to $\frac{1}{480}$ of an inch for every inch in length, it would only be necessary to substitute $\frac{l}{480}$ for f in the given expression, which would then reduce to

$$W + \tfrac{5}{8}\,w = E.\frac{4bd^3\,\frac{l}{480}}{l^3},$$

or

$$W + \tfrac{5}{8}\,w = E.\frac{bd^3}{120l^3},$$

The expression would undergo a similar modification for any other given deflexion.

MASONRY.

Masonry is the art of raising structures, in stone, brick, and mortar.

Masonry is classified either from the nature of the material, as *stone masonry*, *brick masonry*, and *mixed*, or that which is composed of stone and brick; or from the manner in which the material is prepared, as *cut stone* or *ashlar masonry*, *rubble stone* or *rough masonry*, and *hammered stone masonry*; or finally, from the form of the material, as *regular masonry*, and *irregular masonry*.

Cut Stone. Masonry of cut stone, when carefully made, is stronger and more solid than that of any other class; but, owing to the labor required in *dressing* or preparing the stone, it is also the most expensive. It is, therefore, mostly restricted to those works where a certain architectural effect is to be produced by the regularity of the masses, or where great strength is indispensable, as in works exposed to the shocks of the waves, and in the revetment walls of fortifications.

Before entering on the means to be used, to obtain the greatest strength and solidity in cut stone, it will be necessary to give those definitions which may be required to render the subject clearer.

In a wall of masonry, the term *face* is usually applied to the front of the wall, and the term *back*, to the inside, the stone which forms the front, is termed the *facing*, that of the back, the *backing*, and the interior, *the filling*. If the front, or back of the wall, has a uniform slope from the top to the bottom, this slope is termed the *batter*.

The term *course*, is applied to each horizontal layer of stone in the wall; if the stones of each layer are of equal thickness throughout, it is termed *regular coursing*, if the thicknesses are unequal, the term *random* or *irregular coursing*, is applied. The divisions between the stones, in the

courses, are termed the *joints*; the upper surface of the stones of each course is also, sometimes, termed the *bed* or *build.*

The arrangement of the different stones of each course, or of contiguous courses, is termed the *bond.*

The strength and solidity of a mass of cut stone masonry will depend on the size of the blocks in each course, on the *fit* or perfect juxtaposition of the blocks, and on the bond used.

The size of the blocks will depend on the kind of stone, and the nature of the quarry. From some quarries the stone may be obtained of any required dimensions; others, owing to some peculiarity in the formation of the stone, only furnish blocks of small size. Again, the strength of some stones is so great as to admit of their being used in blocks of any size, without danger to the strength of the structure, arising from their breaking; others can only be used with safety, when the length, breadth, and thickness of the block bear certain relations to each other. No fixed rule can be laid down on this point: that usually followed by builders, is to make the breadth at least equal to the thickness, and seldom greater than twice this dimension, and to limit the length to within three times the thickness. When the breadth or length is considerable in comparison with the thickness, there is danger that the block may break, if any unequal setting or unequal pressure should take place. As to the absolute dimensions, the thickness is generally not less than one foot, nor greater than two; stones of this thickness, with the relative dimensions just laid down, will weigh from 1000 to 8000 pounds, allowing, on an average, 160 pounds to the cubic foot. With these dimensions, therefore, the weight of each block will require a very considerable power, both of machinery and men, to set it on its bed.

For the coping and top courses of a wall, the same objections do not apply to excess in length: but this excess is, on the contrary, favorable, because the number of top joints being thus diminished, the mass covered by the coping, will be

better protected, being exposed only at the joints, which it is almost impossible to make water tight, owing to the mortar being crushed by the expansion of the blocks in warm weather, and, when they contract, being washed out by the rain.

The fit of the blocks will depend solely on the accuracy with which the surfaces in contact, are wrought or *dressed;* if this part of the work is slovenly done, the mass will not only present open joints from any inequality in the settling: but from the courses not fitting accurately on their beds, the blocks will be liable to crack from the unequal pressure on the different points of the block.

In arranging the fit, the surfaces of one set of joints, should, as a prime condition, be perpendicular to the direction of the pressure: by this arrangment, there will be no tendency in any of the blocks to slip outwards. In a vertical wall, for example, the pressure being downwards, the surfaces of one set of joints, which are the beds, must be horizontal. The surfaces of the other set must be perpendicular to these, and, at the same time, perpendicular to the face, or to the back of the wall, according to the position of the stones in the mass; two essential points will be attained by this arrangment, the angles of the blocks, both at the top and bottom of the course, and at the face or back, will be right angles; and the block will thus be as strong as the nature of the stone will admit. The principles, here applied to a vertical wall, are applicable in all cases, whatever may be the direction of the pressure and the form of the exterior surfaces, whether plane or curved. There is, however, a modification, which, in some cases, will be requisite, arising from the strength of the stone. It is laid down as a rule, drawn from the experience of builders, that no stone work with angles less than 60° will offer sufficient strength and durability to resist accidents, and the effects of the weather. If, therefore, the batter of a wall should be greater than 60°, which is about 7 perpendicular to 4 base, the horizontal joints, (Fig. 6,)

must not be carried out, in the same plane, to the face or back, but be broken off at right angles to it, so as to form a small abutting joint of about 4 inches in thickness. As the batter of walls is seldom so great as this, except in some cases of sustaining walls for the side slopes of earthen embankments, this modification in the joints will not often occur; for, in a much greater batter, it will generally be more economical, and the construction will be stronger, to place the stones of the exterior in offsets, the exterior stone of one course, being placed within the exterior one of the course below it, so as to give the required general direction of the batter.

Workmen, unless narrowly watched, seldom take the pains necessary, to dress the beds and joints accurately; on the contrary, to obtain what are termed *close joints*, they dress the joints with accuracy a few inches only from the outward surface, and then chip away the stone towards the back or *tail*, (Fig. 7,) so that, when the block is set, it will be in contact with the rest, only throughout this very small extent of bearing surface. This practice is objectionable under every point of view; for, in the first place, it gives an extent of bearing surface, which, being generally inadequate to resist the pressure thrown on it, causes the block to splinter off at the joint; and in the second place, to give the block its proper fit, it has to be propped beneath by small bits of stone, or wooden wedges, which operation is termed *pinning-up* or *under-pinning;* and these props, causing the pressure on the block to be thrown on a few points of the lower surface, instead of being equally diffused over it, expose it to crack.

When the facing is of cut stone, and the backing of rubble, this method of thinning off the block may be allowed for the purpose of forming a better bond between the rubble and ashlar; but, even in this case, the block should be dressed true on each joint, to at least one foot back from the face; and if there exists any cause, which would give a tendency to an outward thrust from the back, then, instead of thinning off all the blocks towards the tail, it will be preferable to leave the tails of some thicker than the parts which are cut.

Various methods are used by builders for the bond of cut stone. The system, termed *headers* and *stretchers*, in which the blocks of one course *break joints* with the courses above and below it, is the most simple, and offers, in most cases, all requisite solidity. In this system, (Fig. 8,) the blocks of each course are laid alternately with their greatest and least dimensions to the face of the wall; those which present the longest dimensions, are termed stretchers; the others, headers. If the header reaches from the face to the back of the wall, it is termed a *through;* if it only reaches part of the distance, it is termed a *binder*. The vertical joints of one course are either just over the middle of the blocks of the next course below, or else, at least four inches on one side or the other of the vertical joints of that course; and the binders of one course, rest as nearly as practicable on the middle of the stretchers of the course beneath. If the backing is of rubble, and the facing of cut stone, a system of throughs or binders, similar to what has just been explained, must be used.

By the arrangement here explained, the facing and backing of each course are well connected; and, if any unequal settling takes place, the vertical joints cannot open, as would be the case were they in a continued line from the top to the bottom of the mass, as each block of one course confines the ends of the two blocks just under it in the course beneath.

In masses of cut stone exposed to violent shocks, as those of which light houses and sea-walls in very exposed positions are formed, the blocks of each course require to be not only very solidly united with each other, but also with the courses above and below them. To effect this, various means have been used. The beds of one course are sometimes arranged with projections, (Fig. 9,) which fit into corresponding indentations, of the next course. Iron cramps in the form of the letter S, or in any other shape that will answer the purpose of giving them a firm hold on the blocks, are let into the top of two blocks of the same course at a vertical

joint, and are firmly set with melted lead, or with bolts, so as to confine the two blocks together. Holes are, in some cases, drilled through several courses, and the blocks of these courses are connected by strong iron bolts fitted to the holes.

The best examples of this method of strengthening the bond of cut stone, are to be found in the works of the Romans, which have been preserved to our time, and in Smeaton's celebrated work, the Eddy-Stone Light House.

When a block of cut stone is to be laid, the first point to be attended to, is to examine the fit, which is done by placing the block on its bed, and seeing that the joints fit close, and the face is in its proper plane. If it be found that the fit is not accurate, the inaccuracies are marked, and the requisite changes made. The bed of the course, on which the block is to be laid, is then thoroughly cleansed from dust, &c., and well moistened, a bed of thin mortar is laid evenly over it, and the block, the lower surface of which is first cleansed and moistened, is laid on the mortar bed, and well settled by striking it with a wooden mallet. When the block is laid against another of the same course, the joint between them is prepared with mortar in the same manner as the bed.

Rubble Stone Masonry. With good mortar, rubble work, when carefully executed, possesses all the strength and durability, required in structures of an ordinary character; and it is much less expensive than cut stone.

The stone, used for this work, should be prepared simply by knocking off all the sharp, weak angles of the block; it is then cleansed from dust, &c., and moistened, before placing it on its bed. This bed is prepared by spreading over the top of the lower course an ample quantity of good ordinary tempered mortar, into which the stone is firmly imbedded. The interstices between the larger masses of stone are filled in, by thrusting small fragments, or chippings, into the mortar. Finally, the whole course should be carefully grouted before another is commenced, in order to fill up any voids, left between the full mortar and stone.

To connect the parts well together, and to strengthen the weak points, throughs or binders should be used in all the courses; and the angles should be constructed of cut or hammered stone. In heavy walls of rubble masonry, the precaution, moreover, should be observed, to lay the stones on their *quarry bed;* that is, to give them the same position, in the mass of masonry, that they had in the quarry; as stone is found to offer more resistance to pressure in a direction perpendicular to the quarry bed, than in any other. The directions of the lamina, in stratified stones, show the position of the quarry bed.

Hammered stone, or dressed rubble, is stone roughly fashioned into regular masses with the hammer. The same precautions must be taken in laying this kind of masonry, as in the two preceding.

Brick Masonry. With good brick and mortar this masonry offers great strength and durability, arising from the strong adhesion between the mortar and brick.

The bond used in brick work, is very various, depending on the dimensions of the walls. The two most usual kinds, are the *English bond* and *Flemish bond.* The first consists, simply, in laying each course as headers and stretchers; the second, in laying several courses of stretchers and one course of headers, alternately.

The mortar bed of brick may be either of ordinary, or thin tempered mortar; the latter, however, is the best, as it makes closer joints, and, containing more water, does not dry so rapidly as the other. As brick has great avidity for water, it would always be well not only to moisten it before laying it, but to allow it to soak in water several hours before it is used. By taking this precaution, the mortar between the joints will set more firmly than when it imparts its water to the dry brick, which it frequently does so rapidly as to render the mortar pulverulent when it has dried.

Foundations. The first point to be attended to in commencing the masonry of a structure, is to procure for the founda-

tions an unyielding bed; for, if this is not done, the structure will possess neither stability nor durability.

This preparatory measure of procuring a stable bed for the foundations to rest on, will depend on the nature of the subsoil; and it may be readily ascertained by proper soundings, made to a suitable depth.

With respect to foundations, soils are usually divided into three classes:

The 1st class consists of soils which are incompressible, or, at least, to so slight a degree, as not to affect the stability of the heaviest masses which may be laid upon them, and, at the same time, do not yield in a lateral direction. Solid rock, some tufas, compact stony soils, hard clay which yields only to the pick or to blasting, belong to this class.

The 2d class consists of soils which are incompressible, but require to be confined laterally, to prevent them from spreading out. Pure gravel and sand belong to this class.

The 3d class consists of all the varieties of compressible soils; under which head may be arranged ordinary clay, the common earths, and marshy soils. Some of this class are found in a more or less compact state, and are compressible only to a certain extent, as most of the varieties of clay and common earth; others are found in an almost fluid state, and yield, with facility, in every direction.

The measures to be taken, to prepare the bed of the foundations, will also depend upon whether the structure is on dry land, or surrounded by water.

To prepare the bed for a foundation on rock, the thickness of the stratum of rock should first be ascertained, if there are any doubts respecting it; and if there is any reason to suppose that the stratum will not offer sufficient resistance to the weight of the structure, it should be tested by a trial weight, at least twice as great as the one it will have to bear permanently. The rock is next properly prepared to receive the foundation courses, by levelling its surface, breaking down all projecting points, and filling up cavities, either with rubble masonry, or with beton; and carefully removing any

portions of the upper stratum which present indications of having been injured by the weather. The surface, prepared in this manner, should, moreover, be perpendicular to the direction of the pressure; if this is vertical, the surface should be horizontal, and so for any other direction of the pressure. If, owing to a great declivity of the surface, the whole cannot be brought to the same level, when the pressure is vertical, it must be broken into steps, in order that the bottom courses of the foundation throughout, may rest on a horizontal surface. If fissures or cavities are met with, of so great an extent as to render the filling them with masonry too expensive, an arch must then be formed, resting on the two sides of the fissure, upon which the walls of the structure will be raised.

The slaty rocks require most care in preparing them to receive a foundation, as their upper stratum will generally be found injured to a greater or less depth by the action of frost.

In stony earths and hard clay, the bed is prepared by digging a trench wide enough to receive the foundation, and deep enough to reach the compact soil which has not been injured by the action of frost: a trench from 4 to 6 feet, will generally be deep enough for this purpose. The bottom of the trench must be perpendicular to the direction of the pressure.

In compact gravel, and sand, where there is no liability to lateral yielding, either from the action of rain, or any other cause, the bed may be prepared as in the case of stony earths. If there is danger from lateral yielding, the part on which the foundation is to rest must be secured by confining it laterally by means of sheeting piles, or in any other way that will offer sufficient security.

In laying foundations on sand, a further precaution is sometimes resorted to, of placing a platform on the bottom of the trench, for the purpose of distributing the whole weight more uniformly over it. This, however, seems to be an unnecessary precaution; for if the botttom courses of the ma-

sonry are well settled in their bed, there is no good reason to apprehend any unequal settling from the effect of the superincumbent weight: whereas, if the wood of the platform should, by any accident, give way, it would leave that part of the foundation without any support.

It sometimes happens, in opening the trench in sand for the bed of a foundation, that water is found at a slight depth, and in such quantity as to impede the labors of the workmen. In this case, if the trench cannot be kept dry by the use of pumps or scoops, a row of sheeting piles must be driven on each side of the space occupied by it, somewhat below the bottom of the bed, the sand is thrown out, on the outside of the sheeting piles, and its place filled with a puddling of clay, to form a water-tight enclosure round the trench. The excavation for the bed is then commenced, but if it be found that the water still makes rapidly at the bottom, only a small portion of the trench will be opened, and after the lower courses are laid in this portion, the excavation will be gradually carried forward, as fast as the workmen can execute the work without difficulty from the water.

The beds of foundations in compressible soils require peculiar care, particularly, if the soil is not homogeneous, presenting more resistance to pressure in one point than in another; for, in that case, it will be very difficult to guard against unequal settling.

In ordinary clay, or earth, a trench is dug of the proper width, and deep enough to reach a stratum, beyond the action of frost; the bottom of the trench is then levelled off to receive the foundation. This may be laid immediately on the bottom, or else upon a *grillage* and *platform*. In the first case, the stones forming the lowest course, should be firmly settled in their beds, by ramming them with a very heavy beetle. In the second a timber grating, termed a grillage, (Fig. 10,) which is formed of a course of heavy beams laid lengthwise in the trench, connected firmly by cross pieces into which they are notched, is firmly settled in the bed, and

the earth solidly packed between the longitudinal and cross pieces; a flooring of thick planks, termed a platform, is then laid on the grillage, to receive the lowest course of the foundation. The object of the grillage, is to diffuse the weight more uniformly over the surface of the trench, and to prevent any part yielding. If the soil is homogeneous, this will be an excess of precaution; but it appears very suitable when the compressibility of the soil is not uniform.

In the case of a soil unequally compressible, it would be better to spread a layer of beton, from 3 to 6 feet in thickness, over the bottom of the trench, to receive the lowest course of the stone work, as it would present more solidity, and would be less subject to accident, than the grillage and platform.

In marshy soils, the principal difficulty consists in forming a bed sufficiently firm, to give stability to the structure, owing to the yielding nature of the soil in all directions.

The following are some of the dispositions that have been made, with success, in this case. Short piles from 6 to 12 feet long, and from 6 to 9 inches in diameter, are driven into the soil as close together as they can be crowded, over an area considerably greater than that which the structure is to occupy. The heads of the piles are accurately brought to a level to receive a grillage and platform; or, else, a layer of clay, from 4 to 6 feet thick, is laid over the area thus prepared with piles, and is either solidly rammed in layers of a foot thick, or submitted to a very heavy pressure for some time before commencing the foundations. The object of preparing the bed in this manner, is to give the upper stratum of the soil all the firmness possible, by submitting it to a strong compression from the piles, and when this has been effected, to procure a stable bed for the lowest course of the foundation by the grillage, or clay bed; by means of which, the whole pressure will be uniformly distributed throughout the entire area. This case is also one in which a bed of beton would replace, with great advantage, either the one of clay, or the grillage.

The purposes to which the short piles are applied in this case, is different from the object to be attained usually in the employment of piles for foundations; which is to transmit the weight of the structure that rests on the piles, to a firm incompressible soil, overlaid by a compressible one, which does not offer sufficient firmness for the bed of the foundation. The use of long piles in such a case, is sometimes the only practicable means of forming a stable bed for the foundation, and the plan is generally more economical, than either to excavate the compressible soil to reach the incompressible one; or to prepare the surface of the incompressible soil, so as to offer sufficient firmness.

To prepare the bed to receive the foundations in this case, strong piles are driven at equal distances apart, over the entire area on which the structure is to rest. These piles are driven, until they meet with a firm stratum below the compressible one, which offers sufficient resistance to prevent them from penetrating farther. The measure of this resistance is estimated by taking the *absolute stoppage*, or the refusal of the pile to penetrate farther than $\frac{2}{10}$ of an inch from the effect of 30 strokes of a ram, weighing 800 pounds, raised to the height of 5 feet at each stroke.

Piles are generally from 9 to 18 inches in diameter, with a length not above 20 times the diameter, in order that they may not bend under the stroke of the ram. They are prepared for driving, by stripping them of their bark, and cutting down the knots, so that the friction, in driving, may be reduced as much as possible. The head of the piles is encircled by a strong iron hoop, to prevent the ram from splitting it; and the foot is fitted into a wrought iron socket, which is made of a suitable form to penetrate the soil.

The number of piles required, will be regulated by the weight of the structure. An allowance of 1000 pounds on each square inch will insure safety. The least distance apart, at which the piles can be driven with ease, is about 2½ feet between their centres. If they are more crowded

than this, they will force each other up, as they are successively driven. When this is found to take place, the driving should be commenced at the centre of the area, and the butt end of the pile should be taken for the foot, or part driven into the soil.

After the piles are all driven, their heads are sawed off to a level, to receive a grillage and platform for the foundation. A large beam, termed a *capping*, is first placed on the heads of the outside row, to which it is fastened by means of wooden pins or *tree-nails* driven into an auger hole, made through the cap into the head of each pile. After the cap is fitted, longitudinal beams, termed *string pieces*, are laid lengthwise on the heads of each row, and rest at each extremity on the cap, to which they are fastened by a dove tail joint and a wooden pin. Another series of beams, termed *cross pieces*, are laid crosswise on the string pieces, over the heads of each row of piles. The cross and string pieces are connected by a notch cut into each, so that, when put together, their upper surfaces may be on the same level, and they are fastened to the heads of the piles in the same manner as the capping. The extremities of the cross pieces rest on the capping, and are connected with it, like the string pieces.

The platform is made of thick planks which is laid over the grillage, with the extremity of each plank resting on the capping, to which, and to the string and cross pieces, the planks are fastened by nails.

The capping is usually thicker than the cross and string pieces by the thickness of the plank; when this is the case, a rabate, about 4 inches wide must be made on the inner edge of the capping, to receive the ends of the planks.

An objection is made to the platform as a bed for the foundation, owing to the want of adhesion between wood and mortar; from which, if any unequal settling should take place, it would expose the foundations to slide off the platform. To obviate this, it has been proposed to replace the grillage and platform by a layer of beton resting partly on the heads

of the piles, and partly on the soil between them. This means would furnish a solid and stable bed for the masonry of the foundations, devoid of the objections made to the one of timber.

To counteract any tendency to sliding, the platform may be inclined if there is a lateral pressure, as in the case, for example, of the abutments of an arch.

In soils of alluvial formation, it is common to meet with a stratum of clay on the surface, underlaid with soft mud, in which case, the driving of short piles would be injurious, as the tenacity of the stratum of clay would be destroyed by the operation. It would be better not to disturb the upper stratum in this case, but to give it as much firmness as possible, by ramming it with a heavy beetle, or by submitting it to a heavy pressure.

Sand has also been used, with complete success, to form a bed for the foundations in a very compressible soil. For this purpose a trench is (Fig. 11,) excavated, from 4 to 6 feet in depth; the trench is filled with sand to the depth of 3 or 4 feet, the sand being spread in layers of about 9 inches, and each layer being firmly settled by a heavy beetle, before laying the next. If water should make rapidly in the trench, it would not be practicable to pack the sand in layers. Instead, therefore, of opening a trench, holes about 6 feet deep, and 6 inches in diameter, (Fig. 12,) should be made, by means of a short pile, as close together as practicable; when the pile is withdrawn from the hole, it is immediately filled with sand. To cause the sand to pack firmly, it should be slightly moistened before placing it in the holes, or trench.

The sand when used in this way, as a bed for foundations, appears to act by transmitting the pressure laid on it, not only over the bottom of the trench, but over the sides, so that, unless lateral yielding takes place, a very great resistance is offered to the pressure, and great stability is secured.

In laying foundations in water, two difficulties have to be overcome, both of which require great resources and care

on the part of the engineer. The first consists in the means to be used in preparing the bed of the foundation; and the second, in securing the bed from the action of the water, to insure the safety of the foundations. The last is, generally, the more difficult problem of the two, for a current of water will gradually wear away, not only every variety of loose soils, but also the more tender rocks, or those of a loose texture belonging to the calcareous and argillaceous classes, particularly if stratified, as well as most varieties of sand stone.

To prepare the bed of a foundation in stagnant water, the only difficulty that presents itself is to remove the water from the area on which the structure is to rest. If the depth of water is not over 4 feet, this is done by surrounding the area with an ordinary water-tight dam of clay, or of some other binding earth. For this purpose, a shallow trench is formed around the area, by removing the soft or loose stratum on the bottom; the foundation of the dam is commenced by filling this trench with the clay, and the dam is completed by spreading successive layers of clay about one foot thick and pressing each layer as it is spread, to render it more compact. When the dam is completed, the water is pumped out from the enclosed area, and the bed for the foundation is prepared as on dry land.

When the depth of stagnant water is over 4 feet, and in running water, of any depth, the ordinary dam must be replaced by the coffer-dam. This construction consists of two rows of plank, termed *sheeting piles*, driven into the soil vertically, forming thus a coffer work, between which, clay or binding earth is filled in, to form a water-tight dam to exclude the water from the area enclosed.

To construct the coffer-dam, (Fig. 13,) a row of ordinary piles is driven around the area to be enclosed, about 4 feet from each other. These piles should take a firm hold of the soil, and, for this purpose, should penetrate from 4 to 6 feet below the bottom. They are connected firmly at top by a string course, formed of stout pieces of scantling, which are

laid horizontally against the row of piles, at least one foot above the surface of the water, and are bolted to each pile. This string course is laid on the side of the row next to the area enclosed.

A second row of piles is then driven parallel to the first, in a similar manner, leaving a distance between the rows, equal to the desired thickness of the dam. This thickness for all depths under 10 feet, should be 10 feet, and for all depths over this, one foot is added to the thickness for every additional depth of three feet. For small depths the dam will have a surplus of strength, but as the top of it is commonly used as a scaffolding or bridge, it will be well to give it this prescribed thickness for that purpose. The second row is connected at top by a string course, placed like that of the first row, but on the off side from the area.

A second string course, but of smaller scantling than the first, is bolted to the piles of each row on the opposite side of the first two. These are intended as supports for the tops of the sheeting piles to rest against, after they are driven.

When this frame work is completed, the sheeting piles are driven within the space between the rows of piles; one row of the sheeting piles resting against the string course of the inner row of piles, the other, against that of the outer course. The sheeting piles are about 9 inches broad, and 3 or 4 inches thick. They should penetrate 3 or 4 feet below the bottom, to take a firm hold on the soil. After the sheeting piles are driven, they are fastened to the string courses, against which they rest, by another string course of thick plank, placed opposite to the first, and fastened to it by bolts or spikes passing through it and the sheeting piles.

Cross pieces of timber are next laid resting on the string courses, to which they are fastened by a notch cut into each piece, and a nail or pin of wood. These cross pieces are placed about 3 or 4 feet apart, according to the dimensions of the scantling used for them. Their object is to connect the two rows of piles together, to prevent them from spreading

outwards, when the earth of the dam is put between the sheeting piles, and, also, to serve as joists for the scaffolding, or bridging, over the dam.

After the coffer work is, in this way, completed, the soft mud, or loose soil, is scooped out from the bottom between the rows of sheeting piles, for the purpose of leaving a compact stratum for the earth of the dam, termed the *puddling*, to rest on. The best puddling to form a water tight mass, is a mixture of pure clay and sand. The puddling is spread in successive layers about one foot thick; each layer is pressed compactly before spreading the succeeding one, care being taken at the same time, to agitate the water as little as practicable.

When the puddling is finished, the area, enclosed by the coffer-dam, is freed from water by pumps, or in any other way most convenient.

The coffer-dam cannot be used with economy on a sandy bottom if the depth of water is above 5 feet; for the exterior water, by its pressure, will, in most cases, force its way under the puddling, so soon as the interior is freed from water. If the bottom is of ordinary soil, or of soft clay, the same inconveniences will be found in depths over 10 feet. In these cases, therefore, there will be no other remedy, than to scoop out all the soft mud, or loose soil, from the enclosed area, and to replace it with a layer of clay from 3 to 6 feet thick, which must be compactly pressed. A flooring of thick plank must next be laid over this artificial bottom, and be confined, by loading it with loose stone, or in any other way, to resist the pressure of the exterior water under it, when the water is pumped from the area. A layer of beton would be still more effectual than the method here explained, and would form a better bed for the foundations.

When the coffer-dam cannot be used with economy, the *floating caisson* must be resorted to. This is a large box, the bottom of which is flat, (Fig. 14,) being made of heavy scantling laid side by side, and firmly connected together, and

serves as a bed for the foundation. The sides of the box are vertical, and formed of a frame work of upright timbers which are inserted into a capping piece, which receives the ends, also of the scantling forming the bottom; the uprights are covered on the outside by thick plank nailed to them. The seams are caulked to render the caisson water-tight. The sides are not permanently attached to the bottom, but are arranged so that they can be detached from it when the masonry is completed. This arrangement of the sides, is effected in the following manner. Strong hooks, of wrought iron, are fixed to the bottom of the caisson at the sides of the capping piece, corresponding to the points where the uprights of the sides are inserted into this piece. Cross pieces of strong scantling are laid across the top of the caisson, resting on the opposite uprights, upon which they are notched. These cross pieces project from 6 to 12 inches beyond the sides, and the projecting parts are each perforated by an auger hole, large enough to receive a bolt of 2 inches in diameter. The object of these cross pieces is twofold; the first is to *stay*, or *buttress*, the sides of the caisson at top against the exterior pressure of the water; and the second is to serve as a point of support for a long bolt, or rod of iron, which has an eye at the lower end, into which the hook on the capping piece is inserted, with a screw at top, to which a nut, or female screw, is fitted, and which, resting on the cross pieces as a point of support, draws the bolt tight, and, in that way, attaches the sides and bottom of the caisson firmly together.

A bed is prepared to receive the bottom of the caisson, either by levelling the soil on which the structure is to rest, if it be of a suitable character to receive directly the foundation; or else large piles are driven through the upper compressible strata of the soil to the firm strata beneath, and the heads of these piles, are sawed off on a level to receive the bottom of the caisson. In fresh water, where wood is not attacked by the worm, the heads of the piles may be sawed off at any

point above the bottom, provided that the bottom of the caisson is always covered with water, to secure the wood from decay. In salt water, the heads of the piles should not project more than one foot above the bottom, as, at this distance, the wood will still be secure from the attacks of the worm.

To settle the caisson on its bed, it is floated to and moored over it, and the masonry of the structure is commenced and carried up, until the weight grounds the caisson. But, as whatever precautions may have been taken to form a level bed, it may not be perfectly so, the caisson should be so contrived, that it can be grounded, and afterwards raised, and any requisite change be made in the level. To effect this, a small sliding gate should be placed in the side of the caisson, for the purpose of filling it with water at pleasure. By means of this gate, the caisson can be grounded, and, by closing the gate, and pumping out the water, it can again be set afloat.

After the caisson is settled on its bed, and the masonry of the structure is raised above the surface of the water, the sides are detached, by first unscrewing the nuts and detaching the rods, and then taking off the top cross pieces. By filling the caisson with water, this operation of detaching the sides, can be more easily performed.

To adjust the piles before they are driven, and to prevent them from spreading outwards, by the operation of driving, a strong grating of heavy timber, (Fig. 15,) formed by notching cross and longitudinal pieces, on each other, and fastening them firmly together, may be resorted to. This grating is arranged in a similar manner to a grillage, only the square compartments, between the cross and string pieces, are larger, so that they may enclose an area for 4 or 9 piles; and, instead of a single row of cross pieces, it would be better to form the grating with a double row, one at top, the other at the bottom, embracing the string pieces, on which they are notched.

The grating may be fixed in its position at any depth un-

der water, by a few provisional piles, to which it can be attached.

Another method of laying foundations in deep water is to enclose the area, on which the structure is to rest, by a strong coffer-work, (Fig. 16,) and to fill this coffer-work with beton to a level within one or two feet of the surface of the water, and then to commence the masonry on this mass, after it has become firm, which will require from 15 days to several months, according to the quality of the beton. The bottom of the area enclosed should be scooped out to a depth of 6 feet, if practicable, before the beton is thrown in.

In a recent publication on mortar by *General* Treussart, of the *French Corps of Engineers*, the following method of securing a firm bed for foundations, in any depth of water, is proposed by the author. The area on which the structure is to rest, is first enclosed by strong sheeting piles, (Fig. 17,) driven sufficiently deep, to take a firm hold on the soil. The bottom, within this area, is next scooped out to a depth of 6 feet, and the soil removed is replaced by a mass of beton of the same thickness. While the mass of beton is still green, a second row of sheeting piles is driven into it, about 6 inches, leaving an interval of 5 feet between it and the first row. This interval is then filled by a compact puddling, care being previously taken to secure the rows of sheeting piles from yielding laterally. A water-tight dam is thus formed, and the water is pumped from the enclosed area. If, from the permeable nature of the soil at the bottom, it is feared, that the pressure of the exterior water, on the under side of the mass of beton, might throw it up, then it would be necessary to lay a provisional weight on this mass, before the water is pumped out; this weight being gradually removed, as the structure advances.

The immersion of beton, for foundations in water, requires great care. The best plan seems to be, to use a bucket with four sides, the top being wider than the bottom; it is provided with an ordinary handle, to which a small rope is

attached, for the purpose of raising and lowering it in the water. A cord is attached to the bottom of the bucket, to upset it and throw out its contents. The beton, broken into blocks of about half a cubic foot, is placed in the bucket, lowered near the spot it is to occupy, and is thrown out. By this means, it is deposited in the mass in a compact state, which is essential to the firmness of the mass; for were the beton thrown into the water at the surface, the greater part of the lime would be separated from the other elements, before it reached the bottom.

To prevent voids in the different layers, each one should be firmly pressed by a ram, whilst the beton is still green, and a fresh layer must not be laid, until the other has partially set.

Another precaution is also necessary to form a perfect union between the layers. Whatever pains may be taken in lowering the beton, some of the lime will wash, and remain suspended in the water; this will, finally, settle into a thin cream on the surface of the layer, and would prevent a union between it and the succeeding one, if not removed. To effect this, when there is a current of water, two holes may be made in the coffer-work just below the surface of the water, to form, when left open, a current through the enclosed area. By agitating the water within the area, the lime held in suspension, will be gradually carried off through the holes by the current. Or if this plan cannot be adopted, the lime, after it has settled on the surface of the layer, may be carefully swept into a corner by means of a broom, and be taken out by a scoop.

Where the area, occupied by a structure, is very considerable, and the depth of water great, the methods which have thus far been explained, cannot be used. In such cases, a solid bed is made for the structure, by forming an artificial island of loose heavy blocks of stone, which are spread over the area, and receive a batter of from one perpendicular to one base, to one perpendicular and six base, according to the expo-

sure of the bed to the effects of waves. This bed is raised several feet above the surface of the water, according to the nature of the structure, and the foundation is commenced upon it.

It is important to observe, that, where such heavy masses are laid upon an untried soil, the structure should not be commenced before the bed appears entirely to have settled, nor then, if there be any danger of further settling taking place from the additional weight of the structure. Should any doubts arise on this point, the bed should be loaded with a provisional weight, somewhat greater than that of the contemplated structure, and this weight may be gradually removed, if composed of other materials than those required for the structure, as the work progresses.

A very striking case of this character, occurred in our own works at the fort, named *Castle Calhoun*, at the entrance of Hampton Roads, Virginia. This fort is building on an artificial bed of the character just described, which was laid on a bar of compact sand, and, as there was every reason to suppose its depth very great, considered incompressible. After the bed was finished, and the structure commenced, a very great settling was observed, which increased so rapidly, that the work had to be suspended, and resort to be had to a provisional weight before proceeding farther. This has produced the expected result, and a final term has taken place in the settling; the cause of which, in the first place, appears to have been owing to the sand bar, resting on a stratum of indefinite depth, formed of a soft mud, which is the common formation along our southern sea board.

To give perfect security to the foundations in running water, the soil around the bed must be protected to some extent from the action of the current. The most ordinary method of effecting this, is to form what is termed an *enrockment* around the bed, by throwing in loose masses of broken stone of sufficient size to resist the force of the current. This method will give all required security, where the soil is not of a shift-

ing character, like sand and gravel. To secure a soil of this nature, it will, in some cases, be necessary to scoop out the bottom around the bed to a depth of from 3 to 6 feet, and to fill this excavated part with beton, the surface of which may be protected from the wear arising from the action of the pebbles carried over it by the current, by covering it with broad flat flagging stones.

When the bottom is composed of soft mud to any great depth, it may be protected by enclosing the area with sheeting piles, to prevent a lateral spread, and then filling in the enclosed space with fragments of loose stone. If the mud is very soft, it would be advisable, in the first place, to cover the area with a grillage, or with a layer of brush wood laid compactly, to serve as a bed for the loose stone, and thus form a more stable and solid mass.

Sheeting piles are often used to enclose an area around a foundation resting on piles. This, however, is a very inadequate method, for the sheeting piles are exposed to the same danger as the piles. The only use to which sheeting piles can be applied in securing the bed of a foundation, is in preventing the soil on which it rests from yielding in a lateral direction, as, for example, in quay walls, where there is an outward thrust from the earth, resting against the back of the wall, which might throw the wall over, were the soil around the foundations, on the exterior of the wall, to yield in the slightest degree laterally.

Construction of Masonry. Having given the most essential principles and details for the security of foundations, the next step in a natural order, will be to explain the construction of the masonry of the different parts of a structure.

Foundations. As the object of the foundations is to give greater stability to the structure by diffusing its weight over a greater surface, their breadth, or *spread*, should be proportioned both to the weight of the structure and to the resistance offered by the subsoil. In a perfectly unyielding soil, like hard rock, there would be no increase of stability by

augmenting the base of the structure beyond what would be strictly necessary to its stability in a lateral direction, whereas in a very compressible soil, like soft mud, it would be necessary to make the base of the foundation very broad, so that by diffusing the weight over a great surface, the subsoil may offer sufficient resistance, and any unequal settling be obviated.

The thickness of the foundation will depend on the spread; it is customary to make it from 3 to 6 feet.

The base of the foundation is made broader than the top from motives of economy. This diminution of the mass, (Fig. 18,) is made either in steps, termed *offsets*, or else by giving a uniform batter from the base to the top. The latter method is now generally used; it presents equal stability with the former with a less mass.

When the foundation has to resist only a vertical pressure, an equal batter is given to it on each side; but if it has to resist also a lateral effort, the spread should be greater on the side opposed to this effort, in order to resist its tendency, which would be to cause a rotation on that side.

The bottom course of the foundations is usually formed of the largest sized blocks, roughly dressed with the hammer; but if the bed is compressible, or the surfaces of the blocks are undulating, it is preferable to use blocks of a small size for the bottom course; because these small blocks can be firmly settled by means of a heavy beetle, into close contact with the bed, which could not be done with large sized blocks, particularly if their under surface were not perfectly plane. The next course above the bottom one should be of large blocks, to connect in a firm manner the smaller blocks of the bottom course, and to diffuse the weight more uniformly over them. When a foundation is laid for a structure resting on isolated supports, like the pillars, or columns of an edifice, an *inverted* or *counter-arch*, (Fig. 19,) should connect the top course of the foundation under the base of each isolated support, so that the pressure on any two adjacent ones may be distributed

over the bed of the foundation in the interval between them. This precaution is obviously only useful in compressible soils. In incompressible soils it would be only necessary to carry up the courses immediately below each support, with great care, to present a stable bed for the base of the support.

The counter arch is also used in compressible soils when an upward pressure on the bottom of the foundations is to be counteracted; as, for example, at the bottom of structures in, or near water, where the water, by finding its way under the bottom, and establishing a communication with a level or head of water considerably higher than the bottom, would cause a very great upward pressure.

The angles of the foundations should be formed of the most massive blocks. The courses should be carried up uniformly throughout the foundation, to prevent unequal settling in the mass.

Hydraulic mortar should be used for the foundations, and the upper courses of the structure should not be commenced until the mortar has partially set throughout the entire foundation.

The stones of the top course of the foundation should be sufficiently large to allow the bottom course of the upper part of the structure to rest on the exterior stones of the top course. The courses of the upper structure should be carried up uniformly throughout, to prevent unequal settling; this precaution is particularly necessary in cases where the facing, backing, and filling of a wall, are not formed of the same kind of masonry. The lower courses should be laid with hydraulic mortar to at least 3 feet above the surface of the ground. If the upper structure has to sustain a heavy weight independently of its own mass, it would be advisable to allow the mortar to set before this weight is laid on.

Walls of ordinary dimensions, in which the back and face are exposed to the action of the atmosphere, may be laid with common mortar. But heavy walls, and those which have to sustain a head of water, or a terrace, must be laid with hy-

draulic mortar. Besides the usual mortar bed for the stone grout should be poured over each course; particularly in rubble work walls, which require to be water-tight, as those used for lock chambers, terrace walls, &c. The grout, if poured in slowly and carefully, will fill up all the interstices which might be left between the stone and the mortar bed.

Heavy walls requiring great solidity have, in some cases, been laid with grout alone, the backing and facing of each course being first laid in full mortar, and the filling being formed of dry stone closely packed, and then grouted. This method has not been found to answer the proposed end; for, in the successive drenchings of the stone with the grout, the lime and sand separate, and form distinct layers, and the grout, after it has set, is found to be very porous and weak.

Although objectionable for stone walls, grout may be used with advantage for heavy brick walls requiring great solidity, as the dry brick will absorb the water rapidly, and will prevent the disunion of the lime and sand. The exterior courses of the wall should, in this case, be laid with very thin tempered mortar, and the grout be not poured in until the exterior joints are sufficiently firm to be water-tight.

Beton has frequently been used as a filling for walls requiring to be water-tight; it presents however no advantages over walls of cut or rubble stone laid in hydraulic mortar, with the additional precaution of grouting each course; and it has the disadvantage of requiring great care in the construction to prevent any unequal settling in the backing, facing, and filling.

In walls where the materials are not of the same character throughout the breadth of the course, it will be very difficult to prevent unequal settling in the different parts of the mass. The only means by which it can be avoided, consist in packing the stone of that part of the course where most mortar is used as closely as practicable, and in carrying up the courses uniformly and slowly throughout, so that the settling may be

equable, and the mortar may partially set, before it has to bear a great weight from the superior courses.

The unequal settling caused by the inequality of weight of the different parts of a mass of masonry, is one of the chief causes of the ruin of structures; for it produces cracks, and a disjunction of the parts, from which lateral pressures arise against some portions, which, in the original plan, were not calculated to sustain any but a vertical pressure; and those portions, not having sufficient strength to withstand this pressure, give way, and cause the destruction of the rest. It is not an easy problem, in all cases, to provide against accidents of this kind. All that can be done is either to build up the heaviest masses first, and allow them to gain their final settling before the others are raised, with which they are to be connected; or else, if the character of the structure admits of it, the whole may be carried up together, leaving the lighter and heavier masses in juxtaposition simply, without any bond between them, so that they may settle independently.

The surface of a wall laid with common mortar, which is exposed to the weather, should have the exterior part of the joint, to the depth of several lines, filled with hydraulic mortar. This operation, which is termed *pointing*, consists in scraping out the common mortar to the requisite depth, and filling the void with hydraulic mortar. Before the pointing is put in, the void should be well cleansed with a dry brush, and afterwards thoroughly moistened.

To protect soft stones from the action of the weather, and the back of terrace walls laid with common mortar from the water which filters through the earth, a thin plastering of hydraulic mortar is laid over the surface to be thus protected. This is termed *flash-pointing*. The same precautions should be used for it as for ordinary pointing.

The pointing should be done as the work progresses; and the flash-pointing of the back of terrace walls should moreover be covered with earth as soon as laid; in order that the

hydraulic mortar, of which the pointing is made, may have the benefit of the moisture in the recent work and in the earth, which will assist its setting, and forming a better union with the body of the masonry than would take place were the pointing done after the work has dried.

The mortar used for pointing should be composed of the finest pure siliceous pit sand of an angular grain, and of the best hydraulic lime and cement in equal parts, or in such other proportions as experiments on the ingredients may indicate. The ingredients should be mixed with great care, and the mortar be used at as hard a temper as it can be conveniently worked with the trowel. The following proportions for pointing have been used with complete success in the public works at Fort Adams,

Sand (in measure,)	3	parts.
Rosendale Cement, ...	1.60	"
Water, ...	0.60	"

Masonry of Arches. Cut stone, and brick, are the only proper materials for heavy arches, the strength of which mainly depends upon the care bestowed upon the fit and bond of the blocks which form the different courses.

The arches which are in most common use for structures, are the simple *cylindrical arch*, and the *groined arch*. The blocks, (Fig. 20,) which form the courses of an arch are termed the *arch stones*, or *voussoirs*. One series of the joints of the voussoirs are perpendicular to the curve of the arch, and they are continued the entire length of the arch without interruption; by this arrangement the voussoirs between any two of these joints have a uniform thickness throughout. Each course of voussoirs between any two of these joints is termed a *string course*. The other series of joints are in the same direction as the curvature of the arch, or perpendicular to the joints of the string courses; but, instead of being continued, they are so arranged that the blocks of the string courses shall break joints. The voussoirs contained between any two of this last series of joints, form what is termed a *ring course*.

By this arrangement of the joints of the string and ring courses, if any settling takes place in the string courses, it will be uniform throughout; whilst the bond between the string courses will counteract any tendency towards their separation.

The under surface of the arch (Fig. 20) is termed the *intrados* or the *soffit*; the upper surface is termed the *extrados* or the *back*. The line in which the arch joins its lateral supports, termed *piers* if they are between two arches, and *abutments* if at the extremity of an arch, is denominated the *springing line*.

The thickness of the arch stones between the joints of the string courses is generally uniform; the thickness between the soffit and back is also usually uniform; and the length between the joints of the ring courses will depend on the strength of the stone and its breadth and thickness.

In laying the arch stones, every precaution must be taken to have the planes of the joints perpendicular to the surface of the soffit; not only should the exact angle which the joints of the string courses make with either a vertical or a horizontal line be marked on each stone, but the position of each joint should also be marked on the timber centre by which the arch stones are supported.

The stones should be laid in very thin tempered mortar, to produce as close a fit as practicable. The *key stone* is sometimes laid dry, and is even, in some cases, driven into its place by a wooden maul; this, however, is objectionable, as it might produce an unequal pressure on the surface of the stone, and cause either the key stone or those in contact with it to crack.

When all the arch stones are laid, the joints at the back should be examined, and if any are found open they should be filled with pieces of strong slate firmly driven in, and then be grouted.

The back of the arch is generally closed in by a mass of rubble masonry, termed a *capping*, (Fig. 20,) which is finish-

ed on top with two slopes, inclining downwards from the key stone, like the roof of a house. As the object of the capping is to render the arch water-tight, the rubble stone used should consist of small flat fragments, such as slate stone, packed very close in hydraulic mortar, and well grouted. The top of the capping is finished off with a layer of hydraulic mortar, like that used for flash pointing, laid in one uniform bed of an inch or two in thickness.

The point of junction of the arch and its pier, or abutment, requires particular attention; especially, in what are termed *segment arches*, (Fig. 20,) or those formed of an arc of a circle less than a semi-circle. The stone on which the lowest voussoir of the segment arch rests is termed an *askew back*, it should form a part of the face of the pier, or abutment, below the springing line, so that the stone itself may not be liable to be crushed by the pressure of the arch on it; which would be the case were the joint of the top course of the pier at the springing line. The askew back should moreover be very firmly connected with the other stones of the top course of the pier to prevent its being thrust back on its bed by the pressure of the arch.

Light brick arches require no other particular care than to have the joints of the string courses perpendicular to the surface of the soffit. The bricks are usually laid on an end on the centre.

Heavy brick arches over 2 feet in thickness, if the curvature of the arch is considerable, require great care in the arrangement of the bond, in order to procure the greatest solidity, by placing the greatest number of bricks in the arch; and the greatest strength, by the connection of the different courses.

Were the bricks so laid that the joints between each string course were continuous from the soffit to the back of the arch, however close the joints might be at the soffit, they would be very open at the back; and the strength of the arch would therefore depend on that of the mortar in the joints,

unless the precaution were taken to fill in the joints with fragments of slate, closely packed as the successive courses of brick are laid from the soffit to the back, an operation which would present considerable practical difficulty in preserving the proper direction of the joints. On the other hand, were the bricks laid in successive courses over the soffit, or as it is termed in *shells*, the greatest possible number of brick would then be laid in the arch, but as there would be no other union between the successive shells than the mortar between them, it is to be apprehended that they might easily separate, should any motion take place in the arch, and the whole mass would therefore offer but little comparative resistance.

To unite, therefore, the advantages of the two methods, the entire arch (Fig. 21) should be divided into several portions, by joints running entirely through from the soffit to the back, the brick being laid in these successive portions alternately in shells, and in blocks with joints running entirely through the arch from the soffit to the back. Any bond may be adopted for the portions laid in shells. If the arch is not over three feet in thickness, a very solid mass can be obtained by dividing the thickness into two equal shells. In the arrangement here explained, the blocks, in which the joints run entirely through, should not consist of more than three or four bricks in thickness estimated along the curve of the soffit.

The bricks which form the key of the arch require to be laid with great care. The first course on the soffit may be formed (Fig. 22) of a thickness of three bricks, laid on their ends in very thin tempered mortar, and well wedged in, if necessary, with pieces of strong slate. The next course should be formed of five bricks, laid also on an end, and forming continuous joints with those below them. This course can be laid in grout; for, by dividing the length of the course into several compartments, separated by a single row of bricks laid in mortar, the grout may be first poured

into these compartments, and the bricks be set in it, and the joints be then filled with pieces of slate. The third and following courses are laid in a similar manner; increasing the thickness of each successive course by two bricks.

The advantage of partly filling the compartments beforehand with grout, and setting the bricks, and the pieces of slate in the joints in it, is obvious.

Influence of Season on Masonry. It may be laid down as a maxim in building, that mortar which is exposed to the action of frost before it has set, will be so much damaged as to impair entirely its properties. This fact places in a stronger light what has already been laid down, on the necessity of laying the foundations and the structure resting on them in hydraulic mortar, to a height of at least 3 feet above the ground; for, although the mortar of the foundations might be protected from the action of the frost by the earth around them, the parts immediately above would be exposed to it, and as those parts attract the moisture from the ground, the mortar if of common lime, would not set in time to prevent the action of the frosts of winter.

In heavy walls the mortar in the interior will usually be secured from the action of the frost; and masonry of this charactor may be carried on until freezing weather commences.

During the heats of summer, the mortar is injured by a too rapid dessication. To prevent this the stone, or brick, should be thoroughly moistened before being laid; and afterwards, if the weather is very hot, the masonry should be kept wet until the mortar gives indications of setting. The top course should always be well moistened by the workmen on quitting their work for any short period during very warm weather.

The effects produced by a high or low temperature on mortar in a green state are similar. In the one case the freezing of the water prevents a union between the particles of the lime and sand; and in the other the same arises from the water being rapidly evaporated. In both cases the mortar when it has set is very weak and pulverulent.

CARPENTRY.

Carpentry is the art of arranging beams of timber for the various purposes to which they are applied in structures.

The term *frame* is applied to any combination of beams firmly connected with each other.

The frame work of a structure may be supported either by suspending it from fixed points above it, or by resting it on fixed points below it. In both cases the arrangement must be such as to present a state of stable equilibrium. When the frame is suspended from fixed points above it, this state of equilibrium will exist, whatever may be the degree of flexibility of the figure of the frame; but when the frame is supported from beneath, the arrangement of the parts of the frame, in order that this state shall exist, must present a figure of an invariable form.

As the frames of structures are generally supported from beneath, the first point to be attended to in their arrangement is to combine the pieces to obtain a figure presenting an invariable form. This is effected by making such a disposition of the principal beams of the frame that the pressures, thrown on its different points, shall be transmitted in right lines, parallel to the fibres of these pieces, directly to the points of support. By this disposition the pressure will be thrown directly on the fixed points, and will have no tendency to change the figure of the frame by an action on those beams which do not immediately rest on these points. As an arrangement of this nature is not always practicable, it will be necessary, in some cases, to adopt a disposition in which counteracting pressures may come into play; by placing the beams in such positions that a pressure whose line of direction does not pass through a fixed point, will be destroyed by an equal one in an opposite direction.

The pressure on each beam in the frame, as well as the line of its direction, can be determined by the laws of statics; and the size of the beam must be so regulated that the effects of the pressure shall not impair its elastic force.

The points of junction of the beams, which are termed the *joints*, must moreover be firmly connected by proper artificial means to prevent any yielding at those points.

The beams composing the frame work of a structure may be either straight or curved. The positions of the straight beams may be either horizontal, vertical, or inclined; and they may rest on one or more points of the support. As the directions of the pressures may be either perpendicular, parallel, or oblique to the fibres of the beams, it will be necessary, in the first place, to determine their numerical values in the different cases here assumed.

The algebraical expressions relative to the flexure and rupture of beams, for the most simple cases, have already been given under the head of the *Strength of Materials*. The following expressions will apply to the laws of rupture in the cases that most usually occur in frame work.

When a beam (Fig. 23) is confined at one end, and is submitted to a strain, arising from a weight, represented by W, at the other, and from a uniform weight, represented by w, on each unit of its length, the relations between the dimensions of the beam and the weights are shown in the annexed expression,

$$R = \frac{6Wl + 3wl^2}{bd^2} \quad . \; . \; . \; . \; . \; . \quad (A)$$

in which b is the breadth, d the depth, and l the length of the beam.

When a beam is supported at its two ends, and is submitted to a similar strain, the weight W being applied at its middle point, the relations between the weights and the dimensions of the beam are as follows,

$$R = \frac{6Wl + 3wl^2}{4bd^2} \quad . \; . \; . \; . \; . \; . \quad (B)$$

in which the distance between the points of support is represented by l; and by b and d, the breadth and depth.

When a beam (Fig. 24) rests on two props, and a weight W, is applied at any point between them, the relations are expressed by

$$R = \frac{6W(l^2 - 4a^2)}{4lbd^2} \quad . \quad . \quad . \quad (C)$$

in which a represents the distance between the middle point of the beam and the point at which the weight is applied, and b, d and l the same as in the preceding cases.

When a beam, (Fig. 25,) confined at one end, rests on a point of support at the other, and is submitted to a strain from a weight applied at the middle point between the supports, the relations are expressed by

$$R = \frac{9Wl}{8bd^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (D)$$

and the prssure on the support under the unconfined end will be expressed by

$$\frac{5}{16}W.$$

By comparing the two expressions (B) and (D), it will appear, that a beam under the circumstances represented by the latter will be stronger than in the former, in the proportion of 4 to 3.

When both ends of a beam are confined, the relations are expressed by

$$R = \frac{6Wl}{8bd^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (E)$$

and comparing this in a similar manner with expression (B), it appears, that the beam in the latter case will be twice as strong as in the former.

When a beam (Fig. 26) rests on three points of support, which are at equal distances from each other, and is submitted to a strain, arising from two unequal weights represented by W, and W′, one applied at the middle point between two of the supports, and the other at the middle point between the other two, their relations are expressed by

$$R = \frac{9(W + W')l}{16bd^2} \quad . \quad . \quad . \quad . \quad (F)$$

in which l is the distance between the points of support.

When the two weights are equal, or $W = W'$, the expression becomes

$$R = \frac{9\,Wl}{8\,bd^2} \quad . \; . \; . \; . \; . \; . \; . \quad (G)$$

an expression which is the same as (D); and which shows, what might have been inferred from the state of the beam, that the parts between the props are in the same state as if they were confined at the middle prop, and rested on supports at the other extremity.

The pressure on the middle prop in the case of the weights being unequal will be represented by

$$\frac{22}{32}(W + W'),$$

or when $W = W'$ by

$$\frac{22}{16} W,$$

which shows that this prop sustains nearly two thirds of the total pressure.

When a beam is laid on four props, at equal distances apart, and the weights applied at the middle points of the intervals are equal, the relations are expressed by,

$$R = \frac{21\,Wl}{20\,bd^2} \quad . \; . \; . \; . \; . \; . \; . \quad (H)$$

the pressures on the extreme props will be

$$\frac{7}{20} W,$$

and on the intermediate props

$$\frac{23}{20} W,$$

When a vertical beam, (Fig. 57,) whose cross section is rectangular, is submitted to a strain, arising from a weight W, which is applied at a given distance c from the axis of the beam, and it is wished that this strain shall not exceed a certain limit, the relations will be expressed by

$$R' = \frac{W\,(d + 6c)}{bd^2} \quad . \; . \; . \; . \; . \quad (I)$$

in which R′ represents a certain number that will produce a given extension of the fibres of the beam, beyond which it

would not be safe to go in practice. The value for R′ is given in the subject of the strength of materials.

When the cross section of the vertical beam is a circle, the relations are expressed by

$$R'_{\prime}=\frac{7\,W\,(r+4c)}{22\,r^{3}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (K)$$

in which r is the radius of the circle.

When a beam, (Fig. 28,) in an inclined position, is confined at its lower extremity, and is submitted to a strain, arising from the weight W, placed on its upper extremity, the relations are very nearly expressed by

$$R'=\frac{W(d\,sin.\,a+6\,l\,cos.\,a)}{bd^{2}} \quad . \quad . \quad (L)$$

in which a is the angle that the beam makes with a vertical line and l is the length of the beam. R′ in this case is determined as in the preceding.

The most ordinary arrangement of inclined pieces in a frame is that in which the lower end rests on a horizontal support, (Fig. 29,) along which it is prevented from sliding by a joint, or an iron strap; and the upper end rests against a vertical support, the pressure of the beam being applied at some intermediate point between the supports.

By examining a beam in this position, it will be seen, in the first place, that the entire pressure, arising from a weight W′ placed on any point of the beam, will be borne by the horizontal support: secondly, that a horizontal pressure will be exerted against the vertical support at the upper end of the beam, and also against the strap, or joint, at the lower end, which pressure will be equal at these points, and be represented by

$$W'\,\frac{c\,tan.\,a}{l}$$

in which a is the angle between the beam and a vertical line; and c, the distance from the point of application of the weight to the lower end of the beam.

The beam may therefore be considered as confined at the

12

point where the weight acts, and acted upon at its lower end by the two pressures,

$$W', \text{ and } W' \frac{c \tan. a}{l},$$

the one vertical, and the other horizontal; and at its upper end by the horizontal pressure

$$W' \frac{c \tan. a}{l}.$$

The expression (L) may therefore be applied to this case, for the part of the beam between the point of application of the weight and the lower end, by replacing *sin. a* in that expression, by

$$\sin. a \left(1 - \frac{c}{l}\right),$$

cos. a by

$$\cos. a \left(1 + \frac{c \tan.^2 a}{l}\right),$$

and W by

$$W' \sqrt{1 + \frac{c^2 \tan.^2 a}{l}}.$$

For the upper end, *sin. a*, *cos. a*, and W, in the same expression, would be respectively replaced by,

$$\frac{c \sin. a}{l} \quad \frac{c \sin. a \tan. a}{l}, \text{ and } W' \frac{c \tan. a}{l}.$$

The value assigned to R′ will be regulated as in the preceding cases.

The foregoing expressions comprehend all the usual cases in which straight timber is used in frame work; and it is only necessary to substitute for R and R′ their values, as given under the head of Strength of Materials, to find the weight which a beam of given dimensions will bear under any of those circumstances.

When curved beams are used for sustaining a pressure, the frame may consist simply of a straight beam bent to a proper curvature, and kept in this state, by being confined between two supports at its two ends; or else the frame may be formed of a series of curved beams.

In the first case, where a bent beam, (Fig. 30,) is confined

between two supports, the expression for the greatest weight W, which, laid on the crown of the curve, can be borne by the beam, is nearly

$$W = \frac{Ebd^3}{c^2} \cdot \sqrt{\frac{l-c}{c}}, \quad . \quad . \quad . \quad (M)$$

in which E is the co-efficient of elasticity, as determined in the tables on the Strength of Materials, l the entire length of the beam, and c the distance between the points of support.

The horizontal pressure on the points of support, occasioned by the weight W, is

$$\frac{3 \quad W^2 c^3}{64 \, Ebd^3 \, (l-c)},$$

Curved frames, or *wooden arches*, are usually formed of several thicknesses of beams laid on each other. The beams of each thickness, or *course*, abut end to end, and break joints with those above and below them; and the whole are firmly connected by iron hoops and bolts. In some cases, instead of a solid beam, formed in this way, the arch is constructed of two curved portions, parallel to each other, with an interval between them. Each of these curved portions consists of several thicknesses of beams, arranged as has just been explained; and the two portions are firmly connected with each other by means of upright and diagonal straight pieces, which prevent any flexure in the one without a corresponding flexure in the other.

A beam which is formed by uniting several thicknesses of beams is termed a *built beam*. The resistance both to flexure and rupture in built beams will depend on the manner in which the several courses of the built beam are arranged.

If the built beam, (Fig. 31,) is formed of several courses, each course consisting of a single beam of equal length and thickness, the beams of each course being simply laid on each other, and confined closely by hoops and bolts, the expressions for the resistance to rupture and flexure, in all the cases which relate to straight beams submitted to a cross strain, will be given by simply writing nbd^3, instead of bd^3,

in the expressions (1), (2), and (3), under the head of Strength of Materials; and nbd^3 for bd^3, in all the other expressions under the same head, and in the preceding expressions (A), (B), &c.: n representing the number of courses.

If the courses of the built beam (Fig. 32) are of equal thickness, but formed of several pieces breaking joints, then, instead of nbd^3, nbd^2, there must be written in the expressions referred to $(n-1)\ bd^3$, and $(n-1)\ bd^2$; in which n represents as before the number of courses.

A built beam of a more solid form (Fig. 33) can be made, by making slight rectangular notches into the top and bottom of the beams of each course, these notches being arranged to lie opposite each other, in order that a block of hard wood, termed a *key*, may be fitted into them, to prevent the different courses from slipping on each other when bent; or, in place of this arrangement, the beams of each course may be formed with indents (Fig. 34) to fit each other, which will, in the same way, counteract any tendency to slipping. A built beam of this construction, when firmly put together with iron hoops and screw bolts, will be nearly as strong as a solid beam; and the expressions for solid beams may be used in estimating its resistance to flexure, or rupture from a cross strain.

When a built beam (Fig. 35) is formed of two others, which are firmly connected by upright and diagonal pieces, the expressions before referred to may still be used, by placing $(d'^3-d''^3)$ instead of d^3, in the expressions of the flexure; and $\frac{d'^3-d''^3}{d'}$ in those relating to rupture; the letter d' in this case represents the entire depth of the built beam; and d'' the distance between the top and bottom beams.

In forming a wooden arch, it is very important that the curvature of the arch should be of such a figure as to present a stable equilibrium; and when such a relation exists between the figure of the arch and the pressure which it bears,

that there will be no tendency to a change of form, or that the arch throughout will be simply in a state of extension or compression; the figure, or curve of the arch, is termed a *curve of equilibrium.*

As the curve of equilibrium depends on the action of the pressure borne by the arch, it will present a different figure for each case. The one which answers to the most usual practical cases is the common parabola, which is the curve of equilibrium for a pressure that acts vertically, the partial pressure on any portion of the curve between vertical lines at equal distances apart, being equal.

The term *span* (Fig. 36) is applied to the horizontal distance between the two extremities of an arch, and the term *rise* to the vertical distance from the crown of the arch to the horizontal line joining its extreme points. If in the parabolic arch the following notation be adopted:

$2s$ = span;

r = rise;

x = the abscissa of the curve, reckoned from the crown;

y = the ordinate corresponding to any abscissa x;

w = the weight on any portion of the arch, corresponding to a unit of length, reckoned along the horizontal line of the span;

P = the total vertical pressure on each point of support of the arch;

Q = the horizontal pressure at each of the same points;

T = the pressure at any point of the arch in the direction of the curve at that point;

Then the following expressions will establish the relations between the lines of the parabola and the different pressures above referred to.

$$y = \frac{r}{s^2} x^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

$$P = ws, \text{ and } Q = \frac{s^2}{2r} w, \quad . \quad . \quad . \quad (2.)$$

$$T = \frac{ws^2}{2r}\sqrt{1+\frac{4r^2x^2}{s^4}}, \quad . \quad . \quad . \quad (3.)$$

The expression (1) gives the form of the curve; the expressions (2) give the horizontal and vertical pressures at the extreme points; and (3) the pressure in the direction of the curve at any point on the arch corresponding to the abscissa x.

In order to apply these expressions to practical purposes, the points of support of the arch must be sufficiently firm to resist the pressures calculated from the expressions (2); and as the pressure on any cross section of the arch may be considered as uniformly distributed over the area of the section, if this area be represented by A, the pressure on a unit of surface will be shown by

$$\frac{T}{A};$$

and in order that this pressure shall not exceed a certain limit represented by R′, which limit has been laid down under the head of Strength of Materials, there must exist the relation expressed by

$$R' = \frac{T}{A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

If, as is usually the case, the section of the arch is a rectangle, of which b is the breadth, and d the depth, the expression (4) becomes, by substituting for T and A, their values,

$$R' = \frac{ws^2}{2bdr}\sqrt{1+\frac{4r^2x^2}{s^4}}, \quad . \quad . \quad (5.)$$

which expresses the relations between the pressure and the cross section of the arch at any point corresponding to the abscissa x.

The foregoing is the most simple case of an arch used for a frame. When, besides the uniform pressure, whose action has just been explained, a parabolic arch is submitted to a pressure arising from a weight attached to any point of it, the foregoing relations become modified, and are represented as follows.

Adopting the preceding notation, and supposing (Fig. 36) a weight W to be suspended from any point N of the arch, at a distance from the crown C, represented by c, on the horizontal line of the span, then

$$y = \frac{r}{s^2}(2cx + x^2) \quad . \quad . \quad . \quad . \quad (6.)$$

will give the relations between the ordinates and abscissas for the part of the curve N M, estimating the abscissa on the horizontal line through N;

$$P' = \tfrac{1}{2}W.\frac{s+c}{s},$$

will represent the vertical pressure on the point of support M;

$$P'' = \tfrac{1}{2}W.\frac{s-c}{s},$$

that on the point M';

$$Q' = \frac{5}{64}W.\frac{5s^4 - 6s^2c^2 + c^4}{s^3r},$$

the horizontal pressure on each of the same points due to W, and

$$T = \frac{ws^2}{2r} + \frac{W}{2}.\frac{2r(s+c)(c+x)}{s^3} + \frac{5W}{64}.\frac{5s^5 - 6s^3c^2 + sc^4}{s^4r}.(7)$$

will be the value of the pressure in the direction of the curve at any point between N and M, corresponding to any abscissa x, reckoned from the point N, along the horizontal line drawn through this point.

The effect of the pressure in the direction of the curve, represented by T, which is due to the two weights w and W, is to produce a certain compression of the fibres, which compression on a unit of surface of the cross section of the arch will be found, by dividing T by the area A of the cross section multiplied by the co-efficient of elasticity E, or expressed algebraically

$$\frac{T}{EA}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

But besides this compression, which is due to that portion of the pressure acting in the direction of the curve, there is

another, which arises from the action of the weight W, whose tendency is to change the figure of the curve, by causing it to bend. To obtain therefore the expression for the total compression, the value of this last must be added to that represented by the expression (8). But as this value of the compression varies with the position of the weight W, that position of the weight must be found which will give the greatest value for this variable compression, and this greatest value must be added to that given by expression (8).

From an investigation of the different values here spoken of, it appears, that the greatest value of the compression will be when the weight is placed at a point on the crown which is rather less than two fifths of the semi-span, or $\frac{2}{5}s$, reckoned on the horizontal line of the span, from its middle point. The expression for the compression in this case is very nearly expressed by

$$0{,}531 s \frac{dW}{Ebd^3}, \quad \ldots \ldots \quad (9.)$$

when the cross section of the beam is a rectangle, of which b is the breadth, and d the depth, E representing the co-efficient of elasticity.

The total value of the greatest compression, when the cross section of the beam is a rectangle, will therefore be given by the expression

$$\frac{T}{Ebd} + 0{,}531 s \frac{dW}{Ebd^3}. \quad \ldots \quad (10.)$$

In order to express the relation between the weights applied, and the dimensions of the arch, so that this compression shall not exceed a given limit, represented by

$$\frac{R'}{E},$$

the following expression obtains,

$$\frac{R'}{E} = \frac{T}{Ebd} + 0{,}531 s \frac{dW}{Ebd^3}, \quad \ldots \quad (11.)$$

in which R′ is the limit of the weight which in practice can be borne with safety on a unit of surface as laid down under the head of Strength of Materials.

built beams, and the other letters represent the quantities as already explained.

In estimating the value of T in expression (11), the quantity c must be replaced by $\frac{2}{5}s$; and the quantity x, by

$$x = -\frac{2}{5s} + \frac{P's^2}{2'Qr},$$

as the point of maximum pressure on the curve corresponds to this value of the abscissa x: in substituting for the values of P′, and Q′, in this value of x, the quantity c must be replaced by $\frac{2}{5}s$.

It is easy to gather, from what has been said on the manner of estimating the strength of built beams, the modifications the expression (11) must undergo in each of the cases referred to. If, for example, the arch was formed of two built beams, connected by uprights and diagonal pieces, each of the built beams being formed of several courses connected either by indents or keys, the expression would become,

$$R' = \frac{T}{bd'''} + 0{,}531s\frac{d'W}{b(d'^3 - d''^3)},$$

in which d'' is the sum of the depths of the upper and lower.

It can be shown that, when the weight is applied at the points just indicated, the beam is more strongly solicited to bend than when the weight is applied at the crown in the proportion nearly of 9 to 5.

The expressions given for the strength of straight pieces suppose that the value of the pressure in each particular instance is known; but in a frame, which usually consists of several pieces placed in horizontal, vertical, and inclined positions, it will be necessary to ascertain, in the first place, from the laws of statics, the direction of the pressure on each of those pieces, and its magnitude, before those expressions can be applied to the several cases which may occur in practice.

The following applications present some of the most simple cases of frames; and they will serve to point out the course to be followed in more complicated structures.

The expressions in this subject, from (A) to (I), show the manner of estimating the pressure, arising from a weight

on a horizontal beam, supported beneath, in the most usual cases that occur in practice.

If a weight W, is suspended from the angular point (Fig. 37) of two inclined pieces, AC and BC, which rest against each other at that point, and are confined at their lower ends, the pressure in the direction of the pieces AC and BC will respectively be represented by

$$W \frac{\sin. q}{\sin. (p+q)}, \text{ and } W \frac{\sin. p}{\sin. (p+q)}. \quad . \quad . \quad . \quad . \quad (A')$$

in which p and q represent respectively the angles made by the pieces with the vertical line through C.

The tendency of the weight W will be to press the pieces together at top, and to thrust them out at the lower ends; this tendency, or the horizontal pressure, will be represented by

$$W \frac{\sin. p \sin. q}{\sin. (w+q)},$$

To apply the expressions (A'), it may be observed, that each of the pieces is simply pressed in the direction of its length by a force represented by (A'); consequently, by substituting these expressions instead of W in the expression (7), under the head Strength of Materials, the value of the least weight for beams, of given dimensions, of the form there considered will be obtained.

If a weight W is suspended from the middle point (Fig. 38) of a horizontal beam, resting on two inclined supports AB and A'B', it is necessary, in the first place, that the angles between the inclined pieces and a vertical line should both be equal, in order that the figure of the frame may be in a state of stable equilibrium. If this angle be represented by p, the expression of the pressure on each of the inclined pieces will be represented by

$$\frac{W}{2 \cos. p}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B')$$

and the force with which the lower ends tend to stretch out horizontally will be represented by

$$\frac{W}{2 \tan. p}.$$

With respect to the horizontal piece BB′, each half of it may be considered in the same state, as if the point C were confined, and a force equal to the expression (B′), acting in the direction AB, or A′B′, were applied at either of the ends. The expression (L) will therefore be applied in this case, by substituting in this expression,

$\frac{1}{2}$ for the *cos. a* ; $\frac{1}{2}$ *tan. p* for *sin. a* ; and $\frac{1}{2}$ W *cos. a* for W.

When a horizontal piece BC, (Fig. 39,) attached to a fixed point B, and supported beneath by an inclined piece AD, termed a *strut*, which rests on the fixed point A, has a weight W suspended from the point C, the following expressions wlll give the magnitude of the forces which act on the two pieces of which the frame is formed: Denote by l the distance BD, by l' that DC, and by p the angle that the strut makes with the vertical line.

Supposing the frame free to turn about the points A and B, it will readily appear that the tendency of the weight, at the point C, is to produce an upward vertical pressure at the point B, which will be represented by

$$W\frac{l'}{l};$$

the point D therefore will have to bear a pressure equal to this added to the weight W, or

$$W\frac{l+l'}{l}.$$

As the pressure on the point D is borne by the strut AD, the total pressure in the direction of the strut will be represented by

$$W\frac{l+l'}{l\,cos.\,p}, \quad . \quad . \quad . \quad . \quad (C')$$

at the same time there will arise from this pressure at D, a tension on the part BD, represented by

$$W\frac{l+l'}{l}tan.\,p, \quad .. \quad . \quad (D')$$

In order, then, that the parts BC, and AD may be sufficiently strong, AD must resist the pressure represented by (C′) ; and

the part BC an effort which is due in part to the tension on BD, represented by (D′), and to the action of the weight W on the part DC, which action tends to bend the part DC. The following expression will then give the limit of the effort on a superficial unit of the cross section of DC, when the section is rectangular,

$$R' = \frac{W}{bd^2}\left(\frac{d\,(l+l')\,tan.\,p}{l} + 6l'\right); \quad . \quad . \quad . \quad . \quad (E')$$

in which R′ represents the number given under the head of Strength of Materials, d the depth, and b the breadth of the beam.

When a horizontal piece BC (Fig. 40) rests upon an upright piece AB firmly confined at the point A, and is supported by a strut DE, and has a weight W suspended from C, the state of the pieces BC and DE, will be the same as in the last case. The upright from E to A, will evidently be compressed by the entire weight W, whilst the part BE will suffer an extension, which will be represented by

$$W\frac{l'}{l},$$

the same notation being adopted as in the last case.

The part EA is therefore in a state of compression, arising from the weight W, acting at a distance $l + l'$ from the axis, the limit of its resistance on a unit of surface when the beam is rectangular will be, from what was shown in the expression (I),

$$R' = \frac{W}{bd^2}\,(d + 6\,(l + l')), \quad . \quad . \quad . \quad . \quad (F)$$

and the same limit for the part BE, which is extended by the force $W\frac{l'}{l}$, acting at the same distance, will be,

$$R' = \frac{W}{bd^2}\,\left(\frac{dl'}{l} + 6\,(l + l')\right), \quad . \quad . \quad . \quad . \quad (G')$$

Let there be a frame (Fig. 41) the same as in the last case, in which the upright, instead of being firmly fixed at its lower extremity A′, is prevented from yielding by either of the struts AF, making an angle with the upright denoted by p.

The whole of the frame above the point A will be in the same state as in the last case. The state of the part below the point A will depend on the position of the foot F, of the strut.

If the line of direction of the weight falls within the foot, then the tendency of the weight will be to turn the whole frame around the point A', and in order that this motion may not take place, the strut AF must after a resistance in the direction FA, which is represented by

$$W\frac{l+l'}{c\,sin.\,p}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (F')$$

in which c denotes the distance AA', and the other letters the same as in the last case. This resistance, in the direction FA, is equivalent to a horizontal force represented by $W\frac{l+l'}{c}$, and to a vertical force acting upwards from A, which will be represented by

$$W\frac{l+l'}{c\,tan.\,p}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (G')$$

The strut therefore will be compressed by a force represented by the expression (F'); whilst the part AA', will be compressed by a force represented by the whole weight diminished by that represented in the expression (G'), or by

$$W\left(1-\frac{l+l'}{c\,tan.\,p}\right), \quad . \quad . \quad . \quad . \quad (H')$$

When the line of direction of the weight falls without the foot of the strut, the tendency of the weight will be to turn the frame around the point F. The strut, in this case, will be compressed by a force represented by the expression (F'); but the part AA' of the upright will be extended by a vertical force represented by

$$W\left(\frac{l+l'}{c\,tan.\,p}-1\right), \quad . \quad . \quad . \quad . \quad . \quad (I')$$

and in order then that the frame may not be overturned, the point A' of the upright must be firmly fixed.

The frame represented in (Fig. 42) consists of a horizontal beam, the extremities of which rest on the two points of support B, B', this beam being supported below by two struts which abut against the fixed points A, and A'.

A weight W, suspended from C, will throw a pressure on the four points of support; and the action of this weight on the frame will cause the horizontal beam to bend, and a compression in the direction of the struts. To ascertain the practical limits in this case the frame may be considered under two points of view. First as composed of the horizontal beam alone without struts; and second as composed of the two struts and the horizontal portion DD' alone. As each of these parts taken alone is less strong than the whole frame, it is clear that if their dimensions are so regulated as to bear the entire weight W, for a stronger reason will the two united be sufficient for the same purpose. To estimate the dimensions of the horizontal beam BB', the expression (5), under the head of Strength of Materials, must be used. As to the part ADD'A', it is evidently in the same state as the frame, (Fig. 38,) and the same expressions found in that case are also applicable to this.

When a frame of the form (Fig. 43), consisting of a horizontal beam BB', resting on two uprights, which are solidly fixed at the points A, and A', and supported from beneath by two struts DE, and D'E', is submitted to the action of a weight W suspended from the middle point C, the same reasoning may be applied as in the last case.

In order that the parts ABB'A' may be sufficiently strong, the dimensions of the horizontal beam must be estimated to support the effort of W suspended at C; and each of the uprights must be strong enough to bear a vertical effort represented by $\frac{1}{2}$W. With respect to the part AEDD'E'A', the struts, in the first case, must be strong enough to bear the pressure in the direction of their length, represented by

$$\frac{W}{2 \cos. p},$$

and this pressure in the direction of the strut will be equivalent to a vertical effort represented by $\frac{1}{2}$W, and to a horizontal effort $\frac{1}{2}$W *tan. p*, both applied at the point E of the upright.

The vertical effort is transmitted to the point A, compressing the part EA of the upright. The horizontal effort is equivalent to two others one applied at the fixed point A, which is destroyed by the resistance of that point; and the other applied at the point B, which is represented by

$$\frac{W\, c\, tan.\, p}{2(c + c')}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (K')$$

in which c and c' are respectively the distances AE, and EB.

The action of this effort (K′) is, in the first place, to produce a corresponding extension on the part BD of the horizontal beam; and, in the second place, to cause the upright to yield at the point E by bending. The upright may therefore be considered as fixed at the point E, and submitted to a vertical effort at the point A equal to $\frac{1}{2}$W, and to a horizontal effort at B equal to the expression (K′). These two efforts will produce a strain on the upright, the limit of which, when the beams is rectangular, will be expressed by

$$R' = \frac{W}{2bd^2}\left(d + \frac{6cc'\, tan.\, p}{c + c'}\right), \quad . \quad . \quad . \quad (L'.)$$

and will serve to regulate the relations between its dimensions and the weight W.

In the arrangement of every system of frame work, as one of the main objects is to procure a figure of an invariable form, such a disposition of the parts must be made, that any pressure on one part, which may have a tendency to produce a change of figure, shall be counteracted by some other part. The simplest manner of producing this effect is to combine the parts of the frame to form a series of triangular figures; for the reason, that no change can take place in these figures without the pieces forming their sides becoming shorter or longer by a force either of compression, or of extension. If therefore any of the main pieces of a frame intersect each other, forming quadrilateral figures, it will be necessary to introduce

other pieces placed in the direction of the diagonals of the quadrilaterals, for the purpose of counteracting any tendency to a change of form in these figures. An arrangement of this character, generally termed *diagonal bracing*, is used for all frame work requiring great strength and stiffness.

The pieces in a system which resist a compressing force are usually termed *struts*; those that counteract a force of extension are termed *ties*; and those which are used to add stiffness to the frame, by preventing any tendency to flexure in the other parts, are termed *braces*.

It is important that the different pieces of a frame should be as little *grain cut* as possible, that is, cut in a direction oblique to the natural fibres, otherwise their strength would be greatly impaired. Beams, the cross sections of which are the same throughout, and of a rectangular form, are, on this account, mostly used for frames. In some cases of resistance to a cross strain, it might be preferable to use a beam whose longitudinal section should be of the form of a solid of equal resistance; but as these solids present less resistance to flexure than rectangular beams having a uniform depth, equal to the greatest depth of the solid of equal resistance, and as stiffness is, in almost every practical case, of as much importance as ultimate strength, the rectangular beam of uniform depth is to be preferred.

Joints. The bearing surface of the joints should be as great as the nature of the case will permit, in order to prevent the joint from being crippled, either by the indentation, or crushing of the fibres of the parts in contact. The bearing surfaces of the joints should, moreover, be perpendicular to the directions of the pressure on them, to prevent any tendency to sliding on either of the surfaces.

In arranging the joints, the simplest forms that are most suitable to the object in view should be adopted, in order that there may be the least inconvenience in obtaining an accurate fit of the parts. The parts should not, in all cases, fit close; but allowance should be made for the settling, arising

from the shrinking of the fibres, and the new direction of the pressures which may arise from this cause. If this allowance were not made, the pieces would frequently be liable to split and give way at the joints. In cases of this character in heavy frame work, it is recommended to make the surfaces of the joints circular, as the surfaces would then continue in contact should any change take place in the position of the pieces from settling, or any other cause.

Built beams. To procure as great strength as the nature of the case will admit of, the different courses of a built beam must be connected in such a way that they will not slide on each other when submitted to a cross strain. This may be effected either by placing pieces of hard wood in notches cut in the beams, as represented in Fig. 33; or else by indenting the beams as represented in Fig. 34. The courses are then firmly connected by screw bolts, or by iron hoops, or else by a stirrup formed as in Fig. 44, which will admit of being tightened if the stirrup works loose from the shrinking of the fibres, or from any other cause.

The keys of hard wood may be either simple blocks of a rectangular form, or else double wedges, (Fig. 45,) which will admit of being driven in the notch for the purpose of bringing the surfaces in close contact. This, however, requires care in practice, as the wedges, if driven in with force, might cripple the fibres.

The position of the indents should be regulated to prevent the courses from sliding. It has been recommended in built beams formed of two courses, to make the upper course (Fig. 46) of two separate pieces, abutting against an iron bolt, termed a *king bolt*, as experiment has shown that a beam sawed across at top, to a depth nearly one third of the entire depth, having a piece of hard wood inserted into the cut of the saw, offered more resistance to a cross strain than a whole beam.

The more perfect the contact between the courses, the stronger will be the beam; but, as the joints between the courses can

never be made so close as to exclude water, it will be best, in cases where the built beam is exposed to the weather, to use keys instead of indents, and to leave sufficient space between each course for the circulation of the air, in order that moisture may not be retained long enough in the joint to cause the rot.

Scarf joints. When it is necessary to unite two beams at their ends, the simplest and strongest method consists in placing the two pieces end to end, and confining them in this position by two or four pieces (Fig. 47) bolted on two or four of the sides, as the case may require. This method is termed *fishing a beam;* it is used only for rough heavy work. The side pieces may be simply confined by screw bolts; or else they may be connected with the main pieces by keys, or indents, (Fig. 48.)

When the beam is required to be of the same thickness throughout, a joint, termed a *scarf*, is used in place of fishing. The form of the scarf will depend on the nature of the strains to which the beam is to be submitted. If this is simply a force of compression, or one of extension parallel to the fibres, the form (Fig. 49) is the most suitable; the joint being confined by an iron plate and screw bolts placed on two or four sides, as the case may demand. These plates are usually straps of wrought iron of sufficient dimensions to resist the strain to which they are submitted. Iron hoops, arranged with screws, would be, in many cases, a better arrangement than plates; as it is obvious that the best arrangement would be a hollow tube of iron within which the two ends could be closely confined, and the hoops might be combined to present nearly as much resistance as a tube.

When the beam is submitted simply to a cross strain, the scarf (Fig. 50) will be a very suitable form, as the fibres are compressed at top, and extended at the bottom. The joint is secured at bottom either by an iron plate, or by a piece of wood confined with keys and bolts.

When the beam is submitted both to a cross strain and to

a strain in the direction of the fibres, tending to pull the parts asunder, the form (Fig. 51) is the most suitable. As the fibres on a part of the surface of the joint will be compressed, and the rest extended, the depth of the indent, both at top and bottom, should be equal to two thirds the depth of the beams; the compressed parts in contact, as well as the parts extended, being one third the whole depth. As the tendency of the compression would be to detach those parts on which it acts, by causing the fibres to separate by sliding, the length of the parts along which this disunion might take place should be sufficient to resist the strain on the compressed parts. This length should be about twice the depth of the beam for oak, and about six times the depth for pine, when the joint is confined by bolts and an iron plate at bottom. If iron hoops are used, these dimensions should be doubled.

Mortise and tenon joint. This joint is used to connect two beams when the end of one rests upon the other either perpendicularly or obliquely.

In the first case (Fig. 52) a hole, termed a *mortise*, is cut into the side of one beam, and the end of the other is shaped to fit closely into it, by what is termed a *tenon*. The tenon is confined in the mortise by a wooden pin driven into an auger hole made through the sides of the mortise and tenon. It is a common practice among workmen to make the hole in the tenon nearer to the surface than that through the mortise, for the purpose of making a close joint. This method is very pernicious, for it produces a great strain on the pin, and on the side of the tenon hole, which might cause one or the other to give way, if an additional strain were to take place, arising from any motion of the two beams. Close joints are desirable, as producing an accurate fit, and bringing the bearing surfaces in juxtaposition, and when this is attained, the pin should only serve to keep the parts in their places.

In the second case a notch (Fig. 53) is cut out of the side of one beam, the surfaces of which should be perpendicular to each other, as they are the bearing surfaces for the end of the

other beam. A mortise is cut in the beam on the longest side of the notch. The other beam is shaped to fit the notch and the mortise. The joint is secured, as in the last ease, by a wooden pin, or if greater security is requisite, a screw bolt or iron strap may be used in the place of it.

Double tenon and mortise joints are very frequently used, but they present no superiority over the simple joint, and are of more difficult workmanship.

Dove tail joint. When the extremity of a horizontal beam rests on another, this joint is sometimes used to connect the two; a notch of a trapezoidal form (Fig. 54) being cut into the lower beam to receive the extremity of the upper, which is shaped to fit it. The joint is secured by a wooden pin. This form of joint is very weak, and soon works loose from the shrinking of the fibres. In all cases of this kind, whenever it is practicable, it will be best to cut a notch in each beam of a rectangular form, and to give an overlength to the top beam.

In heavy frame work the strain on the joints is very great, and every precaution should therefore be taken to prevent the surfaces in contact from being crippled, as well as any displacement of the pieces. This can only be effected by making the bearing surfaces as great as possible, and by securing the pieces by a judicious arrangement of the bolts and straps. In addition to these, it has been proposed to insert pieces of lead, or iron, between the bearing surfaces, to prevent the crippling of the fibres.

In the preceding remarks it was observed, that the best arrangement for securing the joint between two pieces abutting end to end, would be to enclose the ends in contact within an iron tube; but as it would be very difficult practically to procure a close fit of this tube and the pieces, the consequence would be that the strain would principally be felt near the ends of the tube. This view of the subject has led to the following arrangements for such cases. Two cast iron plates are accurately fitted to the sides of the beam, and are

connected at top and bottom by two cross pieces, which are screwed to the plates, or confined to them by nuts as in the ordinary cases of screw bolts. As the strain on the plates, arising from that thrown on the cross pieces, or supports, would only be in certain directions, they need not be of one solid piece but formed as in the following figures.

If a piece is to be joined to another close to a wall, the arrangement (Fig 55) will be a very suitable form to counteract the effect of a weight acting at the end of the beam.

If two beams are to be joined in the middle, the arrangement (Fig. 56) will be a suitable form to counteract a cross strain.

It should be observed, that the cross pieces on which the strain is thrown should fit closely the top and bottom of the beam, and be moreover of sufficient breadth to prevent the fibres of the beam from being crippled by this strain.

ROADS.

The general series of operations preliminary to establishing a line of internal communication, whether it be a road, a canal, or a rail-road, is the same in all cases, and consists, in the first place, of a *reconnaissance*, or examination of the country between the two points to be connected by the line, for the purpose of ascertaining the most favorable direction pointed out by the natural features of the country; and in the second place, of an accurate survey of the various lines which have been fixed upon by the reconnaissance, in order to compare their relative advantages.

Reconnaissance. In taking into view any considerable extent of country, two remarkable features immediately present themselves, which are the valleys of the water courses, and the high grounds by which these valleys are separated. Each of these features would seem, at first sight, to present an endless variety of forms, and combinations; but, upon more careful inspection, it will be found, that the more considerable valleys are the main channels, or drains, for others of

a secondary character, whilst these, in their turn, perform the same functions for others of a still smaller class, and so on, in the descending order of progression, from those immense basins which receive the waters of the largest rivers, to the scarcely distinguishable furrows of the most trifling rills. A similar order of progression will be found to hold with respect to the high grounds, in descending, by successive degrees, from those lofty chains which separate valleys of the first order, to the spurs which, proceeding from their sides, divide the secondary valleys, and throw out, in their turn, others of an inferior order which separate the tributaries of the secondary valleys, and are themselves the main stems of a still inferior order.

Two remarkable classes of lines present themselves in connection with these natural features; they are the water courses, which form the lowest lines of the valleys; and the dividing ridges, which are the highest lines of the main chains and spurs; and each of these classes possess the remarkable properties of being lines of greatest declivity of the surfaces to which they belong.

From this glance at the general configuration of any portion of the earth's surface, it will readily appear, that lines of communication admit of a division into two classes: 1. Those which connect two points of the same valley: 2. Those which connect two points separated by a dividing ridge. And as one of the principal conditions which every line of communication should satisfy, is to connect the two points by the shortest practicable route, it will also readily appear, from what has been said, that this condition will be satisfied for the first class of communications by following a direct line between the two points, since the level between the two does not admit of reduction; whereas in the second class the line must not only be as direct as practicable, between the two points, but must also pass the dividing ridge at the lowest level between them, in order to effect all possible reduction in the height between the two points and that in which the line crosses the ridge.

It is therefore a subject of importance to be able, by a simple recónnaissance, to ascertain the lowest point of a dividing ridge between any two points, since it will abridge the labor of the succeeding operations.

In the common maps of every portion of a country it will be found, that the water courses are usually laid down with sufficient accuracy to show the direction in which the valleys lie; that of the rising grounds by which they are separated; and even the approximate position of the ridges. With these approximate data the engineer is furnished with a guide to direct him to the points which present the greatest probability of a favorable result. For there exists a necessary co-relation between the water courses and ridges, as the lines of greatest declivity of the surfaces to which they belong, from which the highest and lowest points of those surfaces can be readily determined. A few familiar illustrations will serve to place this in a clear point of view, without entering into the strictly mathematical reasoning upon which it rests.

From the physical facts of water always seeking what is commonly termed its lowest level, and that by the shortest line, or the line of greatest declivity of the surface along which it flows, it follows, that the water courses mark out the lowest points of the valleys, and are also their lowest lines of greatest declivity. If then, on a map of any portion of a country it be found that the water courses all diverge from, or converge towards one point, it will indicate, without farther examination, that this point is in the first supposition the highest, and, in the second, the lowest point of that portion of country.

If two water courses flow in opposite directions from a point, it will indicate that this is the lowest point of the ridge of the rising ground by which their valleys are separated; for, from what has just been laid down, the ridge must evidently decline on both sides to this point.

If two water courses flow in the same direction and parallel to each other, it will simply indicate a general inclination of the ridge between them, in the same direction as that of

the water courses. The ridge, however, may present in its course elevations and depressions, which will be indicated by the points in which the water courses of the secondary valleys, on each side of it, intersect each other on it; and these will be the lowest points at which lines of communication, through the secondary valleys, connecting the main water courses, would cross the dividing ridge.

If two water courses flow in the same direction, and parallel to each other, and then at a certain point assume divergent directions, it will indicate that this is the lowest point of the ridge between them.

If two water courses flow in parallel but opposite directions, there is nothing to indicate the direction of the inclination of the ridge between them, if any exists; but the meeting of the water courses of the secondary valleys on the ridge, or an approach towards each other, at any point, of the two principal water courses, will indicate the points of depression in the ridge.

Survey. The surveys which succeed the reconnaissance consist of several *trial lines*, between the points, fixed upon in the reconnaissance, which are generally run with the chain and compass, and are then levelled with the spirit level, throughout their entire extent; for the purpose of determining the undulations of the ground along the lines. Besides the longitudinal levels, a series of cross levels are made, at equal distances apart, perpendicular to the directions of the trial lines, in order to show the inclination of the ground, on each side of the trial lines, for a width greater than that which the line of communication will probably occupy.

Map and Memoir. After these surveys are made with all desirable accuracy, a map, exhibiting the topographical features of the ground, and the profiles in the direction of the different levels, is carefully drawn up from the notes taken on the ground. As there are many other points, upon which accurate information is desirable in such cases, that cannot be shown on the map, it should be accompanied by a descrip-

tive memoir, in which should be set forth the character of the natural features of the country along the lines which are deemed favorable or otherwise to the construction of the road, as the nature of the soil, that of the water courses, &c. &c.

Survey of a Common Road. In laying out a common road, the engineer is less restricted in the direction of his line than in any other kind of communication, owing to the character of the conveyance used upon it; nevertheless he should confine himself, as far as economy of expenditure will permit, to the most direct line between the two points, and the one which offers the least height to be overcome. The obstacles with which he will meet to prevent this are hills, valleys, marshes, and water courses.

When a hill intervenes between the two points to be connected, the principal object to be attended to, is to give the road such a slope that, in the descent with the usual speed, there shall be no danger to the carriages from the accelerating force of gravity in the direction of the road; and this will be accomplished by not making the slope greater than what is termed *the angle of friction* for the particular kind of a road covering used, whether it be a pavement, a broken stone surface, or a gravel road. For when the slope of the road is equal to the angle of friction, the friction of the carriage wheels will be in equilibrium with the component of the force of gravity in the direction of the road, and this component will, therefore, have no tendency to increase the velocity of the carriage, which it would do were it greater than the force of friction, as the difference between the two forces would then act as an accelerating force on the carriage.

To determine the angle of friction, direct experiments have been made, by allowing carriages to descend freely on a road of variable inclination until the friction overcame the force which caused the motion, (*Note* 1); and also by the *force of traction* on a level road, or the fractional part of the weight of the carriage which, when applied to it, would be just sufficient to overcome the friction and set the carriage in motion.

The following are the results of those experiments, the load moved being one ton, or 2240 pounds.

No. 1.	*Well made pavement,* . . .	33 *lbs.*
" 2.	*Broken stone surface laid on an old flint road,*	65 "
" 3.	*Gravel road,*	147 "
" 4.	*Broken stone surface on a rough pavement bottom,*	46 "
" 5.	*Broken stone surface on a bottom of beton,*	46 "

From this it appears that the angle of friction in the first case is represented by $\frac{33}{2240}$, or $\frac{1}{68}$ nearly; and that the slope of the road should therefore not be greater than one perpendicular to sixty-eight in length, or that the height to be overcome must not be greater than one sixty-eighth of the distance between the two points measured along the road, in order that the force of friction may counteract that of gravity in the direction of the road.

A similar calculation will show that the angle of friction in the other cases will be as follows:

No. 2, . . .	1 to . . .	35 nearly.
" 3, . . .	1 " . . .	15 "
" 4 and 5, . .	1 " . . .	49 "

In laying out therefore a road, when one point is higher than another, or when it is necessary to pass a ridge at a point higher than either of the extreme points, the line followed should be direct between the two points, so long as the ascent is within the foregoing limits, according to the character of the road covering, and no other obstacles intervene which would render necessary a change of direction. If, owing to any of these causes, a change of direction should become necessary at any point, it will be made, and be continued in the new direction until the direction towards the point of arrival can be resumed. An examination of the line between the point where the original direction is resumed and the first point will remain to be considered in comparison with those already laid out.

To render this somewhat clear by a diagram, let A and B be the points to be connected, A being the *point of departure*, B that of *arrival*. A direction is first assumed between A and B, and is continued to D, where it becomes necessary to assume a new direction DC, owing to some impediment; this new direction is continued to C, at which point it is found practicable to assume the direction CB, as the shortest to the point of arrival. The line passing through the points ADCB, is then examined on that part between A and C, to ascertain whether the direction AC, which is the most direct, will also satisfy the other necessary conditions; and, if it does, it will be taken as the corrected line.

In all cases, so far as it can be done with a due regard to economy in the outlay of construction, a uniform ascent should be obtained between the points of departure and arrival, to avoid useless ascents and descents, which occasion a loss of power. Cases of this character not unfrequently present themselves, as for example, where the points of departure and arrival, lying on opposite sides of a hill, can be connected by a straight line by crossing the ridge at a level higher than either of the points; or else by taking a circuitous direction around the base, by which the ascent between the two would be made uniform. In such cases the choice of the engineer must be governed by his judgment, founded on a comparison of the expense of the two lines, and the advantages which they severally offer with respect to the time and means of conveyance.

A uniform ascent, within the prescribed limits, can always be obtained, whenever there is a continual rise of the ground between the point of departure and the point where the road crosses the ridge, by making frequent changes of direction in a zigzag line between the two points. The length of each zigzag will depend on the nature of the surface, and will in no manner affect the total length of the road if the ascent be uniform; for it is a property of lines of the same inclination which connect two different levels to be of the

same length, whatever change of direction the lines may assume.

It may be stated in this place that the straight portions of the line are connected at the points of change of direction by an arc of a circle (*Note 2*,) tangent to the two lines which it connects. This curved part should not be so abrupt as to require any considerable diminution of speed in the carriage whilst passing over it, and, as a farther precaution for safety in descents, the slope of the road at these points should be less than along the straight portions of the line.

When a valley intervenes between the two points, the same principles should guide the engineer in the choice of his line as in the case of a hill; for the valley must be descended on the side towards the point of departure to be ascended on the opposite side; and the ascent and descent should be as uniform as practicable, and in no part greater than the limits already laid down.

The case may also here present itself, whether it will be best to pursue the direct line between the two points, or to deviate from it, by crossing the valley at a higher level, nearer the head of it, by a more circuitous line. Other cases may persent themselves where the width of the valley, being much less at some lower level, might offer a great economy in the embankments; or, finally, where undulations in the ground, if taken advantage of, would produce the same effect, by cutting down a portion of the elevated parts to fill up the depressions between them. All of these cases require careful examination before any line between the points is definitively adopted.

When a marshy, or very wet soil intervenes between the points, it will generally be best to change the direction of the line to avoid crossing it, owing to the difficulty of obtaining a firm bottoming for the road covering in such localities, which can only be effected by a thorough system of drainage.

When a water course intervenes between the points, the point at which the line should intersect it will depend on the

character of the stream, and that of the approaches to it; as both economy of construction, and safety to the structure, over which the road is carried, may designate some other point than that in which the direct line intersects the water course.

As the general direction of a road will usually be the same as the valleys of the water courses between the points of departure and arrival, the advantages presented by both sides of the valley, for constructing the road, should be carefully balanced. The principal favorable points are, few secondary water courses intersecting the line of the road, a firm soil, a free exposure of the road surface to the action of the sun and wind, and facilities for procuring the necessary materials to form the road covering.

A careful comparison of the surveys of the different trial lines will usually enable the engineer to decide, without farther labor, upon the one which combines the most desirable advantages.

Equalizing the Excavations and Embankments, &c. The next important point to be adjusted is *balancing*, or *equalizing* the excavations and embankments; that is to make such a disposition of the slopes, and such partial changes in the main direction of the line, that the parts which will require to be excavated shall furnish just sufficient earth to form those portions which must be filled in. The solution of this problem, in the cases which most usually occur in practice, is of a very indeterminate character, and the engineer is obliged to resort to a system of successive approximations, by assuming different slopes, within the prescribed limits, and by shifting the position of the line to the right, or left, until he arrives at the most favorable result. In conducting these trials, the whole line should be subdivided into several portions, and the equalization of these portions should be attempted independently of each other, instead of trying a general equalization throughout the entire line.

In balancing the excavations and embankments their solid

contents are calculated, by subdividing them into the most simple geometrical solids, such as prisms, prismoids, wedges, and pyramids, whose solidities can be determined by the ordinary rules for the mensuration of solids. This subdivision will present but little difficulty when a sufficient number of cross profiles have been taken. Each cross profile will present a quadrilateral, or a triangular figure, of which the line of the road way, the side slopes, and the line of the surface of the ground, will form the sides. The three first will be right lines, and the last a curved line, but generally of such slight curvature that it may be regarded as a right line. The same remark may be made with respect to the surface of the ground, which, although in fact a curved surface, may be regarded as a plane surface, between any two of the consecutive planes of the profiles; provided these profiles be not taken too far apart. This manner of regarding the surface of the ground will only give an approximate result, in calculating the contents of the solids; but this approximation will be sufficiently accurate for all practical purposes, and will avoid more complicated methods, which, although more rigorous in their results, are less suited to the solution of problems of this nature.

In determining the relations between the volumes of the embankments, and the excavations by which they are to be furnished, it must also be borne in mind that earth in its natural state occupies less space than when broken up; and as the embankments, when first formed, are in the state of earth newly broken up, an allowance must be made according to the nature of the soil. This allowance will generally vary between one twelfth and one eighth; that is earth when first broken up will occupy from one twelfth to one eighth more bulk than it does in its natural state.

After the excavations and embankments are equalized, the changes caused by the operation in the direction of the line are laid down on the map previously to laying out the line on the ground. This operation is performed by running the

new line as laid down on the map, and marking the middle line, or *axis*, of the road, by stakes, or pickets, placed at equal intervals apart, and numbered to correspond with the same points, on the map. The width of the road way, and the lines on the ground corresponding to the side slopes of the excavations and embankments, are laid out in a similar manner, by stakes, placed along the lines of the cross profiles.

Besides the numbers marked on the stakes, to indicate their position on the map, other numbers, showing the depth of the excavations, or the height of the embankment, from the surface of the ground, accompanied by the letters *Cut. Fill.* to indicate a *cutting*, or a *filling*, as the case may be, are also added to guide the workmen in their operations. The positions of the stakes on the ground, which show the principal points of the axis of the road, should, moreover, be laid down on the map with great accuracy, by ascertaining their bearings and distances from any fixed and marked objects in their vicinity, in order that the points may be readily found should the stakes be subsequently misplaced.

Detailed maps of the different divisions of the road, made to a suitable scale, should accompany the general map. The object of these maps being to give with the utmost accuracy the longitudinal and cross sections of the natural ground, and the road way, by exhibiting all the parts of the road surface, and of the excavations and embankments, with the horizontal dimensions of all the parts, and the vertical heights of the different points above one general plane of level, termed the *plane of comparison*, numbered with great care.

Detailed drawings of the road covering, of the masonry and carpentry of the bridges, culverts, &c., to which are added written *specifications* of the manner in which the embankments, excavations, masonry, &c., is to be executed, should accompany the division maps, to guide the superintending engineer of the division in the performance of his duties.

Before breaking ground, to commence the *grading*, or

forming the excavations and embankments, another problem of a very intricate character, which has occupied the attention of the first mathematicians, and has called into play all the resources of the higher analysis, remains to be solved. This is the removal, and the disposition of the different volumes so as to present the greatest economy in transportation and expense. The results which have been arrived at in the different cases which have been treated, seldom present themselves in practical operations. There is, however, one general principle which is applicable to all cases, which is, that to make the transportation a minimum between the points from which the earth is taken, and that where it is deposited, the lines passed over by the centres of gravity of all the particles must neither cross each other in a horizontal, nor in a vertical direction. To apply this principle to practice, the entire volumes of the embankments and excavations should be subdivided into several others, (*Note* 3,) by planes in the direction of the transportation, and these partial solids, should be removed within the boundaries marked out by these planes. The farther the subdivision is carried, the greater will be the accuracy of the result.

Grading. In forming the excavations, the inclination of the side slopes demand peculiar attention. This inclination will depend on the nature of the soil, and the action of the atmosphere and internal moisture upon it. In common soils, as ordinary garden earth formed of a mixture of clay and sand, compact clay, and compact stony soils, although the side slopes would withstand very well the effects of the weather with a greater inclination, it is best to give them two base to one perpendicular, as the surface of the road way will, by this arrangement, be well exposed to the action of the sun and air, which will cause a rapid evaporation of the moisture on the surface. Pure sand and gravel may require a greater slope according to circumstances. It is not usual to use any artificial means to protect the surface of the side slopes from the action of the weather; but it is a precaution which, in

the end, will save much labor and expense in keeping the road-way in good order. The simplest means, which can be used for this purpose, consist in covering the slopes with good sods, (Fig. 57,) or else with a layer of vegetable mould about four inches thick, carefully laid and sown with grass seed. These means will be amply sufficient to protect the side slopes from injury when they are not exposed to any other causes of deterioration than the wash of the rain, and the action of frost on the ordinary moisture retained by the soil.

When the side slopes are not protected in this manner, it will be well, in localities where stone is plenty, to raise a small wall of dry stone at the foot of the slopes, to prevent the wash of the slopes from being carried into the road-way.

A covering of brush wood, or a thatch of straw, may also be used with good effect, but, from their perishable nature, they will require frequent renewal and repairs.

In excavations through solid rock, which does not disintegrate on exposure to the atmosphere, the side slopes might be made perpendicular; but as this would exclude, in a great degree, the action of the sun and air, which is essential to keeping the road surface dry and in good order, it will be necessary to make the side slopes with an inclination, varying from one base to one perpendicular, to one base to two perpendicular, or even greater, according to the locality; the inclination of the slope on the south side in northern latitudes being greatest, to expose better the road surface to the sun's rays.

The slaty rocks generally decompose rapidly on the surface, when exposed to moisture and the action of frost. The side slopes in rocks of this character may be cut into steps, (Fig. 58,) and then be covered by a layer of vegetable mould sown in grass seed, or else the earth may be sodded in the usual way.

The stratified soils and rocks, in which the strata have a *dip*, or inclination to the horizon, are liable to *slips*, or to give way by one stratum becoming detached and sliding on

another, which is caused either from the action of frost, or from the pressure of water, which insinuates itself between the strata. The worst soils of this character are those formed of alternate strata of clay and sand, particularly, if the clay is of a nature to become semi-fluid when mixed with water. The best preventives that can be resorted to in these cases, are to adopt a thorough system of drainage, to prevent the surface water of the ground from running down the side slopes, and to cut off all springs which run towards the road-way from the side slopes. The surface water may be cut off by means of a ditch (Fig. 57) made on the up-hill side of the road, to catch the water before it reaches the slope of the excavation, and convey it off to the natural water courses most convenient, as, in almost every case, it will be found that the side slope on the down-hill side is, comparatively speaking, but slightly affected by the surface water. To cut off the springs, it will be necessary to sink a ditch, or drain, on the side of the road from which the water flows, sufficiently deep to intercept the springs, and to fill this drain with broken stone, loosely thrown in, to offer an easy water way, and to prevent the drain from filling in with earth. The drain, thus arranged, must have an outlet towards some natural water course.

Neither of these precautions, however, will suffice in some cases where the soil is very loose, or is of a marly or chalky character; and all that can be done will be to give the wash a wide berth, and to allow the side slopes to assume with time their natural inclination, removing the fragments of the slips as they are deposited at the foot of the slope, except such as may form a kind of buttress for the slopes.

In forming the embankments, (Fig. 59,) the side slopes should be made with a less inclination than that which the earth naturally assumes, for the purpose of giving them greater durability, and to prevent the width of the top surface, along which the road-way is made, from diminishing by every change in the side slopes; as it would were they made

with the natural slope. To protect the side slopes more effectually they should be sodded, or sown in grass seed, and the surface water of the top should not be allowed to run down them, as it would soon wash them into gullies, and destroy the embankment. In localities where stone is plenty, a sustaining wall of dry stone may be advantageously substituted for the side slopes.

To prevent, as far as practicable, the settling which takes place in embankments, they should be formed with great care ; the earth being laid in successive layers of about four feet in thickness, and each layer well settled with beetles. As this method is very expensive, it is seldom resorted to except in works which require great care, and are of trifling extent. For extensive works, the method usually followed, on account of economy, is to embank out from one end, carrying forward the work on a level with the top surface. In this case, as there must be a want of compactness in the mass, it would be best to form the outsides of the embankment first, and to gradually fill in towards the centre, in order that the earth may arrange itself in layers with a dip from the sides inwards; this will in a great measure counteract any tendency to slips outwards.

A common method also, but a very bad one, is to make what is termed a *side forming*, which is done, by raising the whole embankment at once, commencing at one side and filling towards the other. This method is only used when the road is partly in cutting and partly in filling ; it is unstable, and the earth settles greatly and is liable to slips.

Besides the necessary embankments for the road surface, there are others, termed *spoil banks*, which consist of the surplus earth from excavations that is deposited in some convenient locality as near to the point from which the earth is taken as safety to the side slopes of the excavation will permit. The spoil banks are generally formed parallel to the road-way, and some feet back from the top of the side slope, to prevent the weight of the mass from crushing in the side slope. The down-hill side will, in most cases, be the most

suitable locality; and, if it be thought necessary, a slight surface drain may be placed at the foot of the slope, towards the road to prevent the water from the spoil bank from making its way to the side slope of the excavation.

In the cases of what are termed *side cuttings* where the road-way is partly in excavation and partly embankment, the formation of the side slopes and the excavations and embankments, with the precautions to preserve them from damage, will be arranged as has already been explained. But if the inclination of the natural surface of the ground, on which the embankment rests, is so great as to endanger its stability, it will be necessary to form the surface into steps (Fig. 60) to give the embankment a stable bed.

In side cuttings along a natural surface of great inclination, the method of construction just explained will not be sufficiently secure, and sustaining walls must be substituted for the side slopes, both of the excavations and embankments. These walls may be made simply of dry stone, when the stone can be procured in blocks of sufficient size to render this kind of construction of sufficient stability to resist the pressure of the earth. But when the blocks of stone do not offer this security, they must be laid in mortar, (Fig. 61,) and hydraulic mortar is the only kind which will form a safe construction. The batter of the walls may vary between twenty-four perpendicular to one base, and six perpendicular to one base. It should never be less than this latter, otherwise the mortar between the joints, near the surface, will be washed out by the rain, and the seeds of vegetables lodging in the voids, will germinate, and, in a short time, destroy the adhesion between the mortar and stone by the penetration of their roots between them. The wall which supplies the slope of the excavation should be carried up as high as the natural surface of the ground; the one that sustains the embankment should be built up to the surface of the road-way; and another wall, of less thickness and about four feet high, termed a *parapet wall*, should be raised upon it, to secure vehicles from accidents in passing from the line of the road-way.

A road may be constructed partly in excavation and partly embankment along a rocky ledge, by blasting the rock, when the inclination of the natural surface is not greater than one perpendicular to two base; but with a greater inclination than this, the whole should be in excavation.

There are examples of road constructions, in localities like the last, supported on a frame work, consisting of horizontal pieces, which are firmly fixed at one end, by being let into holes drilled in the rock, and are sustained at the other by an inclined strut underneath, which rests against the rock in a shoulder formed to receive it.

Drainage. A system of thorough drainage, by which the water that filters through the ground will be cut off from the soil beneath the road-way, to a depth of at least three feet below the bottom of the road covering, and by which that which falls upon the surface will be speedily conveyed off, before it can filter through the road covering, is essential to the good condition of a road.

The surface water is conveyed off by giving the surface of the road-way a slight transverse convexity (Fig. 62) from the middle to the sides, where the water is received into the gutters, or *side channels*, from which it is conveyed by underground aqueducts, termed *culverts*, built of stone or brick, and usually arched at top, into the main drains that communicate with the natural courses. This convexity is regulated by making the figure of the profile an ellipse, of which the semi-transverse axis is 15 feet, and the semi-conjugate axis nine inches; thus placing the middle of the road-way nine inches above the bottom of the side channels. This convexity, which is as great as should be given, will not be sufficient in a flat country to keep the road surface dry; and in such localities, if a slight longitudinal slope cannot be given to the road, it should be raised, when practicable, three or four feet above the general level; both on account of conveying off speedily the surface water, and to expose the surface better to the action of the wind.

To convey the water from the subsoil in a level country, ditches, termed *open side drains*, (Fig. 63,) are made parallel to the axis of the road, and at some feet from it on each side. The bottom of the side drains should be at least three feet below the road covering; their size will depend on the nature of the soil to be drained. In a cultivated country the side drains should be on the field side of the fences.

As open drains would be soon filled up along the parts of a road in excavation, by the washings from the side slopes, covered drains, built either of brick or stone, must be substituted for them. These drains (Fig. 64) consist simply of a flooring of flagging stone, or of brick, with two side walls of rubble, or brick masonry, which support a top covering of flat stones, or of brick, with open joints, of about half an inch, to give a free passage way to the water into the drain. The top is covered with a layer of straw, or brushwood, and clean gravel, or broken stone, in small fragments, is laid over this, for the purpose of allowing the water to filter freely through to the drain, without carrying with it any earth or sediment which might, in time, accumulate and choak it. The width and height of covered drains will depend on the materials of which they are built, and the quantity of water to which they yield a passage.

Besides the longitudinal covered drains in cuttings, other drains are made under the road-way which, from their form, are termed *cross mitre drains*. Their plan is in shape like the letter V, the angular point being at the centre of the road and pointing in the direction of its ascent. The opening of the angle should be so regulated that the bottom of the drain shall not have a greater slope along either of its branches, than one perpendicular to one hundred base, to preserve the masonry from damage by the current. The construction of mitre drains is the same as the covered longitudinal drains. They should be placed at intervals of about 60 yards from each other.

In some cases surface drains, termed *catch water drains*,

are made on the side slopes of cuttings. They are run up obliquely along the surface, and empty directly into the cross drains which convey the water into the natural water courses.

When the roadway is in side cutting, cross drains (Fig. 66) of the ordinary form of culverts are made, to convey the water from the side channels and the covered drains into the natural water courses. They should be of sufficient dimensions to convey off a large volume of water, and to admit a man to pass through them, so that they may be readily cleared out, or even repaired, without breaking up the road-way over them.

The only drains required for embankments are the ordinary side channels of the road way, with occasional culverts, to convey the water from them into the natural water courses. Great care should be taken to prevent the surface water from running down the side slopes, as they would be soon washed into gullies by it.

Very wet and marshy soils require to be thoroughly drained before the road-way can be made with safety. The best system that can be followed in such cases, is to cut a wide and deep open main drain on each side of the road, to convey the water to the natural water courses. Covered cross drains should be made at frequent intervals to drain the subsoil of the road-way. They should be sunk as low as will admit of the water running from them into the main drains, by giving a slight slope to the bottom each way from the centre of the road to facilitate its flow.

Independently of the drainage for marshy soils, they will require, when the subsoil is of a spongy elastic nature, an artificial bed for the road covering. This bed may, in some cases, be formed by simply removing the upper stratum to a depth of several feet, and supplying its place with well packed gravel, or any soil of a firm character. In other cases, when the subsoil yields readily to the ordinary pressure that the road surface must bear, a bed of brushwood, from 9 to 18

inches in thickness, must be formed to receive the soil on which the road covering is to rest. The brush wood should be carefully selected from the long straight slender shoots of the branches or undergrowth, and be tied up in bundles, termed *fascines*, from 9 to 12 inches in diameter, and from 10 to 20 feet long. The fascines are laid in alternate layers crosswise and lengthwise, and the layers are either connected by pickets, or else the withes, with which the fascines are bound, are cut to allow the brushwood to form a uniform and compact bed.

This method of securing a good bed for structures on a weak wet soil has been long practised in Holland, and experience has full tested its excellence.

Road Coverings. The object of road coverings is to diminish the resistances offered to the force of traction, by the friction and collision of the wheels, along the road surface, and to reduce, as far as practicable, the wear and tear of the surface, occasioned by the passage of vehicles, and the action of the weather. To effect this in the most perfect manner, the road covering must be formed of some material which is smooth, hard, and durable, and be laid on an unyielding bed. For each of the causes of the bad qualities of roads depends more or less on these properties of the materials and of the bed on which they are laid.

If the subsoil, or bed, is not of an unyielding character, the materials of the road covering will be soon worked into it by the passage of vehicles. The subsoil will thus be forced to the surface, where, becoming mixed up with the materials, it will greatly tend to increase their wear and tear, by keeping them always moist, from the quantity of mud which will be constantly forming, and from the disunion which this will cause between them.

If the materials are not hard and durable the road covering will rapidly wear into ruts, and will thus increase the friction and collision, which depend on the evenness of the surface.

To satisfy these conditions of a good road, both theory and experience are in favor of a road covering of hard stone, of sufficient thickness to prevent a weight laid on any point of the surface from pressing the materials into the subsoil.

Among the various methods in which stone has been used for road coverings, that which is termed a *paved road*, or *pavement*, has been found to be the best where the road is constantly traversed by heavy vehicles.

Pavements are variously constructed, according to the more or less care required by the nature of the transportation. The method in most general use in our country, is to excavate the soil to a suitable depth, to receive a bed, or *form*, of clean sand, which is made from one foot to several feet in thickness; into this form, stones of a round shape, and of various sizes, which have received the common name of *paving stones*, are set as close together as they can be packed, and are firmly settled by a heavy beetle manœuvred by one or two men. The stones are driven until their tops are even with the surface of the road-way, which is convex and higher in the middle than at the sides; the road surface is then covered over with a layer of clean sand two or three inches in thickness, which is gradually worked in between the stones by the combined action of the wheels and the weather.

The defects of this kind of pavement are obvious at a glance. The road surface is of the worst kind to diminish the friction and collision; and although the form is of a material which is perfectly firm, when prevented from yielding laterally, still the shape of the stones is the most favorable to produce this lateral yielding. The consequences therefore to be apprehended, and which are fully verified by experience, are, that the force of traction will be very great, and the roadway will soon get out of order by wearing rapidly into ruts. The most of these defects would be removed by substituting cubical blocks, about nine inches in thickness, for the round stone, as they would form a more solid and a smoother co-

vering, besides being of a better shape to resist being forced into the form.

The best system of pavement is that which has been partially put in practice in some of the commercial cities of England, the idea of which seems to have been taken from the excellent military roads of the Romans, vestiges of which remain at the present day in a good state.

In constructing this pavement a bed (Fig. 65) is first prepared, by removing the surface of the soil to the depth of a foot or more, to obtain a firm stratum; the surface of this bed receives a very slight convexity, of about two inches to ten feet, from the centre to the sides of the road-way. If the soil is of a soft clayey nature, into which small fragments if broken would be easily worked by the wheels of vehicles, it should be excavated a foot or two deeper to receive a form of sand or of clean fine gravel. On the surface of the bed thus prepared a layer of small broken stone, four inches thick, is laid; the dimensions of these fragments should not be greater than two-and-a-half inches in any direction; the road is then opened to vehicles until this first layer becomes perfectly compact, care being taken to fill up any ruts with fresh stone, in order to obtain a uniform surface. · A second layer of stone, of the same thickness as the first, is then laid on, and treated in the same manner; and finally a third layer. When the third layer has become perfectly compact, and is of a uniform surface, a layer of fine clean gravel, two-and-a-half inches thick, is spread evenly over it to receive the paving stones. The blocks of stone are of a square shape, and of different sizes, according to the nature of the travelling over the pavement. The largest size are ten inches thick, nine inches broad, and twelve inches long; the smallest are six inches thick, five inches broad, and ten inches long. Each block is carefully settled in the form, by means of a heavy beetle; it is then removed in order to cover the side of the one against which it is to rest with hydraulic mortar; this being done the block is replaced, and properly adjusted. The blocks of the

different courses across the road-way should break joints. The surface of the road is convex; the convexity being determined by making the outer edges six inches lower than the middle for a width of thirty feet.

As pavements of this character would only be required in cities, they must be accompanied by side walks, and crossing places, for foot passengers. The side walks are made of large flat flagging stone, at least two inches thick, laid on a form of clean gravel well rammed and settled. The width of the side walks will depend on the street being more or less frequented by a crowd. It would, in all cases, be well to have them at least twelve feet wide; they receive a slope, or pitch, of one inch to ten feet, towards the pavement to convey the surface water to the side channels. The pavement is separated from the side walk by a row of long slabs, set on their edges, termed *curb stones*, which confine both the flagging and paving stones. The curb stones form the sides of the side channels, and should for this purpose, project six inches above the outside paving stones, and be sunk at least four inches below their top surface; they should, moreover, be flush with the upper surface of the side walks, to allow the water to run over into the side channels, and to prevent accidents which might happen from their tripping persons passing in haste.

The crossings should be from four to six feet wide, and be slightly raised above the general surface of the pavement, to keep them free from mud.

The kind of stone selected for pavements should be hard, with a compact texture, and not possessing the property of receiving a polish.

The system of pavement which has just been described fulfils in the best manner all the conditions of a good road; presenting a hard even surface to the wheels, and reposing on an unyielding bed, formed by the bottoming of broken stone, which is protected, as far as practicable, from the action of water by the mortar in the joints; though it may reasonably admit of a doubt, whether the mortar would not be soon

destroyed by the action of the vehicles, and the expansion and contraction of the stone which, in our climate, is found to be very considerable in exposed positions.

The middle of roads in the vicinity of large cities should, for economy, be paved, for a width of eight feet on each side of the centre of the road, in the same manner as the streets, if it be found that more than three inches of the ordinary road covering of broken stone is worn down annually by the travelling over the road. The wings of the road, on each side of the pavement, may be made of broken stone or clean coarse gravel. Great attention will be requisite to prevent ruts from forming where the pavement and wings unite.

Wooden pavements, formed of blocks of a hexagonal or square shape, cut into lengths of nine inches, have been for some period in use in Russia, and are now undergoing experiment with us. The blocks are first well seasoned, and, in some cases, prepared by immersing them in some one of the solutions which have been found to be preventives of the rot. After they are laid, the joints and top surface are coated with hot tar, or pitch, and a thin layer of fine gravel is spread over the surface before the road is thrown open to vehicles. In some instances a wooden pin is inserted into each block to connect it firmly with the one next to it which has a hole in it to receive the pin. This seems however an excess of precaution, for if the bottoming is firm, the blocks cannot work into it, and there is but little tendency in the pressure at the surface to throw the blocks up.

The wear and tear of this kind of pavement, when made of the harder kinds of wood, will, in all probability, be less than that of stone. Its durability will depend upon the efficacy of the preventives to protect the wood from the rot. As to its cost, it will be greater, probably, even with us, than stone, owing to the preparation of the material. One objection urged against it, is the danger that may be apprehended from the decomposition of the wood in our hot climates. This objection is one of great moment, particularly when coupled

with the slight durability of wood under such circumstances.

The ordinary road covering for common roads, is formed of a coating of small broken stone, laid either on a paved bottom, or else on the surface of the subsoil which forms the bed.

The paved bottom road covering (Fig. 66) is formed, by excavating the surface of the ground to a suitable depth, and preparing the form for the pavement with the precautions as for a common pavement. Small blocks of stone of an irregular pyramidal shape are selected for the pavement, which should be seven inches thick in the centre, and three inches thick at the sides, for a road-way 30 feet in width. The base of each block should not measure more than five inches, and the top not less than four inches.

The blocks are set by the hand, with great care, as closely in contact at their bases as practicable, and blocks of a suitable size are selected to give the surface of the pavement a slightly convex shape from the centre outwards. The spaces between the blocks are filled with chippings of stone compactly set with a small hammer.

A layer of broken stone, four inches thick, is laid over this pavement for a width of nine feet on each side of the centre; no fragment of this layer should measure over two-and-a-half inches in any direction. A layer of broken stone of smaller dimensions, or of clean coarse gravel, is spread over the wings to the same depth as the centre layer.

The road covering, thus prepared, is thrown open to vehicles until the upper layer has become perfectly compact, care having been taken to fill in the ruts with fresh stone, in order to obtain a uniform surface. A second layer, about two inches in depth, is then laid over the centre of the road-way; and the wings receive also a layer of new material laid on to a sufficient thickness to make the outside of the road-way nine inches lower than the centre, by giving a slight convexity to the surface from the centre outwards. A coat-

ing of clean coarse gravel, one inch and a half thick, termed a *binding*, is spread over the surface, and the road covering is then ready to be thrown open to travelling.

The stone used for the pavement may be of an inferior quality, in hardness and strength, to that placed at the surface, as it is but a litte exposed to the wear and tear occasioned by travelling. The surface stone should be of the hardest kind that can be procured. The gravel binding is laid over the surface to facilitate the travelling, whilst the under stratum of stone is still loose; it is, however, unserviceable, as, by working in between the broken stones, it prevents them from setting as compactly as they would otherwise do.

If, from any circumstance, the road-way cannot be paved the entire width, it should, at least, receive a pavement for the width of nine feet on each side of the centre. The wings, in this case, may be formed entirely of clean gravel, or of chippings of stone.

For roads which are not much travelled, like the ordinary cross roads of the country, the pavement will not demand so much care; but may be made of any stone at hand, broken into fragments of such dimensions that no stone shall weigh over four pounds. The surface coating may be formed in the manner just described.

A road covering formed entirely of broken stone is made precisely in the same manner, and with the same precautions to obtain a compact solid mass, as the bottoming of broken stone for a paved road. None of the fragments of broken stone should measure more than two inches and a half in any direction.

A good road covering may be formed in a similar manner of clean gravel. The first layer should be four inches thick, and be spread over a level form. The other layers are laid on in successive thicknesses of three inches, until the covering has attained the entire thickness of sixteen inches in the centre, with the ordinary convexity at the surface.

Each pebble which measures over one inch and a half in

diameter must be broken into smaller fragments before being laid on the road. The largest sized gravel should be spread over the centre of the road for a width of at least fifteen feet.

The system of road coverings, with a paved bottoming and broken stone surface, has been for a long period in use in France, and, within late years, has been generally adopted in England. The experience in both of these countries is decidedly in favor of this system, as possessing the properties of a good road covering in a degree superior to any other formed of small fragments of stone. The arrangement of the paved bottoming is the most suitable to distribute any pressure at the surface over a considerable bearing surface of the subsoil, and thus to prevent it from working up between the blocks, whilst, owing to the pyramidal shape of the blocks, the tendency of the pressure will be to keep them in their places.

In road coverings of broken stone alone, which are commonly known as *M'Adamized* roads, from the name of the gentlemen who first brought them into notice, the bed of the subsoil must be of a very firm nature, otherwise the small fragments will be easily forced into it, by the pressure on the surface of the road-way, because these fragments are not of a suitable size or shape to distribute the pressure over a large bearing surface of the bed. The consequences arising from this will be, that the fragments of stone will be kept in a loose moist state, owing to the mud formed by the subsoil, and the road-way can never be kept in good order, because the wear and tear of the fragments will be greatly increased, from the want of compactness in the mass, ruts and large quantities of mud will form rapidily, and the road covering will be readily broken up by even a slight frost.

The M'Adamized road recommended itself, when first introduced, partly from its real superiority to the methods then in use, and partly owing to the ease with which the road covering could be formed, and from its supposed economy over all other methods. When compared with a good paved bot-

tom road its inferiority to it has been fully established, except in localities where the subsoil is perfectly unyielding. As to the economy of the two methods, the first will, in most cases, be in favor of a M'Adamized road, owing to the care with which the paved bottoming requires to be set. The cost of repairs will generally be found greatly in favor of the paved bottom road.

Wherever the paved bottom road has been attempted in our own country, a most injudicious method has been followed, of placing very large fragments of stone loosely arranged at the bottom, and covering them with a layer of broken stone; the consequences of which have been, that the bottom stones are forced up, and gradually work to the top, from the unequal pressure occasionally thrown on their corners, by which the smaller fragments are forced beneath them.

Road coverings of broken stone are not suited to the streets of cities, because they do not offer sufficient resistance to the wear and tear occasioned by the constant travelling on such thronged thoroughfares. They present, moreover, a great inconvenience from the dust which is formed in dry, and the mud in wet weather.

Cross dimensions of Roads. A road thirty feet in width is amply sufficient for the carriage way of the most frequented thoroughfares between cities. A width of forty, or even sixty feet, may be given near cities, where the greater part of the transportation is effected by land. For cross roads, and others of minor consideration, the width may be reduced according to the nature of the case. The width should be at least sufficient to allow two of the ordinary carriages of the country to pass each other with safety. In all cases, it should be borne in mind, that any unnecessary width increases both the first cost of construction, and the expense of annual repairs.

Very wide roads have, in some cases, been used, the centre part only receiving a road covering, and the wings, termed *summer roads,* being formed on the natural surface of the subsoil. The object of this system is to relieve the road co-

vering from the wear and tear occasioned by the lighter kind of vehicles during the summer, as the wings present a more pleasant surface for travelling in that season. There is but little gained by this system under this point of view, and it has the inconvenience of forming during the winter a large quantity of mud which is very injurious to the road covering.

There should be at least one foot-path, (Fig. 62,) from five to six feet wide, and not more than nine inches higher than the bottom of the side channels. The surface of the foot path should have a pitch of two inches, towards the side channels, to convey its surface water into them. When the natural soil is firm and sandy, or gravelly, its surface will serve for the foot-path; but in other cases the natural soil must be thrown out to a depth of six inches, and the excavation be filled with fine clean gravel.

To prevent the foot-path from being damaged by the current of water in the side channels, its side slope, next to the side channel, must be protected by a facing of good sods, or of dry stone.

As it is of the first importance, in keeping the road-way in a good travelling state, that its surface should be kept dry, it will be necessary to remove from it, as far as practicable, all objects that might obstruct the action of the wind and the sun on its surface. Fences and hedges along the road should not be higher than five feet; and no trees should be suffered to stand on the road-side of the side drains, for independently of shading the road-way, their roots would in time throw up the road covering.

Repairs and Materials. The very friable rocks, which are easily worn down by attrition, should not be used for the surface coating of the road covering, when harder materials can be procured. The materials should be collected in small depots, at convenient intervals along the road-side, to be always at hand when wanted. Ruts and hollows should be filled in with fresh materials immediately, by successive thin coatings, a

fresh one being laid so soon as the other is thoroughly worked into the mass.

The best seasons for general repairs throughout the line, are in the wet portions of the autumn and spring, between October and April. The whole line should be put in complete order in each of these months. Besides these general repairs the side channels should be cleared out from time to time, and the mud should be removed from the road surface, so soon as it has formed half an inch thick, to some convenient point on the road-side, where it will not obstruct the road-way or side channels.

BRIDGES.

A bridge is a structure raised to sustain a road-way above a water course.

Bridges are classified, either from the character of the structure, as *fixed bridges*, and *moveable bridges*; or from the nature of the material of which the structure is formed, as *stone bridges*, *wooden bridges*, &c.

Definitions. The portions of the road-way, at each extremity of the bridge, which lead to it, are termed the *approaches*. The extreme points of support of the bridge, at each shore, are termed the *abutments*, and the intermediate points of support, the *piers*. The interval, or water-way, between any two points of support is termed a *bay*. The walls which sustain the embankments of the approaches, where they join the bridge, are termed the *wings*. The faces of the bridge, up and down stream, are termed the *heads*. The projections of the piers beyond the faces are termed the *starlings*, or *cut-waters*. The centre line of the bridge between its extremities is termed the *axis*.

Survey. Before any plan for a bridge is adopted, a careful survey must be made of the stream itself, its bed, its shores, and the most accessible points of approach to it, for some distance above and below the point where it is intersected by the line of the road-way. The object of this survey is to ascer-

tain the regimen of the stream, the nature of its bed, and the character of the soil along its banks, for the purpose of determining the probable changes which will take place in its regimen, from the construction of the bridge, and also the kind of structure which will be most suitable to the character of the soil.

A complete topographical map, exhibiting the natural features of the surface, should be made. A set of profiles, both lengthwise and crosswise, should be added, to the map, showing the slope of the bed, the character of the substrata, as ascertained by soundings along the lines of the profiles, and the water lines at the different epochs of ordinary high and low water, and of freshets.

A descriptive memoir, containing all the information that can be obtained with respect to the changes that the bed has undergone at different epochs, the effects of freshets upon it, the velocity of the current at the season of freshets, and also at the time in which the survey is made, as observed at the different stations of the cross profiles, should accompany the map.

Basis of the Plan. The two principal points which claim the attention of the engineer in arranging the plan of a bridge, are, 1:—to procure a suitable water-way for the passage of the stream, in order that the water, being partially dammed back by the bridge, may not rise sufficiently high to damage the grounds above it, nor receive, from the same cause, a velocity so great near the bridge as to change the bed; 2:—to adopt such a system of foundations as shall secure the durability of the structure.

These points will require a careful examination and study of the locality, of the water-way under the bridge, and of the form and size of the bays. After this preliminary study is made, the forms and dimensions of the abutments, piers, and arches, can be regulated according to the nature of the materials, and such modifications can be made in the locality, water-way, and bays, as may be required in regulating these parts.

Locality. When the point where a bridge is to be erected is not determined by the position of a line of communication which cannot be changed, the engineer will select such a point as offers the most security for the foundations, and the greatest economy in the construction. As a general rule a straight reach of the stream should be preferred, as the regimen there is more uniform than at or near elbows. And a firm soil, which will not yield to the increased velocity of the current, caused by the obstruction of the piers, should be chosen for the foundations.

The axis of the bridge should be placed perpendicular to the thread of the stream, when the locality will admit of its being done; when this cannot be effected, the bridge is termed *askew*, or *oblique*, because the *sides* or *faces* of the piers, which must be parallel to the thread of the current, in order that they may not be exposed to the effects of the water striking them obliquely, are oblique to the axis.

The obliquity, or angle between the axis and the thread of the stream, should not exceed 70°, both on account of solidity and economy.

Water-way. The durability of the bridge will essentially depend on its having a water-way of suitable dimensions.

Observations made on the regimen of streams have shown, that where the volume of water is nearly the same, and no sudden changes are made in the form and size of the bed, the stream acquires a uniform regimen; or, in other words, that the longitudinal slope, the cross section and the velocity of the current at any point, remain sensibly the same. If the cross section is diminished, the water will be partially dammed back; this will cause the water to rise to a certain height above the point, and will occasion a fall, or *shoot*, and an increased velocity at the point; from these causes danger may arise to agriculture during great freshets, from the overflowing of the banks above the point, and to navigation, from the shoot; and the bed will be gradually worn away, and will expose to ruin any structure raised upon it. If, on the contrary, the cross section is enlarged, the velocity of the cur-

rent will be decreased, and the bed will gradually fill up with deposits occasioned by this cause, until the stream acquires its natural uniform regimen. By enlarging therefore the natural water-way, the effects would be only temporary, and might eventually prove destructive to the bridge, should the deposites be formed principally in a few bays, because during a freshet the velocity might be so greatly increased at other points, from this cause, as to undermine the foundations.

The artificial water-way should therefore be so regulated that the velocity will not effect any sensible change in the bed. In order to ascertain this suitable velocity, the mean velocity of the stream should be ascertained, both above and below the point, at the time of the highest water; and also the effects of the current upon the bed, particularly at any points in the vicinity where the water-way is diminished either by natural or artificial obstructions.

The mean velocity, by which is understood that average velocity of the current at any point, which multiplied by the area of the cross section of the water-way at that point, will give the volume of water which flows through in a given unit of time, can be found by measuring the velocity at the surface of the middle thread of the stream, which is done by noting the time it takes a light ball, or *float*, composed of some substance whose specific gravity is nearly the same as that of water, as, for example, *white wax*, or *camphor*, to pass between two fixed points, and taking four-fifths of it for the mean velocity.

Having found the mean velocity of the natural water-way, that of the artificial water-way will be obtained from the following expression,

$$v = m \frac{\mathrm{S}}{s} \mathrm{V},$$

in which s and v represent, respectively, the area and mean velocity of the artificial water-way, S and V, the same data of the natural water-way, and m a constant quantity, which, as determined from various experiments, may be represented by them ixed number 1,097.

With regard to the effect of the increased velocity on the bed, there are no experiments which directly apply to the cases usually met with. The following table is drawn up from experiments made in confined channels, the bottom and sides of the channel being formed of rough boards.

Stages of accumulation termed	Velocity of river in feet per second.	Nature of the bottom which just bears such velocities.	Specific gravity of the material.
Ordinary floods	3·2	Angular stones, the size of a hen's egg	2·25
	2·17	Rounded pebbles, one inch in diameter	2·614
	1·07	Gravel of the size of garden beans . . .	2·545
Uniform tenors	0·62	Gravel of the size of peas	2·545
	0·71	Coarse yellow sand	2·36
Gliding	0·351	Sand, the grains the size of aniseeds	2·545
Dull	0·26	Brown potter's clay	2·64

Form and size of the bays. As the bay is the space included between the bottom of the stream, the sides of the piers, and the soffit of the arch, it will chiefly depend on the form and dimensions of the last, as the piers are regulated by the size of the arches.

The most usual forms of the intrados of cylindrical arches, are the *full centre*, or semi-circle, in which the rise is equal to one half the span; the *segment arch*, which is formed of a circular arc less than a semi-circle, the rise being less than half the span; and the *oval*, or *basket-handle arch*, in which the rise is some fractional part of the span less than one half, and the general form of the curve is like the ellipse, the tangent to it at the springing line being vertical.

The full centre (Fig. 67) is the oldest form of the cylindrical arch; it is of easy construction, and presents great solidity. The principal objection to it for bridges arises from its rise being equal to half its span, requiring either very small arches, or else, if large arches are used, of giving such a height to the road-way as to demand excessive embankments for the approaches; or, if the road-way is lowered, to avoid heavy embankments, of offering a great obstruction to the stream, from the great width of the *spandrels*, or parts of the heads of the bridge above the piers.

The segment arch presents the same objections as the full centre, unless the amplitude of the arc be small. In that case, (Fig. 68,) the springing lines of the arch may be placed near the high water line, thus procuring a wide water-way, and, at the same time, but slight embankments for the approaches. The slight convexity of the arch will, however, present less strength than the full centre, and will, moreover, cause a great pressure, or *horizontal thrust*, against the abutments of support.

The oval arch (Figs. 69 and 70) may be either an ellipse, or else a continuous curve formed of arcs of circles of unequal radii which are tangent to each other. The latter is generally preferred to the ellipse, because the form of the arch stones can be more easily arranged, and with the same rise and span, it can be constructed to give a greater water-way than the ellipse.

The oval arch may be described with three, or a greater odd number of centres. The number of centres will depend on the relation between the span and rise; when the latter is one third, or a greater fractional part of the former, three centres may be used; but if the rise is less than one third of the span, five or a greater number of centres must be taken, in order to obtain a curve of a pleasing form to the eye. (*Note* 4.)

To give a greater water-way than the ellipse, the curve at the springing lines should include the ellipse, or, in other words, its curvature at those points should be less than that of the ellipse. To present sufficient strength throughout, the radius of the arc at the crown must not be greater than once and a half the span. With these conditions the oval arch will present all of the advantages, and none of the defects of the full centre arch.

With regard to the size of the arches, or the dimensions of the span and rise, so much will depend on local circumstances that no positive rules can be laid down for them.

For streams with a gentle current, which are not subject

to heavy freshets, small arches, or those of a medium size, may be adopted, because, even a considerable diminution of the natural water-way will not greatly affect the velocity under the bridge, and the foundations therefore will not be liable to be undermined. The difficulty, moreover, of laying the foundations in streams of this character is generally very inconsiderable. For streams with a rapid current, and which are moreover subject to great freshets, large arches will be most suitable, in order to give a water-way as great as practicable, and to diminish the danger to the points of support, by placing as few in the stream as practicable. The size of the arches will also depend on the materials used in their construction. If they are of the first quality, large arches may be constructed with safety: if they are of an inferior quality, it will be most prudent to adopt small arches, or those of a medium span.

It is usual to give the same rise and span to all the arches, and to place their springing lines on the same level throughout. When this is done, the road-way and parapet wall of the bridge will be level throughout. This arrangement is thought by some, to give a bolder appearance, and a more pleasing effect to the structure. It has the positive inconveniences of requiring large embankments for the approaches, when the banks of the stream are low, and the rise of the arches is considerable; and it prevents the road surface from being as thoroughly drained as it would be, were there a longitudinal slope each way from the middle to the extremities of the bridge. These inconveniences may be avoided by adopting arches of unequal dimensions which will admit of a suitable longitudinal slope; or else, if the arches receive the same dimensions, by placing their springing lines lower from the centre to the extreme arches.

The crown of the arches should be at least three feet above the water line of the highest freshets, in order to allow floating bodies, as trees, &c., to pass through the bridge at those epochs.

Dimensions of the Arches. The *length* of the arches, or the distance between the heads of the bridge, will depend on the breadth of the road-way, which is regulated like any other thoroughfare. The thickness of the arch at the crown, or the *depth* of the key-stone, will depend on the horizontal thrust, which is dependent on the rise and span, (*Note 5*,) and on the strength of the stone used for the structure. It is difficult to regulate this depth by calculation, owing to the practical difficulty of so constructing an arch that no settling shall take place at the crown, and that the joints of the voussoirs shall remain in juxtaposition throughout, thereby preserving a uniform pressure over the entire surface of the joint. If this practical difficulty could be got over, it would then only be necessary to give the key-stone such a depth that the surface of the joint should resist the pressure of the horizontal thrust, which depth is found by calculation to be much less than that allowed in practice.

From an examination of a great number of existing bridges, which have withstood, for some period back, all the usual causes of destructibility, it appears that the depth of the key-stone for medium sized arches need not be greater than two and a half feet, and for large arches not more than four feet. The ring courses may have a uniform depth throughout, or they may increase or decrease in depth, from the key-stone to the springing lines, as the nature of the structure may seem to demand.

Dimensions and form of Piers. As the horizontal thrust of the two half arches, on each side of a pier, counteract each other, the only effort that the pier will have to sustain, will be that arising from the weight of the two half arches, and the accessory weight that may be placed over them on the road surface. The thickness of the pier at top, or at the springing line, might therefore be only sufficient to sustain this pressure; and it would, therefore, depend on the strength of the stone, and the depth of the ring courses if uni-

form. The least thickness, therefore, should be twice the depth of the ring course.

When calculated from these data, the thickness of the piers is found much below what is usually allowed in practice; because allowance must be made for the wear and tear of the exterior surface of the piers by the current, for shocks from floating bodies, for the mixed character of the masonry, the exterior courses being usually of cut, and the interior of rubble stone, and finally for slovenly workmanship.

The common practice, at present, is to give the same thickness to all the piers; although some are in favor of forming what are termed *abutment piers*, that is, piers having sufficient thickness to resist alone the horizontal thrust of the arches, in order, should one arch give way, to prevent the others from yielding. This last plan is objectionable on account of greatly diminishing the natural water-way, from the large dimensions of the piers, and thereby endangering the foundations; and also from a very considerable addition to the expense of the structure. Its advantages are the one just stated, and an economy in the centres, as each arch in this system may be finished separately, and one centre therefore be made to serve for all if necessary.

From an examination of a great number of the most celebrated bridges, it appears, that the thickness of the piers at top (Fig. 67, &c.) varies between one-eighth and one-seventh of the span. The boldest structure of this character, which has existed for some period back, is the bridge of Neuilly, near Paris, built by Perronet, the most celebrated architect of bridges of modern times. The thickness of its piers is one ninth of the span.

The faces of the piers usually receive a slight batter to give them greater stability. This batter should not be greater than ten perpendicular to one base, and may approach as nearly a vertical as the character of the structure may seem to demand.

The spread of the foundations will depend on the nature

of their bed; the same rule being followed in their case as for other structures in masonry. The foundation courses (Fig. 67, &c.) are carried up with offsets, from six to twelve inches in width, instead of a uniform batter. The object of the offsets, is to afford firm points of support for the frame work of the centres.

The projections of the ends of the piers, beyond the heads of the bridge, or the starlings, will depend on circumstances; usually, this projection is only half the thickness of the pier. The form of the starling may present either two plane surfaces, (Fig. 71,) with an angle up or down stream; or two curved surfaces, (Fig. 72,) with an angle in the same direction; or else its plan (Figs. 73 and 74) may be a semi-circle, or semi-ellipse.

The object of the starling being to prevent ice, and other floating bodies, from damaging the head of the bridge, and to present the most suitable cut-water to prevent whirls, occasioned by the back-water, from undermining the foundations, its form should subserve both of these purposes. From various experiments it appears, that the semi-ellipse is the most suitable form for both purposes.

The starlings, like the piers, receive a batter. This batter may be either in a right line, or the starling may receive a swell between the top and bottom like a column. In streams where the bridges are exposed to injury from the breaking up of the ice in the spring, it has been proposed to give the up-stream starlings a very considerable batter, thereby presenting an inclined plane, along which the ice would ascend, and breaking by its own weight, would pass through the arches.

The starlings are carried up above the springing lines to the highest water line. They are finished usually at top by a flat conical surface, (Fig. 67, &c.,) formed of a single stone, which rests on a cushion stone, that projects beyond the face of the starling, and serves the purpose of a coping to throw the water clear of the face. This capping is termed the *hood* of the starling.

Arrangement of the facing of the bridge. The arrangement of the voussoirs, and the horizontal courses of the starlings and spandrels, requires some attention, to produce a pleasing architectural effect. This arrangement will depend on the curve of the intrados, and the materials used for the constructure.

When the facing of the bridge is entirely of cut stone, the most usual arrangement consists (Fig. 69) in giving the voussoirs near the springing lines unequal thicknesses, in order that the extremities of their joints may meet those of the horizontal courses of the starlings, which are of equal thickness throughout, thus presenting the appearance of a uniform connection between the arch and starlings.

In some cases the same stone forms the voussoirs and a part of the horizontal course, making what is termed an *elbow-joint.* This arrangement is bad, except in very light arches, because the elbows are liable to crack from any settling of the arch.

The arrangement of the spandrel courses with the voussoirs in oval arches (Figs. 69 and 70) will depend on the thickness of the blocks in each case. The top extremities of the voussoirs are cut to form a part of the horizontal and vertical joints of the spandrel courses.

In very flat segment arches, (Fig. 68,) where the depth of the spandrel is not great, the voussoirs may be continued to the cornice of the bridge.

In bridges of mixed masonry, where the ring courses are of cut stone and the spandrels of rubble, or brick, (Fig. 67,) the voussoirs receive either a uniform depth, or else are made alternately of unequal depths, according to the taste of the architect.

Superstructure. A *cornice* is placed on a level with the top of the key-stone, to serve as a coping, to protect the facing below it from the rain. The form and projection of the cornice will depend on the height of the bridge, and the style of its architecture. In small bridges of a common

character the cornice should consist simply of a projecting coping, with, at most, a simple supporting moulding beneath. In large bridges, a cornice composed of several members, or mouldings, with supporting modillions, (Fig. 75,) may be required. The top and bottom surfaces of the cornice should be of a suitable form to throw the water from the facing.

A parapet wall is placed above the cornice to secure vehicles and foot passengers from accidents. It is formed of two or more courses, (Fig. 75,) the top course being rounded at top, and the blocks being secured from accidents, either by means of iron or copper clamps, or else by a mortise or dovetail joint. In highly finished bridges, a stone ballustrade has been used instead of the ordinary parapet wall. (Fig. 77.)

The height of the parapet wall above the foot-path need not be more than four feet; its thickness may be from one to two feet.

Style of Architecture. The character of the style of architecture most suitable for bridges will depend, to a certain extent, on the locality; bridges in cities, near buildings of a sumptuous character, requiring, evidently, a more ornate style than those in rustic, or other localities. But the character of the ornaments must be suitable to the purposes to which they are supposed to be applied, and also to the object and general character of the structure itself. As the object of a bridge is to bear heavy loads, and to withstand severe shocks, its general character should be that of strength, and it should therefore present a massive appearance. Whatever then does not add to this character of strength, and at the same time is not of obvious utility, should be rejected as superfluous and unsuitable. From these principles it will obviously appear, that a suitable and an agreeable form given to the starlings, a proper combination of the voussoirs and spandrel courses, and a handsome projecting cornice with a plain parapet, are all that can be attempted in the way of ornament, without detracting from that general appearance of strength which is the most suitable characteristic of such structures.

In some modern bridges small columns with an entablature have been placed above the starlings; an arrangement which is obviously of no possible utility, for the mind is left in doubt whether the columns have been added for the entablature, or the latter for the columns. Besides this, the effect of the broad plane masses of the spandrels, which by themselves present a massive form, is destroyed by these small ornaments, which are not in character, even in size, with the rest of the structure. The architectural effect produced by a level bridge is considered, by some, as a great beauty, from its giving a character of great boldness to the structure; but this beauty rests on rather contestable grounds, for a structure, whose principal character is strength, should not only be solid, but should appear so; and there can be no doubt that to the eye, a bridge, which rises from the two extremities to the middle, presents an appearance of more security than one on a dead level throughout.

Masonry of the Bridge. The same principles apply to this branch of the construction of bridges as have been laid down for structures of masonry in general, both as regards the foundations and the superstructure.

The facings of the piers and abutments in large arches (Fig. 20) should be of cut stone: in other cases, the ends, or other parts most exposed to shocks from floating bodies, may alone be of cut stone. The filling in, for both cases, may be of good rubble, laid with care in full mortar, and grouted. The top course of the piers and abutments, upon which the arches immediately rest, should be of cut stone in large blocks, well clamped and bonded.

The form and dimensions of the abutments are different from the piers, as they sustain not only the weight of half of the extreme arches, but also the horizontal thrust of those arches. Their thickness (*Note* 5) should therefore be regulated to counteract this thrust, whose tendency is to cause the abutments to yield, either by sliding on the bed of the foundation, or by oversetting around the exterior foot of the foun-

dation. Besides this thickness, *buttresses*, or *counterforts*, (Fig. 76,) are added to the abutments of large arches, where the horizontal thrust is very great. A buttress is placed at each end of the abutment, and one, in some cases, in the middle.

The voussoirs should be laid with great care, and the open joints at the back be well wedged with slate and grout. The spandrels (Fig. 20) between the head walls should be constructed either of hammered stone, or of the best rubble, in order to obtain a very compact mass between the voussoirs of the two contiguous arches. This interior mass is usually built up to a height of one fourth of the rise above the piers. This height should be regulated by the form and dimensions of the arches, and the greater or less strength which they will demand at this point. In some modern bridges this interior mass is built of cut stone, (Fig. 77,) the lower courses being horizontal, and the top course an inverted arch. There seems to be an excess of precaution in this arrangement, as, in the ordinary form of cylindrical arches, there will be, if any, but a very slight tendency to throw up the horizontal courses.

The backing of the arch (Fig. 20) is usually finished off with a plane surface; the capping of two contiguous arches forming a gutter over the piers, where the water which filters through the road-way, is received, to be conveyed off by an iron conduit, leading through the areh itself to the soffit, or else through the mass of the pier to the low water level. The latter is the more recent arrangement, and is considered the best, as the masonry is not exposed to injury from the water trickling along its surface.

In some cases (Fig. 77) the surface of the backing is curved. The masonry of the backing is usually of rubble laid with extreme care to obtain an impermeable mass. The capping (Fig. 20) is formed of the best hydraulic mortar prepared as for flash pointing.

The facing of the head walls is usually of cut stone,

and the backing of rubble, or brick masonry. The blocks of the cornice, when practicable, should be each of a single stone, and the joint between the cornice and lowest course of the parapet wall should be above the level of the foot-path, to secure the mortar at the joint, from injury from the water, which may collect on the foot-path.

The thickness of the head walls will depend on the construction used to sustain the road-way. The ordinary method, in most of the old bridges, was to fill in between the head walls with earth or rubbish. When this is done, the head walls should be regulated to sustain the pressure of the earth resting against them. In the bridges of recent construction the road-way usually rests on a flooring of large flagging stone, or one of rubble or brick masonry, supported beneath either by a system of walls parallel to the head walls, or by a system of groined arches. The latter system (Fig. 78) is used when the road-way rests on a flooring of masonry, and the former, (Fig. 78,) when flagging stones are used for the same purpose. The object of this construction is chiefly to relieve the piers of the weight of a mass of heavy materials, which is often desirable when the subsoil of the foundations is not very firm. It is most serviceable in arches where the rise is great. The head walls in this case being relieved from all lateral thrust, except a comparatively small one when groined arches are used, may be less thick than when they sustain the pressure of the filling in of earth, and they may, therefore, be regulated as in ordinary structures, an allowance, however, being made both for their not receiving any lateral support, and for the shocks to which they are exposed from floating bodies, and the passage of heavy vehicles over the bridge. It is usual in practice to give the head walls a mean thickness of one fifth of their height.

Approaches. The arrangement of the extremities of the bridge will depend on the directions of the lines of approach to them; the width and the position of the road surface with respect to the natural surface of the ground; and the locali-

ty. The principal objects to be kept in view are, to make the extremities of easy and safe access to vehicles, and to place as little obstruction as practicable, to the navigation under the extreme arches.

In small unimportant bridges, where the approach is direct, and of the same width as the bridge, and the road surface is but slightly raised above the natural surface, the simplest arrangement will be to prolong the head walls (Fig. 79) so far into the embankment of the approach that the foot of the embankment towards the river shall be thrown back two or three feet from the top of the river bank. The side slope of the embankment, adjoining the bridge, is rounded off to form a conical surface, which is usually faced with dry stone, or with sods.

When there are several lines of approach, or when a single approach is wider than the bridge, and the road surface is above the natural surface, it will be necessary to widen the access to the bridge, and to sustain the embankments, where they join the bridge, by building walls from the bridge, which are termed *wing-walls*.

The simplest arrangement of a wing-wall consists in prolonging the head wall (Fig. 80) gradually outwards, in the form of an arch, as far as the outward edge of the embankment, and from this point giving its top surface the same slope downwards as the side slope of the embankment. The parapet wall is also prolonged to the outward edge of the road surface.

The more ordinary arrangement of a wing-wall consists in building a strait wall, (Fig. 81,) making an angle, or *flare*, with the head wall. The top of this wall is continued on a level with the road surface to its exterior edge; from this point it receives the same slope as the side slope of the embankment. The parapet wall is arranged as in the preceding case.

As the wing-walls sustain a heavy pressure from the embankments, they should receive a batter, regulated like other

sustaining walls, (*Note* 6,) and their thickness should be sufficient to resist the action of this pressure. The facing of the wall may be of cut stone, and the backing of rubble or brick masonry. The sloping top surface should be covered with a coping arranged in the manner that has been laid down for walls with a great batter. The extremity of the wall, at the foot of the embankment, should be terminated with a *buttress* or *newel*, formed of heavy blocks of stone.

The preceding are the most ordinary arrangements of wing-walls. Other arrangements (Fig. 82) of a more complicated character will depend on the nature of the locality, and will demand a particular study.

Where the navigation of the river is partly effected by towing, an arrangement must be made to carry the tow-path either under the extreme arch, through its abutment, or over the road surface of the approach. If the arch is a flat segment arch, the tow-path may, in most cases, be carried under it. With full centres and oval arches it will be necessary to form an arched opening through the abutment for the tow-path, unless the access from the river bank to the road surface of the approach is so easy as to present no obstruction to passing from one side of the bridge to the other.

Water-Wings. To secure the river banks near the bridge from the action of the current, and to prevent the foundations of the abutments from being undermined, the banks should be faced with a revetment of dry stone, and an enrockment should be made around the abutment. In some cases it may be necessary to secure the foot of each of these constructions by a row of piles, or of sheeting piles, which will serve the double purpose of preventing the current from undermining the foot, and of sustaining the pressure of the facing and the earth behind it. These structures are termed *water-wings*. When the abutments project into the stream, the water-wings should gradually narrow the width of the stream from some point on the banks above the bridge to the face of the abutment.

Enlargement of Water-Way. As the obstruction to the passage of the water during freshets is very great in the full centre and oval arches, owing to the size of the spandrels, attempts have been made to remedy this defect. For the bridge is greatly exposed at these epochs from the pressure of the water against the head, from its upward pressure against the soffit of the arch, and from the decreased weight of the bridge due to the difference of the specific gravity of its materials and water. The means which have been resorted to for this purpose consist either in enlarging the water-way at the heads, in order to accommodate it to the contraction of the fluid vein; or in forming an additional water-way of a cylindrical form through the spandrel spaces. The former method has been applied to some of the French bridges, and consists, simply, (Figs. 70 and 83,) in removing a portion of the masonry which forms the edge between the soffit of the arch and the facing of the head walls, thus forming a new surface suitable to the object in view. The latter method is found in some of the old bridges, and is nothing more than an arched conduit of brick or stone through the spandrel spaces.

Centres of the Arch. The wooden frame by which the arch stones are supported, during the construction of the arch, is termed a *centre.*

The centre is an indispensable accessary in the construction of an arch, because the voussoirs will not be retained in a state of equilibrium, by the friction along their beds, beyond a certain inclination of the bed to the horizon; which limit is found by ascertaining the angle of friction for each kind of material.

For ordinary cut stone, laid dry, the angle of friction is 30° with the horizon; if laid in thin tempered mortar, the angle of friction will be between 34° and 36°; and for very porous stones, laid in full mortar, it will be nearly 45°. The voussoirs, (*Note* 7,) therefore, above the joint whose inclination is the same as the angle of fricton must be supported by a centre until the key stone is laid.

Centres for small light arches (Fig. 84) usually consist of several light frames, made of plank in two or more thickness solidly nailed together, shaped on the exterior like the curve of the intrados, and which are covered by horizontal strips of boards, upon which the voussoirs rest. The frames are termed *ribs*, and the horizontal strips *bolsters*. Each thickness of the rib may be formed of two or more pieces of plank abutting end to end, the pieces of one thickness breaking joints with those of the other.

Centres for large heavy arches require to be framed of heavy timber. The frame consists, as in the preceding case, of several ribs, shaped on the back to the form of the intrados, and of bolsters of scantling.

The frame of a rib consists, essentially, of several strong pieces of timber, which abut end to end, and are shaped to the form of the intrados; these are supported by inclined pieces, or struts, that rest on points of support, either at the foundations, or on the ground between the piers, as may be most convenient. The struts are prevented from yielding in a lateral direction by braces.

The same principles apply to the framing of centres as for other heavy structures of timber. Two points, however, requre peculiar care in centres: the whole frame should be perfectly stiff, in order that it may undergo no other change, from the pressure of the arch, than what must arise from the elasticity of the wood; and the connection of the pieces should be of the simplest character, in order that the frame may be readily set up and taken apart.

The arrangement of the struts and braces will depend on the position of the points of support; and as these may be either at the foot of the piers, or at intermediate points between them, the systems of centering may be divided into two classes.

For the first class, where there are intermediate points of support, the curved parts, (Fig. 85) which receive the bolsters, are supported by struts. The struts are normal, or perpendicu-

lar, to the curve of the intrados, and rest on a horizontal beam which is supported beneath by props or shores. The shores are uprights which rest on the points of support. Any tendency to lateral flexure in the struts may be counteracted by braces, formed of pieces which are notched on the struts, and are bolted together in pairs, so as to clamp the struts between them.

For the second class, the points of support, on which the struts immediately rest, are formed by placing a beam on each side of the arch, in a horizontal, or an inclined position, near the joint which makes the angle of friction ; this beam being supported beneath by props, which rest on the offsets of the foundations. The arrangement of the struts may be either the one shown in Fig. 86, in which one end of each strut rests on the beam, whilst the other end rests against a horizontal beam placed between the corresponding struts on each side of the arch ; or a strong frame, (Fig. 87,) consisting of two struts and a horizontal beam, may be set up on the points of support to receive the ends of the struts which support the curved pieces ; or finally (Fig. 77) the end of each strut may rest on one point of support, whilst the other end rests against the end of another strut, sustained by the opposite point of support.

In centres of the first class, the frame of the rib will remain in a state of stable equilibrium in whatever way the courses of the voussoirs may be carried up ; for the pressure, which is normal to the intrados, being directly transmitted by struts to the points of support, will have no tendency to change the shape of the frame, except what arises from the compression of the fibres.

In the second class, the two first systems will be in a state of equilibrium only when the courses of the voussoirs are carried up regularly on both sides of the arch at once ; for, otherwise, the opposite pressures, in the direction of the horizontal beams, being unequal, and not transmitted directly to the points of support, will have a tendency to change the form of the frame. In the last system, (Fig. 77) the pressure at

each point being directly transmitted to the point of support, there will be no tendency to a change of form, in whatever way the voussoirs are carried up.

The braces used for the second class generally consist of pieces normal to the intrados, which are notched on the struts and horizontal beams, and are bolted together in pairs.

The frame of each rib for large arches usually consists of two distinct parts: the one fixed, the other movable. The fixed part (Figs. 77 and 88) is composed of the beams which form the point of support, with their shores; the movable part is the portion of the rib above these beams, which is so arranged that it can be slightly raised or lowered as circumstances may require. The means used, to effect this purpose, consist in forming indents, or steps, on the upper surface of the beam which rests immediately on the shores, to receive another movable beam, the top and bottom surfaces of which are formed into indents, corresponding to those in the fixed bottom beam, and which give the movable beam a wedge-like shape. Another beam, of the same size and shape as the fixed bottom beam, the bottom surface of which is indented to fit the top of the movable wedge, is laid on the movable wedge; and the end of the struts either rest immediately on the top of this last beam, or else are received into a cast iron socket which is fastened to it.

By this simple arrangement it is evident that the movable part of the centre can be lowered by driving back the movable wedge, or can be raised by the reverse operation. As the arrangement is principally serviceable in taking down, or *striking* the centre, the indented beams are termed the *striking plates.* The surfaces of the plates, in contact, may be covered with copper, or some other metal, and be kept well greased, to facilitate the operation of striking the centre.

A striking plate is placed at the point of support for each rib, or else all the ribs may rest on several striking plates, which run the entire length of the arch between the heads. The latter is considered the safer arrangement, as accidents

might happen to the workmen under the arch should the frame work give way in the act of striking.

The wedges are driven back by mauls, handled by men; or, if this method should prove insufficient, or its effects be too irregular, a heavy beam may be suspended from the crown of the centre, and sufficient momentum be given to it, by a motion of oscillation, to drive the wedges back equally.

The parts of each rib (Figs. 77 and 88) are firmly connected by means of iron bolts and straps; and, wherever it may be necessary, by joints of the strongest and simplest forms. A rib is placed under each exterior ring course; and intermediate ribs are placed, from four to six feet apart, according to the weight of the arch, and the strength of the bolsters and ribs. The ribs are connected by horizontal ties, running the entire length of the arch, which are notched on the braces and struts, and are bolted in pairs like clamping pieces. Besides the ties, diagonal braces are placed between each pair of ribs to counteract any tendency to warping laterally.

WOODEN BRIDGES.

A wooden bridge is composed of three distinct parts: 1st. The points of support of the superstructure: 2d. The frame work which sustains the road-way: 3d. The roadway.

The points of support for the superstructure, which are the abutments and piers, are formed either of wood or of stone; the choice of the two systems depending, principally, on the character of the stream, and the nature of the bed. Stone piers and abutments are generally preferable to those of wood, from being of a less perishable nature, and offering more resistance to floating bodies and the action of freshets.

The general arrangement of the stone piers and abutments (Fig. 89) of wooden bridges differs in nothing, except in a few details, required by the character of the superstructure, from those of the stone bridges. The starlings are carried

up above the level of the highest water, and the portion of the supports above them, on which the road-way rests, is a simple pillar with plane faces. The lowest points of the frame work, abutting against the pillars, should be above the level of the highest water, to preserve the wood work from decay, arising from exposure to alternate dryness and moisture.

Wooden abutments may be formed by constructing what is termed a *crib-work*, which consists of large pieces of square timber laid horizontally over each other, to form the upright or sloping faces of the abutment. These pieces are halved into each other at the angles, and are otherwise firmly connected together by iron bolts. The space enclosed by the crib-work, which is usually built up in the manner just described only on three sides, is filled with earth carefully rammed, or with dry stone, as circumstances may seem to require.

A wooden abutment of a more economical construction may be made, by partly imbedding large pieces of timber placed in a vertical, or an inclined position, at intervals of about four feet from each other, and forming the facing to sustain the earth behind the abutment of thick plank. Wooden piers may also be made according to either of the methods here laid down, and be filled with loose stone, to give them sufficient stability to resist the forces to which they may be exposed; but the method is very clumsy, and is inferior, under every point of view, to stone piers, or to the methods which are about to be explained.

The simplest arrangement of a wooden pier consists (Fig. 90) in partly imbedding large pieces of timber, which are placed in a vertical position from two to four feet apart. These upright pieces are connected at top by a horizontal beam, termed a *cap*, which is either mortised to receive a tenon made in each upright, or else is fastened to the uprights by bolts or pins. Other pieces, which are notched and bolted in pairs on the uprights, are placed in an inclined, or diagonal position, to brace the whole system firmly. The several uprights of the pier are placed in the direction of the

thread of the current. If thought necessary, two horizontal beams, arranged like the diagonal pieces, may be added to the system just below the lowest water level. In a pier of this kind, the place of the starlings is supplied by two inclined beams on the same line with the uprights, which are termed *fender-beams.*

A very capital objection to the system just described, arises from the difficulty of replacing the uprights when in a state of decay. To remedy this defect, it has been proposed to drive large piles in the positions to be occupied by the uprights, (Fig. 91,) to connect these piles below the low water level by four horizontal beams firmly fastened to the heads of the piles, which are sawed off at a proper height to receive the horizontal beams. The two top beams are arranged with large square mortises to receive the ends of the uprights, which rest on those of the piles. The rest of the system may be arranged as in the former case. By the arrangement here explained two points are gained; the uprights, when decayed, can be readily replaced, and they rest on a solid substructure not subject to decay; shorter timber can be used for the piers than when the uprights are driven into the bed of the stream.

In deep water, and especially in a rapid current, it is thought that a single row of piles would prove insufficient to give stability to the uprights; and it has therefore been proposed to give a sufficient spread to the substructure to admit of bracing the uprights in two directions, that is, to add, besides the diagonal braces already described, struts on each of the other sides. To effect this, three piles (Fig. 92) should be driven for each upright; one just under its position, and the other two on each side of this, on a line perpendicular to that of the pier. The distance between the three piles will depend on the inclination and length that it may be deemed necessary to give the struts. The heads of the three piles of each upright are sawed off, and connected by two horizontal clamping pieces below the lowest water level. A square mor-

tise is left in these two pieces, over the middle pile, to receive the uprights. The uprights are fastened together at the bottom by two clamping pieces, which rest on those of the heads of the piles, and are rendered more stable by the two struts.

In localities where piles cannot be driven, the uprights of the piers may be secured to the bottom by means of a grillage, arranged in a suitable manner to receive the ends of the uprights. The bed, on which the grillage is to rest, having been suitably prepared, the grillage is floated to its position, and sunk either before or after the uprights are arranged to it, as may be found most convenient. The grillage is retained in its place by an enrockment. As a farther security for the stability of the piers, the uprights may be covered by a sheeting of boards; and the spaces between the sheeting be filled in with gravel.

As wooden piers are not of a suitable form to resist heavy shocks, ice-breakers should be placed in the stream, opposite to each pier, and at some distance from it. In streams with a gentle current, a simple inclined beam (Fig. 93) covered with thick sheet iron, and supported by uprights and diagonal pieces, will be all that is necessary for an ice-breaker. But in rapid currents a crib-work, having the form of a triangular pyramid, (Fig. 94) the up-stream edge of which is covered with sheet iron, will be required to offer sufficient resistance to shocks. The crib-work may be filled in, if it be deemed advisable, with loose blocks of stone.

Frames. In no branch of constructions has more diversity of arrangement, or greater boldness of design been shown, than in the frames of wooden bridges. Wherever ingenious practical carpenters have been found, structures of this kind have been raised, which, for judicious arrangement throughout, have called forth the admiration of the most scientific. Our own country can produce many very remarkable wooden bridges which have gained even an European celebrity.

Whatever, however, of apparent diversity may seem to ex-

ist in the great number of wooden bridges, they can all be reduced to two general classes; each of which admit of two subdivisions. In the first class may be placed all the combinations which are composed of straight timber; and in the second those which form wooden arches. The subdivisions of each arise from the position of the road-way; it may either rest on the frame, or else be suspended from it.

First Class. The simplest arrangement of the first subdivision of this class consists (Fig. 95) in a system of longitudinal beams termed *sleepers*, or *string pieces*, laid parallel to the axis of the road-way. They are slightly notched on the caps of the piers, and are fastened to them with bolts. The sleepers receive the cross joists of the road-way on which the boards and other road covering are laid. The distance between the sleepers will depend on their strength, and on that of the cross joists. It will seldom admit of being over six feet. This system can seldom be used with safety for a carriage road where the width of the bays is over twelve feet.

In bays of twelve to twenty feet wide, short pieces, termed *corbels*, (Fig. 96,) must be placed on the caps of the piers. The sleepers rest on the corbels, which serve the purpose of diminishing the bearing which may be considered in this case as the distance between the ends of the corbels.

When the bay is over twenty feet wide, the corbels will be subject to spring, or bend downwards, if made sufficiently long to give an effectual support to the sleepers; they must therefore, in this case, be strengthened by struts (Fig. 97) mortised into them near their ends, and abutting against the uprights of the piers. This method may be used for bays thirty feet wide.

Beyond thirty feet, and within forty, it will be better to replace the corbels by a beam placed under the middle portion of the sleepers, which is termed a *straining beam*. The straining beam (Fig. 98) is sustained by two struts which abut against it. The struts in this system may be so long

as to be liable to *sag*, or bend downwards; to prevent which, inclined clamping pieces, termed *stirrup pieces*, are fastened to them and to the string pieces.

For bays above forty feet, it will be necessary to use both corbels and straining beams, (Fig. 99,) with a suitable combination of struts and stirrup pieces. The struts in this case may be parallel to each other, or not, as may be found most suitable; the angle, however, between them and the straining beam should not be greater than 150°, to give sufficient stability to the system.

The simplest arrangement of the frames in the second subdivision consists (Fig. 100) in a horizontal sleeper, termed a *tie-beam*, the extremities of which rest on the points of support of two inclined pieces, which are mortised into the tie beam near the points of support; and abut end to end at the other extremities, or else against an upright, termed a *king-post*, placed at the middle of a tie beam, to which it is fastened by a stirrup of iron, or by a tenon and mortise. The cross joists are laid on the tie beams, and with it are suspended from the inclined pieces, by the intermedium of the king-post. This last piece may also be made like the ordinary stirrup-pieces, being notched on the tie-beam. A combination of this kind cannot be well arranged for bays over forty feet in width.

For bays between forty and one hundred feet, a similar system (Fig. 101) may be arranged, by placing a straining beam between the upper ends of the inclined pieces, and suspending the tie-beam and road-way from these points by two stirrup pieces, termed *queen-posts*. It will be necessary in widths over forty feet to place diagonal braces in the space between the queen-posts, stirrup pieces, and tie-beam.

When the bay is over one hundred feet, a longitudinal beam (Fig. 102) is placed at top parallel to the tie-beam, and at a suitable distance from it, several inclined pieces abut against several straining beams placed under this longitudinal beam, and stirrup pieces are placed at all the points of

junction between the inclined and straining pieces. This combination may be extended to very wide bays. It is the same as that used for the celebrated bridge of Schaffhausen, one of the bays of which was 193 feet span.

Another combination of straight pieces for wide bays, consists of a large built-beam, as described under the head of carpentry, which is formed of two parallel built-beams, firmly secured in their places by uprights and diagonal braces; or else simply of a series of diagonal pieces placed close together, and firmly secured to each other, forming a kind of lattice work. Each of these systems is now in extensive use in our country. The road-way in these systems may be placed either at top, at bottom, or at any intermediate point between these two.

In frames belonging to this subdivision, not more than four frames, or *ribs*, can be set up for the road-way; two exterior, and one or two interior, dividing the total width into two equal parts, each of which may be only wide enough for the passage of one carriage and for foot passengers.

Second class. The simplest arrangement of the frames of this class consists (Fig. 103) in slightly bending a large beam, and confining it in this state between two fixed points of support, so that it may be made to support the middle point of the sleepers for the cross joists. This system may be applied to bays from twenty to forty feet wide, where timber of sufficient dimensions can be procured.

The most usual form of wooden arches consists (Fig. 89) in making the arch of several concentric courses of timber bent to a suitable curvature; the different courses being firmly united by keys of hard wood, and stirrups, or hoops of iron. The sleepers rest on the crown of the arch, and are supported, between this point and their extremities, by vertical or inclined stirrup pieces, which transmit their weight and that of the road-way to the arch.

The position of the stirrup pieces is not a matter of indifference; the upright, or vertical position, being superior to the

inclined, in which the pieces are perpendicular to the arch; for, in this last position, the inclined pieces not only transmit the vertical weight of the road-way to the arch, but also a horizontal compression. This compression is caused by the tension on the sleepers, which acting on the upper ends of the stirrup pieces, causes an equal action on the arch at their lower extremities. Besides this, the crown of the arch is subjected to a greater portion of the weight of the bridge when the pieces are inclined, than when they are in a vertical position; and as this part is the weakest, it will therefore require additional strength to resist this additional pressure.

The arch may be relieved of a portion of the weight of the road-way by inclined pieces or struts, which abut against the points of support, and sustain the weight on the portions of the sleepers nearest those points. This arrangement, which is very judicious, can be made when the road-way rests on the arch. (*Note* 8.)

For very large spans the arch may be formed of two concentric built beams, united by cross stirrup pieces and diagonal braces.

The second subdivision of this class embraces those cases where the sleepers and road-way are suspended from the arch by upright stirrup pieces of wood or wrought iron. Each arch in these cases may abut against two fixed points of support, (Fig. 104); in which case the sleepers may be either on a level with the points of support, or else be suspended at some point above them; or the arches may be let into the sleepers near their extremities. The sleepers in this last arrangement, (Fig. 105,) acting as ties, must either be of one entire length, or be formed of a strong built beam, arranged with indents, or with wooden keys and iron fastenings. In this subdivision, the road-way must be divided into two paths by one or two arches which rise in the centre of it.

Wooden arches may be made to span very wide bays; but in general it would be well to restrict the span to 300 feet,

and for streams over this width, to divide the space into two or more bays, as circumstances may point out.

The arrangement of the frame work of such considerable structures requires great judgment, skill, and care, on the part of the engineer. Simplicity should be regarded as an essential condition, so that any part may be easily taken out, and be replaced, without deranging the rest. The points where the frames rest against the support should be above the highest water level, to preserve the essential parts from decay; and it would be a judicious arrangement to leave an open joint between all the courses of built beams, or other heavy essential parts, which rest on each other, and are connected by bolts or hoops, to allow a free circulation of air around the pieces, and prevent the accumulation of moisture between them.

The different ribs must be firmly connected by horizontal ties, formed as clamping pieces, which are bolted in pairs to the frames, and by diagonal braces, to prevent lateral motion, caused by the action of the wind, or the warping of the frames.

Road-way. The road-way is variously formed, according to the greater or less care which it may be deemed necessary to bestow on the structure. The best arrangement consists in laying a system of cross joists on the sleepers, to receive a flooring of thick boards on which the road covering rests. The joists are slightly notched on the sleepers, and if the road covering is intended to be of plank, a second thickness of boards is nailed across the first to form the road surface.

The foot-paths (Fig. 106) receive a necessary elevation above the road surface, and consist simply of a common flooring of boards laid on joists.

The parapet of the bridge may consist of a simple hand railing supported on uprights, and braced by inclined pieces, or else something of a more ornamental character may be arranged either of wood or iron, according to the locality.

The principal objection to wooden bridges arises from the perishable nature of the material. As this objection applies only to structures which are alternately exposed to moisture and dryness, a remedy may be readily found in covering the bridge with a roof and sides of shingles, or with the common weather-boarding of frame houses. In bridges, where the road-way rests on the frames, some difficulty might arise in arranging the roof, but a substitute could be found for it, by covering the planking of the road-way with a metallic covering, to protect the frames at top, and by covering the sides in the ordinary way. Besides these conservative means, the parts of the structure most exposed should be covered with paint, pitch, or any other coating which may be found most efficacious.

RAIL ROADS.

It was shown in treating the subject of common roads, that a great portion of the resistance to the force of traction arose from the friction and shocks occasioned by the inequalities of the road surface; and as these causes of retardation to the motion of vehicles are found, in a greater or less degree, in all ordinary road coverings, it seemed natural to seek a remedy for them by trying some other material not liable to the same objections. Various means have accordingly been tried, with greater or less success; in some cases a way or *track*, has been prepared for each wheel to roll on, formed of long narrow blocks of stone, presenting a uniform even surface; in others, large beams of timber have been applied in a similar manner, with an occasional coating of sheet iron on the surface, where the wear and tear was greatest; finally, iron was substituted for wood, and that system of road covering, now so well known as the *rail-road*, resulted from this last improvement.

A rail-road consists, then, of two ways, or tracks, for the carriage wheels, the surfaces of which are slightly raised above the general surface of the road-way; the rails of each

track being firmly attached to solid supports, imbedded below the road-way surface.

Form and material of Rails. The first attempts made in this kind of road covering were with cast iron rails, termed the *tram* or *plate rail*, the cross section of which presented a bed, or plate, of sufficient width to receive the wheel, with a raised edge, cast on the exterior edge of the plate, to confine the wheel to the track. This plan was found to be subjected to several inconveniences, not the least of which arose from the mud thrown on the plate, and which suggested the idea of what is termed the *edge rail*. The form first adopted for the edge rail consisted in a cast iron bar three feet long, which, in side elevation, presented a semi-ellipse, the cross section of which was of the form presented in Fig. 109. This form was found better than the tram rail, as the wheel was made with a suitable tire to run on the track with safety; but it was observed to have several defects, arising from the nature of the material itself. These were, principally, in the great liability of the rails to break; the great number of joints; the difficulty of uniting the rails so firmly at their ends as to preserve a uniform surface for the track; and, finally, the wear and tear at the surface, which was observed to be very uneven so soon as the hard exterior of the surface was worn off. These defects led to experiments on rails of wrought iron, which resulted completely in their favor, under every point of view, and in the rejection of cast iron.

The wrought iron rails are all formed on the edge rail plan, and are made of rolled iron, fashioned by machinery suitably arranged for the purpose. The form of the rail, both in cross section and side elevation, has undergone several modifications pointed out by theory, and confirmed by experiment. The form first tried, and which is still in use, was similar to that of the cast iron edge rail (Fig. 107); the bars were rolled into lengths of eighteen feet, every three feet of the length presenting in side elevation a semi-elliptical figure. This form, commonly termed the *fish-bellied* rail, being pointed out

by theory as the one which presents equal strength throughout; but, although very suitable for the intended end, under this point of view, the semi-ellipse does not offer throughout the same stiffness as other forms, in which the depth is uniform; and both experiment and theory show that the *straight*, or *parallel*, edge rail, with a cross section, as shown in Fig. 108, is the most suitable form both for strength and stiffness, besides presenting greater stability, with the same area, of the various figures that can be given to the cross section. For, admitting the usually received hypothesis of the contraction and elongation of the fibres of a solid submitted to a cross strain, the expression for the strength and stiffness of this figure will be found by substituting respectively, for the terms bd^3 and bd^2, in the expressions given in the subject of carpentry for the strength and stiffness of rectangular beams laid horizontally, and submitted to a strain at any point, the following terms,

$$bd^3 - b'd'^3, \text{ and } \frac{bd^3 - b'd'^3}{b},$$

in which b is the total breadth of the cross section, d its total depth, b' the total breadth of the projections of the *flanges*, or of the top and bottom *tables* of the rail, and d' the depth between them.

The bottom of the fish-bellied rail is not a perfect semi-ellipse, as that form would present some practical difficulties in the rolling of the bar, and in fitting it upon its supports. The total depth of the rail is usually five inches, (Fig. 107,) and its depth at the points of support from three to three and three-quarter inches. The upper table is usually two inches wide at the top, which is slightly convex, and has its angles rounded off, and about half an inch thick. The rib is nearly one inch thick, being made in some cases plain, and in others with a slight projection on one side near the bottom.

In the straight rail (Fig. 108) the depth is uniform; the upper table is of the same form and dimensions as in the fish-bellied rail, the rib is plain and varies between six and eight-

tenths of an inch in thickness, the lower web is in some examples not so wide as the upper table, by nearly half an inch, and in others it is somewhat wider. The best proportions for these three parts of the cross section still remain to be established by experiment, as well as the total depth, which, at present, varies between four-and-a-half and five inches.

The straight rail is now coming into very general use, owing to the advantages which it presents over the fish-bellied rail, not only as regards stiffness and stability, but also from the ease with which it can be cut to any suitable length for the turnings in a track.

In our own country, motives of economy in the first cost of such works, directed attention to the combination of wood and iron for rail-ways; and the method which has usually been followed consists in fastening a narrow bar of rolled iron on long string pieces of timber (Fig. 110) to form the tracks. The string pieces are usually from six to nine inches in depth, and six inches broad, and of any suitable length. The bars are generally two inches wide, with the top surface very slightly convex; their depth varies from three-eighths to three-quarters of an inch, and their length is usually eighteen feet. The bars are fastened on the top of the beam, somewhat without the inner edge, by means of strong screws, which pass through holes of an elliptical form made in the bars, the top of the holes being enlarged, so that the head of the screw may not project above the top of the bar. A small plate of zinc or iron is let into the string pieces, under the joint between two bars, to prevent their ends from crushing the wood, when the wheels of the cars pass over them. The joint between the bars is in some cases like a mitre joint, in others like a tenon and mortise, the object in both cases being to prevent the shock in passing from one bar to the other over an open joint.

A combination of bars, of the form just described, fastened to stone sills, or string pieces, has also been tried with us;

but experience has shown, that it is inferior to the method just explained, from the bars wearing loose from the stone, owing to the disintegration of the latter, arising from the action of the wheels on the bars.

Supports of Rails. The ordinary substructure for the rails, consists either of large stone blocks of a cubical form, which are placed at equal intervals apart, along the line of rails; or of wooden sleepers (Fig. 108) laid crosswise to the track, also at equal intervals apart. To fasten the rails to their supports, a contrivance, termed a *chair*, is used, which consists of a bed of cast, or wrought iron, so arranged that it can be fastened to the supports, and can receive the rail, which is fastened to it by inserting a wedge between the sides of the chair and rail.

Various forms have been given to the chair, the object in every case being to give stability to the rail, by preventing any motion, except that which arises from the elasticity of the metal, and at the same time, to allow sufficient play for the contraction and expansion of the rail between the ordinary limits of temperature. To effect these objects, the cast iron chair has usually been cast in one piece, (Fig. 109,) presenting a horizontal part, or bed, which rests on the support, and two upright parts, or sides, so fashioned on the interior as to fit the sides of the rail, and allow of its being confined by the insertion of an iron wedge. The cast iron chair presents two inconveniences, which have been found of a very serious character; the first arises from the brittleness of the material, and the accidents to which it is exposed from shocks; the second grows out of the difficulty of fastening the rails so firmly to each point of support as to prevent all motion, and at the same time to allow for the contraction and expansion of the metal, which occasion a strain of a very powerful nature on the fastenings. The last of these inconveniences is partly remedied by fastening the rail firmly to the middle chair, so that the expansion and contraction may act equably from this point to the ends. The first is also in process of

improvement, by the substitution of wrought iron for cast iron. The chair in this case, (Fig. 108,) consists of three pieces, a bed on which the rail rests, and two side pieces to confine the rail on the bed, and keep it stable, which are fastened to the bed by screw-bolts and nuts.

To fasten the chairs to the stone supports, two holes are drilled into the block to receive strong oak pins, which are firmly driven into them; an iron bolt is driven into each of these pins, through holes which are left for them in the bed of the chair. This method is bad under two points of view, the iron bolts are apt to work loose, and the blocks are liable to split, from the great strain on the sides of the holes, arising from the successive drivings of the pins and bolts.

With wooden sills, a firmer fastening is made, by inserting the iron screw bolts in holes bored entirely through the sill, the head of the bolts being beneath the sill, and confining the chair to the sill by means of nuts screwed on the projecting ends of the bolts.

When the rail consists of a combination of wood and iron, the string pieces are laid either on stone blocks, as for the solid iron rail, or on cross sleepers. In the first case, the chair is formed simply of two uprights of cast iron, which are fastened in the usual manner to the block, and the string piece is confined to the chair by means of a screw bolt, which is passed through the holes made in the uprights of the chair and the string pieces, and confined by a nut. In the second case (Fig. 110) a notch is cut in the cross sleeper to receive the string piece which is confined in the notch by a wooden wedge.

The blocks, or cross sills, as the case may be, are either laid on the natural soil, a hole or trench of sufficient size being dug to receive them, and are well secured by ramming the fresh earth around them; or else the hole, or trench, is made several inches larger in breadth and depth, than the block, or sleeper, and receives a layer of broken stone, which serves both as a bed for the support, and as a drain for the

water, in order to counteract the action of the frost were the subsoil, near the supports, to imbibe water.

In new embankments the rails should, at first, be always laid on cross sleepers, to counteract the effects of unequal setting; and as a farther precaution, the cross sleepers should rest on longitudinal string pieces, placed directly under the lines of the rails, these pieces being prepared with slight notches to receive the sleepers. By this precaution, the weight between any two sleepers will be distributed over a greater surface, and any unequal settling at one point will be less felt at the others, and will cause but a slight derangement in the position of the two lines of rails.

Too much precaution cannot be taken to effect a thorough draining of the subsoil under the track; and, if it be deemed necessary, the whole of the natural soil should be removed, and its place be supplied by a soil which will give a freer passage to the water, and be less subject to break up from frost.

In low swampy localities, subject to overflow during heavy rains, the rails may be raised on trestles, or any other similar structure; but as the frame work, in such a case, would require to be very solidly put together, and would be subject to very rapid decay, the question would present itself, whether it would not be better to raise an embankment on a brush foundation.

Arrangement of the Tracks. A rail-way is arranged with either a single or a double track of rails, according to the exigencies of the trade. The rails of each track are placed from four-and-a-half to four-and-three-quarters feet apart, according to the length of the axle-trees of the wheels, a free play of two inches being left for the flanges of the wheels, when the cars are running on a straight track, to prevent the rubbing of the flanges against the rails. In a double track the width between the two will depend on the width of the cars; it should be such, that two cars passing each other should, under no possible contingency, short of flying the track entirely, come into contact.

When the track is straight, the upper surface of the rails should be on a level. In some cases this surface of each rail has received a slight pitch inwards, to keep the wheels from running up on either rail; but this is unnecessary in a straight track, as the form of the tire of the wheel would counteract any tendency to lateral deviation.

The horizontal changes of direction in the track are made by an arc of a circle tangent to the two straight branches. The curvature of the circle (*Note* 2) should, in all cases, be made as small as a judicious regard to the excavations and embankments will permit, both on account of the tendency of the centrifugal to throw the car from the track, thereby creating a great friction of the flange on the side of the exterior rail, and also to diminish, as much as possible, the sliding of the exterior wheel on its line of rail, caused by the greater length of this line. Each of these defects are partly counteracted by the form of the tire of the wheel, (*Note* 9,) and in addition to this, the outside line of rails, along the curve, (*Note* 9,) should be sufficiently raised above the interior line to allow the component of gravity, in the cross direction of the track, to counteract the centrifugal force of the car in passing the curve with the greatest admissible velocity.

Experiments are wanting to show the exact retardation arising from the curvature of the track. From some made on the Baltimore and Ohio Rail-Road, it would seem, that the resistance on a curve of four hundred feet radius, was about once-and-a-half as great as that on a straight track. In changing the direction from a straight to a curved track, it should be gradually effected, and it may be done (*Note* 4) by forming the curve of several aries of circles of different curvatures, which shall be tangent to each other at the points of passage from one to the other, the radii gradually decreasing from the arc which is tangent to the straight portion to that of the summit of the curve.

Various experiments have been made to determine the angle of friction on rail-roads, under the ordinary circumstan-

ces of travelling, and different results (*Note* 1) have been arrived at; but, from the far greater number it appears, that the angle varies between one perpendicular to two hundred and sixty in length, estimating along the track, and one to two hundred and eighty; or from a rise of 18,8 to 20,3 feet, nearly, for one mile in length. In this resistance to the motion down an inclined plane, is included, not only that due to the friction of the cars, but also that which is due to the resistance of the air to the motion of the cars.

The term *inclined plane* is properly applied to any ascent on a rail-road. If the ascent is such that it will not require a stationary power to draw up and lower the cars, the line of the track, in a horizontal direction, may be either straight or curved, as circumstances may demand; but if stationary power is to be used, then there are several important conditions to which the inclined plane must be subjected. The plane, in the first place, must be straight in a horizontal direction, otherwise the wear and tear of the ropes, used in transmitting the power of the stationary machinery at the top of the plane, and the increased friction, would render the passage up and down the plane very unsafe, if not almost impracticable. The inclination of the plane should, if practicable, be uniform, or, in the contrary case, the inclination near the top should be greater than towards the bottom of the plane. The change, moreover, from one inclination to another, should be gradual, or, in other words, the passage from one declivity to the next should be curved. In this way the velocity of the cars, in descending, will be gradually retarded, or be made uniform, by the gradual increase of resistance, offered by the decreased inclination of the plane; and there will be no loss of power, nor other inconveniences, arising from abrupt changes from one declivity to another. As the cars are usually brought to a state of rest at the top and bottom of the plane, at the commencement and end of their passage up and down, there should be a level portion of the track at these two points of sufficient length for the service of the

inclined plane, and these levels should, moreover, be in the same line of direction as the plane itself.

Arrangements for passing from a track. When a railroad consists of a single track, an arrangement, termed a *siding*, (Fig. 111,) is made to pass from the track at one point, and enter it at another, for the purpose of allowing cars to pass each other. A siding is, therefore, a portion of a track laid along side of the main track, at a suitable distance from it, and connected with it, at each extremity, by a curved portion, which is arranged in such a way that the cars can proceed either along the main track, or pass into the siding, as circumstances may require. If the part of the main track, where a siding is required, be straight, it may be placed on either side of the track, as may be most convenient; but if it be curved, the siding must be placed on the convex side of the track.

The angle contained between the line of direction of the main track, and a line drawn from the point of the main track where the siding enters it by its curved portion, to the point where this curved portion joins the straight part of the siding, is termed the *angle of deflection of the siding*. There is no positive limit laid down for this angle. It will evidently depend on the distance between the siding and main track, and the longest train of cars that it is supposed may travel over the track at one time. It should, however, be as small as possible, in order to make the change from the main track to the siding as gentle as practicable. The curved portion must evidently, in the case of a straight main track, be composed of two arcs of circles, one tangent to the straight line of the main track, the other tangent to the straight line of the siding, and the two tangent to each other mid-way between these two points, but presenting their convexities in opposite directions.

An arrangement similar to a siding, which is termed a *crossing*, (Fig. 112,) is made for a communication between the two tracks of a double track. The position and

curvature of the crossings are arranged on the same principles as for sidings. Crossings and sidings are placed at such points of the track as may be required by the service of the road.

The points where a siding, or crossing, quits the rails of the main track, and also intersects them, are so arranged, that a car can be kept in the main track, or be turned into the siding, or crossing, at pleasure. The method usually pursued for this purpose, is to make a portion of the track, or of the siding, at the point of separation, movable, (Fig. 111,) so that it will form in one position a part of the track, and, being displaced, will allow the car to turn aside into the siding. This movable portion is termed a *switch*. At the point where the rails of the crossing, or siding, cross those of the main track, a disposition, termed a *turn-out*, (Fig. 112,) is so arranged, that no impediment will be offered to the wheel in its course, either along the main track, or along the crossing. The arrangement and play of the switches and turn-outs will be best understood by a reference to the figures.

The surveys, and other labors required in establishing a rail-way, are of precisely the same character as for a common road. The formation of the road surface, or, as it is termed, the *grading*, to receive the rails and their supports, demands the greatest care, owing to the character of the substructure, its excellence depending principally on the firmness of the supports, and the stability of the rails. To lay the supports and rails, cross and length measures of suitable form and dimensions are used, together with the ordinary mason's level, to determine the level, or the inclination, of the top surface of the rails, the position of the supports, rails, &c.

CANALS.

Canals are artificial channels for water, applied to the purpose of inland navigation; for the supply of cities with water; for draining; for irrigation, &c. &c.

Navigable canals are divided into two classes: 1st. Canals which are on the same level throughout their entire length, as those which are found in low level countries: 2d. Canals which connect two points of different levels, which lie either in the same valley, or on opposite sides of a dividing ridge. This class is found in broken countries, in which it is necessary to divide the entire length of the canal into several level portions, termed *reaches*, the communication between the reaches being effected by some artificial means. When the points to be connected lie on opposite sides of a dividing ridge, the highest reach, which crosses the ridge, is termed the *summit level*.

1st Class. The surveying and laying out a canal in a level country, are operations of such extreme simplicity as to require no particular notice in this place; since these operations have been fully explained in the subject of Common Roads. The line of the canal will be run in a direct line between the two points to be connected, unless it be found necessary to deflect it at any intermediate points; in which case the straight portions will be connected by arcs of circles of sufficient curvature to allow the boats used in the navigation to pass each other at the curves without any diminution of their ordinary rate of speed.

The cross section of this class (Fig. 113) presents usually a *water-way*, or channel of a trapezoidal form, with an embankment on each side, raised above the general level of the country, and formed of the excavation for the water-way. The level, or surface of the water, is usually above the natural surface, sufficient thickness being given to the embankments to prevent the filtration of the water through them, and to resist its pressure. This arrangement has in its favor the advantage of economy in the labor of excavating and embanking, since the cross section of the cutting may be so calculated as to furnish the necessary earth for the embankment; but it exposes the surrounding country to injury, from accidents happening to the embankments.

The relative dimensions of the parts of the cross section may be generally stated as follows; subject to such modifications as each particular case may seem to demand.

The width of the water-way, at bottom, should be at least twice the width of the boats used in navigating the canal; so that two boats, in passing each other, may, by sheering towards the sides, avoid being brought into contact.

The depth of the water-way, should be at least eighteen inches greater than the draft of the boat, to facilitate the motion of the boat, particularly if there are water plants growing on the bottom.

The side slopes of the water-way in compact soils should receive a base at least once-and-a-half the altitude, and proportionally more as the soil is less compact.

The thickness of the embankments, at top, is seldom regulated by the pressure of the water against them, as this, in most cases, is inconsiderable, but to prevent filtration, which, were it to take place, would soon cause their destruction. A thickness from four to six feet, at top, with the additional thickness given by the side slopes at the water surface, will, in most cases, be amply sufficient to prevent filtrations. A pathway for the horses attached to the boats, termed a *tow-path*, which is made on one of the embankments, and a foot-path on the other, which should be wide enough to serve as an occasional tow-path, give a superabundance of strength to the embankments.

The tow-path should be, at least, twelve feet wide, to allow the horses to pass each other with ease; and the foot-path at least six feet wide. The height of the surfaces of these paths, above the water surface, should not be less than two feet, to avoid the wash of the ripple; nor greater than four-feet-and-a-half, for the facility of the draft of the horses in towing. The surface of the tow-path should incline slightly outwards, both to convey off the surface water in wet weather, and to give a firmer footing to the horses, which naturally draw from the canal.

The side slopes of the embankment vary with the character of the soil: towards the water-way they should seldom be less than two base to one perpendicular; from it, they may, if it be thought necessary, be less. The interior slope is usually not carried up unbroken from the bottom to the top, but a horizontal space, termed a *bench*, or *berm*, about one or two feet wide, is left, about one foot above the water surface, between the side slope of the water-way and the foot of the embankment above the berm. This space serves to protect the upper part of the embankment, and is, in some cases, planted with such shrubbery as grows most luxuriantly in aquatic localities, to protect more efficaciously the banks by the support which its roots give to the soil. The side slopes are better protected by a revetment of dry stone. Aquatic plants of the bull-rush kind have been used, with success, for the same purpose; a row of them being planted on the bottom, at the foot of the side slope, serving to break the ripple, and preserve the slopes from its effects.

The earth of which the embankments are formed should be of a good binding character, and perfectly free from vegetable mould, and all vegetable matter, as the roots of plants, &c. In forming the embankments, the vegetable mould should be carefully removed from the surface on which they are to rest; and they should be carried up in uniform layers, from nine to twelve inches thick, and be well rammed. If the character of the earth, of which the embankments are formed, is such as not to present entire security against filtration, a puddling of clay, or better still, of sand, two or three feet thick, may be laid in the interior of the mass, penetrating a foot below the natural surface. Sand is particularly useful in preventing filtration caused by the holes made in the embankments near the water surface by insects, moles, rats, &c.

Side drains must be made, on each side, a foot or two from the embankments, to prevent the surface water of the natural surface from injuring the embankments.

2d Class. This class will admit of two subdivisions: 1st, Canals which lie throughout in the same valley. 2d, Canals with a summit level.

Location. In laying out canals, belonging to the first subdivision, the line of direction of the canal should be as direct as practicable between the two points. As the different reaches, however, must be laid out on one of the side slopes of the valley, their lines of direction will be nearly the same as the horizontal curved line in which the natural surface of the ground would be intersected by the water surface of the canal produced; the variations in direction from this curve depending on the character of the cuttings and fillings, both as to the advantages which the one may present over the other as regards filtration, and the economy of construction.

With respect to the side slope of the valley along which the canal is to be run, the engineer must be guided in his choice by the relative expense of construction on the two sides; which will depend on the quantity of cutting and filling, the masonry for the culverts, &c., and the nature of the soil as adapted to holding water. All other things being equal, the side on which the fewest secondary water courses are found will, generally speaking, offer the greatest advantage as to expense; but, it sometimes may happen that the secondary water courses will be required to feed the canal with water, in which case it will be necessary to lay out the line on the side where they are found most convenient, and in most abundance.

As to the points in which the line of direction should cross the secondary valleys, the engineer will be guided by the same considerations as for any other line of communication; crossing them by following the natural surface, or else by a filling in along a right line, as may be most economical.

Cross section. The side formation of excavations and embankments require peculiar care, particularly the latter, as any crevices, when they are first formed, or which may take place by settling, might prove destructive to the work. In most cases, a stratum of good binding earth, lining the water

way throughout to the thickness of about four feet, if compactly rammed, will be found to offer sufficient security, if the substrueture is of a firm character, and not liable to settle. Even this has not, however, been found to answer in all cases, particularly where the substructure is formed of fragments of rocks offering large crevices to filtrations, or when it is of a marly nature. In such cases it has been found necessary to line the water-way throughout such parts with solid masonry, laid in hydraulic mortar. A lining of this character, (Fig. 114,) both at the bottom and sides, about six inches in thickness, formed of flat stones, about four inches thick, laid on a bed of hydraulic mortar, one inch thick, and covered by a similar bed, has been found to answer all the required purposes. This lining should be covered, both at bottom and on the sides, by a layer of good earth, at least three feet thick, to protect it from the shock of the boats striking either of those parts.

The cross section of the canal and its tow-paths in thorough cutting (Fig. 115) should be regulated in the same way as in canals of the first class; but when the cuttings are of considerable depth, it has been recommended to reduce both to the dimensions strictly necessary for the passage of a single boat. By this reduction there would be some economy in the excavations; but this advantage would, generally, be of too trifling a character to be placed as an offset to the inconveniences resulting to the navigation, particularly where an active trade was to be carried on.

The precautions recommended generally against slips in excavations, require to be carefully observed in those for canals. Where the earth cannot be prevented from caving in, a wide berth should be given to it, and it should be allowed to regulate itself.

Summit Level. The choice of the line of direction of canals with a summit level will be regulated in the manner just explained, for the two branches which are separated by the dividing ridge. In the selection of the summit level, the

engineer will be guided by the principles laid down in the subject of roads; with regard to the lowest points of the dividing ridge, as pointed out by the position of the valleys to be connected, and that of their secondary valleys. Independently of the economy of construction, arising from the line of direction being the shortest, and the height to be ascended being the least, which results from placing the summit level at the lowest point of the dividing ridge, another very important result is obtained, with respect to the supply of water for the summit level; this supply being greatest when the summit level is lowest, as the water for this point must be brought from the ground which is above it.

Deep Cuts and Tunnels. To obtain an adequate supply of water for the summit level, the engineer is often obliged to place the reach so low that a very deep cut, or a tunnel, will be required for the passage of the dividing ridge. The choice between the two methods will depend on the nature of the soil at the dividing ridge, and the comparative expense of the two methods: in general, it is said, that tunneling is to be preferred to deep cutting when the depth to be excavated is above sixty feet; and, as a general rule, also, when the cost of a running yard of each is the same, it is said that deep cutting is to be preferred, from the greater facility and despatch with which it can be done.

Where the point for the summit level is not rigorously fixed, by some necessary condition, the engineer should make a careful examination of all the ground adjacent to the point approximately determined. The object of this examination is to ascertain the nature of the soil,—the height of the ridge above the summit level,—the length of the tunnel required at different points,—the dimensions of the deep cuts by which the tunnel must be approached,—and the greater or less liability of the side slopes of these cuts to slips. In some cases a long tunnel, with short deep cuts to approach it, may be preferable to a short one with long deep cuts. In other cases it may be best to prefer a short tunnel, passing under a high

point of the ridge, to a long one passing under a lower point. The problem is evidently of that mixed character which will require repeated approximations and comparisons for its solution.

A *tunnel* is an underground excavation made for the passage of any line of communication; the character of its construction will depend on the nature of the soil.

Before commencing the excavations, a profile of the ground must be accurately made, along the vertical plane passing through the axis, or central line of the tunnel. This profile is carefully laid out on the ground itself, by means of a level, or other more accurate instrument, and pegs are placed in the ground, along this line, at equal intervals apart. When this preliminary operation is completed, the positions of the *working-shafts*, which are vertical pits, sunk above the crown of the tunnel, through which the excavations are taken off, are laid out, and also those of smaller shafts termed *air-shafts*, which supply the tunnel with fresh air. The excavations are now commenced by sinking the shafts to a proper level; and by cutting away the two faces of the hill at the entrance of the tunnel, to form what is termed a *breast* for the commencement of the internal excavations.

The positions of the shafts will depend on the nature of the soil. If it is sufficiently firm not to require arching, the shafts are sunk directly over the crown, and to a depth of six or nine feet below it; a small excavation, termed a *heading*, is driven from the bottom of each shaft to connect them, and to form a communication through the whole line for carrying out the excavated soil; a part of which, if it be found necessary, may still be sent up through the working shafts. After the heading is finished, the tunnel receives its proper form and dimensions, the work being carried on from the top downwards.

In soils which required to be arched, it is seldom safe to sink the working shafts directly over the crown, as they would weaken the earth, and might occasion cavings-in. It

24

is therefore recommended, in such cases, to mark out the lines of the piers of the arch, and to sink the working shafts ten or fifteen feet on the outside of these lines. The shafts are sunk to a level with the springing lines of the arch, and cross headings are driven from them to the positions of the piers; from these points a heading is driven in the direction of each pier, being sunk as low as the bottom of the foundations. So soon as these two last headings are of sufficient dimensions, the masonry of the piers is commenced, and it is carried up to the springing lines of the arch. Whilst this operation is going forward, or if it be deemed safer after its completion, cross headings, about six feet wide, are driven at suitable points between the piers. In these cross headings, when finished, centres are set up for turning the arches. A portion of the arch is turned over each centre, and the centres, which are so arranged as to be readily taken apart, are removed farther towards each end of the parts thus finished, and the masonry and excavations are thus carried forward uniformly until the whole arch is completed. The part of the excavation which may remain to be completed is carried out through the tunnel, or through working shafts, sunk over the crown of the arch, which communicate with the tunnel by holes left for the purpose in the crown.

Water is the most formidable obstacle with which the engineer meets in tunneling; and it will require all the resources of machinery to keep the work free from it, particularly if it commences to make in the working shafts. In some cases, it will be necessary to commence the heading considerably above the springing lines of the arch, and to carry it through with a downward slope each way, from the middle to the outlets, to give an outlet to the water. In other cases, it may be impracticable to use working shafts; in which case no other means can be resorted to but to commence the tunnel at each end, and to carry it forward by gradually working inwards.

When the soil is of so loose a texture as to require to be

sustained by some artificial means, the working shafts may be faced, like ordinary wells, with dry stone, or else a coffer work, formed of square frames placed horizontally within the shaft, about four feet from each other, and covered in by boards, termed a *sheeting*, may be used. For the headings a coffer work of a similar character may be used; the frames, being of suitable dimensions, are placed three or four feet apart, and are covered on the top and sides by a suitable sheeting. It is not thought necessary to enter here into the practical details for setting up and removing the coffer work, as works of this kind should be intrusted only to intelligent miners who are thoroughly conversant with all the resources of their art.

The height and cross dimensions of a tunnel will depend on the kind of communication. For a canal the width should be sufficient for the passage of a single boat; the water-way and tow path being suitably arranged for this end. In some cases the width has been made only sufficient for the boat, which is propelled in the tunnel by the boatmen with the assistance of iron rings fixed to the crown or the sides of the arch. This arrangement saves expense, but is very inconvenient for the navigation.

The form of the tunnel will depend on the nature of the soil. In solid rock the sides are usually vertical, and the crown is of an arched form. In less firm soils, where masses detach themselves from the crown, owing to the filtration of water, or from other causes, the crown alone need be arched, the sides forming the abutments of the arch. If the earth is loose, both at top and on the sides, it will be necessary to build abutments of masonry for the arch; these abutments may be either upright, or slightly curved outwards in an arched form, according to the nature of the soil. In tunneling through a soft marshy soil, or one exposed to filtrations, both from the sides and bottom, it will be necessary to form the crown and sides, as has just been explained, and to turn an inverted arch at bottom. With regard to the curvature

of the arches, that of the crown should be parabolic, if the pressure is simply that arising from the vertical weight of the mass above the crown, as this is the proper curve of equilibrium in such cases. But when the soil (Fig. 116) is marshy, or of a semi-fluid nature, the pressure may be regarded, without sensible error as normal to the curve, in which case the proper curve of equilibrium is the circle.

The arches of tunnels are usually built of good brick, as this material presents more advantages for such constructions than any other, and offers all requisite security as to its strength and durability. The centres should be of the most simple character, and be framed with an especial attention to be easily taken to pieces and set up. For this purpose they should consist of two or more parts of a solid construction which can be readily put together.

Supply of Water. The quantity of water required for canals with a summit level, may be divided into two portions: 1st. That which is required for the summit level, and those reaches which draw from it their supply: 2d. That which is wanted for the reaches below those, and which is furnished from other sources.

The supply of the first portion, which must be collected at the summit level, may be divided into several elements;—1st, the quantity required to fill the summit level, and the reaches which draw their supply from it;—2d, the quantity required to supply losses, arising from accidents; as breaches in the banks, and the emptying of the reaches for repairs:—3d, the supplies for losses from surface evaporation, from leakage through the soil, and through the lock gates;—4th, the quantity required for the services of the navigation, arising from the passage of the boats from one reach to another. Owing to the want of sufficient data, founded on accurate observations, no precise amount can be assigned to these various elements upon which the engineer could found a rigorous calculation.

The quantity required, in the first place, to fill the summit

level and its dependent reaches, will depend on the size of those reaches, an element which can be readily calculated; and upon the quantity which would soak into the soil, which is an element of a very indeterminate character, depending, as this quantity must, on the nature of the soil in the different reaches.

The supplies for accidental losses are of a still less determinate character.

To calculate the supply for losses from surface evaporation, correct observations must be made on the yearly amount of evaporation, and the quantity of rain that falls on the surface, as the loss to be supplied will be the difference between these two quantities.

With regard to the leakage through the soil, it will depend on the greater or less capacity which the soil has for holding water. This element varies not only with the nature of the soil, but also with the shorter or longer time that the canal may have been in use; it having been found to decrease with time, and to be, comparatively, but trifling in old canals. In ordinary soils it may be estimated at about two inches in depth every twenty-four hours, for some time after the canal is first opened. The leakage through the gates will depend on the workmanship of these parts; generally, it may be estimated equal to the quantity which would flow through an orifice of one inch and four-tenths in area, with a constant head of water of five feet above the orifice.

In estimating the quantity of water expended for the service of the navigation, in passing the boats from the level of one reach to that of another, through the locks which connect the reaches, two distinct cases require examination;—1st, where there is but one lock between two reaches, or in other words, when the locks are isolated; 2d, when there are several contiguous locks, or as it is termed a *flight* of locks between two reaches.

The *lock* is a small basin just large enough to receive a boat, in which the water is usually confined on the sides by

two upright walls of masonry, and at the ends by two gates, which open and shut, both for the purpose of allowing the boat to pass, and to cut off the water of the upper reach from the lower, as well as from the lock whilst the boat is in it. To pass a boat from one reach to the other,—from the lower to the upper for example,—the lower gates are opened, and the boat having entered the lock they are shut, and water is drawn from the upper reach, by means of valves, to fill the lock and raise the boat to the level of the upper reach; when this operation is finished, the upper gates are opened, and the boat is passed out. To descend from the upper reach, the lock is first filled, the upper gates are then opened, and the boat passed in, these gates are next shut, and the water is drawn from the lock, by valves, until the boat is lowered to the level of the lower reach, when the lower gates are opened and the boat is passed out.

In the two operations just described, it is evident, that for the passage of a boat, up or down, a quantity of water must be drawn from the upper reach to fill the lock to a height which is equal to the distance between the surface of the water in the two reaches; this height is termed the *lift* of the lock, and the volume of water required to pass a boat up or down is termed the *prism of lift*. The calculation, therefore, for the quantity of water requisite for the service of the navigation, will be simply that of the number of prisms of lift which each boat will draw from the summit level in passing up or down.

Let a boat, on its way up, be supposed to have arrived at the lowest reach supplied from the summit level; it will require a prism of lift to ascend the next reach above, and so on in succession, until it reaches the summit level, from which one prism of lift must be drawn to enable the boat to enter it. From this it appears that but one prism of lift is drawn from the summit level for the passage of a boat up. Now, in descending on the other side, the boat will require one prism of lift to take it to the next lower reach, and this prism of

lift will carry it through all the successive locks, if their lifts are the same. For the entire passage of one boat then, two prisms of lift must be drawn from the summit level.

This boat will thus leave all the locks full on the side of ascent, and empty on the side of the descent. Now the next boat may be going in the same, or in an opposite direction, with respect to the first. If it follows the first, it will evidently require two prisms of lift for its entire passage, and will leave the locks in the same state as they were. If it proceeds in an opposite direction, it will require a prism of lift to ascend to the summit level; but, in descending, it will take advantage of the full lock, left by the preceding boat, and will therefore not draw from the summit level for its descent to the next reach; the same will take place at every reach until the last, where it will carry out with it the prism of lift, which was drawn from the summit level for the preceding boat, so that in this case it will draw but one prism of lift from the summit level. If the two boats had met on the summit level, the same would have taken place; therefore, when the boats alternate regularly, each will require but one prism of lift for its entire passage. But as this regularity of alternation cannot be practically carried into effect, an allowance of two prisms of lift must be made for the entire passage of each boat.

In calculating the expenditure for locks in flights, a new element, termed the *prism of draught*, must be taken into account. This prism is the quantity of water required to float the boat in the lock when the prism of lift is drawn off; and is evidently equal in depth to the water in the reaches, unless it should be deemed advisable to make it just sufficient for the draught of the boat, by which a small saving of water might be effected.

Locks in flights may be considered under two points of view, with regard to the expenditure of water;—the first where both the prism of lift, and that of draught, are drawn off for the passage of a boat;—or second where the prisms of

draught are always retained in the locks. The expenditure of course will be different for the two cases.

To ascertain what will take place in the two cases, let a case be supposed, in which there is a flight of locks on each side of the summit level, to connect it with the two next lower reaches. In the first case, a boat, arriving at the foot of the flight, finds all the locks of the flight entirely empty, except the lowest, which must contain a prism of draught to float the boat in. To raise the boat, then, to the upper level, all the locks of the flight must be filled from the summit level, which will require as many prisms of lift as there are locks, and as many prisms of draught as there are locks less one;—or representing by L the prism of lift;—D the prism of draught; and n the number of locks in the flight, the total quantity of water, for the ascent of the boat, will be represented by

$$n\text{L} + (n-1)\text{D}; \quad . \quad . \quad . \quad . \quad . \quad (1).$$

In descending, on the opposite side, the boat will require a prism of lift and one of draught at the first lock; but to enter the second another prism of draught in addition will be required, and this entire quantity will be sufficient to take it through all the remaining locks of the flight, this quantity will therefore be represented by

$$\text{L} + 2\text{D}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

so that for the entire passage of the boat, the total expenditure will be represented by

$$(n+1)\text{L} + (n+1)\text{D}. \quad . \quad . \quad . \quad (3).$$

The flight, on one side, is thus left full after the passage of the first boat, and on the other side, empty. If a second boat, then, follows directly after the first, the prism of lift must be drawn from the lowest lock to admit the boat, this prism is then supplied from the lock next above, and so on to the summit level; so that but one prism of lift will be drawn off for the ascent of this boat, and it will require one of lift, and two of draught, to carry it down the opposite flight. If therefore the total number of boats which follow in this order, including

the first, be represented by m, the total expenditure will be represented by,

$$(n+1)\,L + (n+1)\,D + (m-1)\,2L + (m-1)\,2D. \quad . \quad . \quad . \quad (4).$$

If the second boat, instead of following the first, arrives in the opposite direction, or alternates with it, the expenditure for its ascent will be represented by the expression (1), and for its descent it will be nothing, since it finds the opposite flight filled, as left by the first boat; but if the locks had been emptied, then the passage of the second boat would have taken place under the same circumstances as that of the first.

It will be unnecessary here to go farther into these calculations for the various cases that may occur, under the different circumstances of passage of the boats, or of empty or full flights; the preceding gives the spirit of the method, and will give the means for entering upon a calculation to allow for the loss or gain by the passage of freighted or empty boats, following any prescribed order of passage. These refinements are, for the most part, more curious than useful; and the engineer should confine himself to making an ample allowance for the most unfavorable cases, both as regards the order of passage and the number of boats.

Feeders and Reservoirs. Having ascertained, from the preceding considerations, the probable supply which must be collected at the summit level, the engineer will next direct his attention to the sources from which it may be procured. Theoretically considered, all the water of the country, adjacent to the summit level, which lies above a horizontal plane passed through this point, might be conveyed to it: but it is found in practice that channels for the conveyance of water must have certain slopes, and that these slopes, moreover, will regulate the supply furnished in a certain time, all other things being equal. In making, however, the survey of the country, from which the water is to be supplied to the summit level, all the ground above the horizontal plane, just referred to, should be examined, leaving the determination of

the slopes for after considerations. The survey for this object, consists in making an accurate delineation of all the water courses above the summit level, and in ascertaining the quantity of water which can be furnished by each in a given time. This survey, as well as the measurement of the quantity of water furnished by each stream, which is termed the *gauging*, should be made in the dryest season of the year, in order to ascertain the minimum supply.

The usual method of collecting the water of the sources, and conveying it to the summit level, is by feeders and reservoirs. The *feeder* is a canal of a small cross section, which is traced on the surface of the ground with a suitable slope, to convey the water either into the reservoir, or direct to the summit level reach. The dimensions of its cross section, (*Note* 10,) and the longitudinal slope of the feeder, should bear certain relations to each other in order that it shall deliver a certain supply in a given time. The smaller the slope given to the feeder, the lower will be the points at which it will intersect the sources of supply, and therefore the greater will be the quantity of water which it will receive. This slope, however, has a practical limit which is laid down at four inches in 1000 yards, or nine thousand base to one altitude; and the greatest slope should not exceed that which would give the current a greater mean velocity than thirteen inches per second, in order that the bed of the feeder may not be injured. Feeders are furnished, like ordinary canals, with contrivances to let off a part, or the whole, of the water in them, in cases of heavy rains, or for making repairs.

A *reservoir* is a large pond, or body of water, held in reserve for the necessary supply of the summit level. A reservoir is usually formed by choosing a suitable site in a deep and narrow valley, which lies above the summit level, and erecting a dam of earth, or of masonry, across the outlet of the valley, or at some more suitable point, to confine the water to be collected. The object to be attained, in this case, is to embody the greatest volume of water, and at the same time

present the smallest evaporating surface, at the smallest cost, for the construction of the dam.

It is generally deemed best to have two reservoirs for the supply, one to embody the greater quantity of water, and the other, which is termed the *distributing* reservoir, for regulating the supply to the summit level. If, however, the reach at this point is very capacious, it may be used as the distributing reservoir.

The dams of reservoirs have been variously constructed; in some cases they have been made entirely of earth (Fig. 117); in others entirely of masonry; and in others of earth packed in between several parallel stone walls. It is now thought best to use either earth or masonry alone, according to the circumstances of the case, particularly as regards the comparative expense of the two methods.

Earthen dams should be made with extreme care, of the best binding earth, well freed from every thing that might cause filtrations. A wide trench should be excavated to the firm soil, to receive the base of the dam; and the earth should be carefully spread and rammed in layers not over a foot thick. As a farther precaution, it has been thought necessary to place a wide stratum of the best clay puddling in the centre of the dam, reaching from the top to three or four feet below the base. Fine sand would serve all the purposes of clay, and be a better security against injury from water-rats, &c. The slope of the dam towards the pond should be from three to six base to one perpendicular; the reverse slope need only be somewhat greater than the natural slope of the earth. The pond slope should be faced with dry stone to preserve it from the action of the surface ripple.

To draw the water from the dam, an arched culvert, large enough for a man to enter it with ease, is made near the base at some suitable point. The culvert is arranged so as to be closed by one or more valves that can be easily opened. In some cases these valves are placed near the entrance to the culvert towards the pond, which is the better plan for secu-

rity to the dam, as the pressure of the water on the sides of the culvert, for considerable depths, will be very great. In other cases, the valves are placed near the middle of the culvert. In both cases, suitable arrangements must be made to enable the valves to be manœuvred and examined with safety.

Dams of masonry are nothing more than water tight walls of suitable forms and dimensions to prevent filtration, and resist the pressure of water in the reservoir. The most suitable cross section is that of a trapezoid, the face towards the water being vertical, and the exterior face inclined, with a suitable batter to give the wall sufficient stability. The wall should be at least four feet thick at the water line, to prevent filtration, and this thickness may be increased as circumstances may seem to require. Buttresses, or counterforts, have been, in some cases, added to the exterior facing, to give the wall greater stability; but this arrangement is of doubtful utility, and is inferior to throwing the mass of the buttresses into a uniform additional thickness to the wall. Valves are placed at the inner ends of small arched openings made in the wall to draw off the water. These openings should be made at different depths below the surface, in order that the pressure of water on the valve may not be too great to impede its manœuvre. The distance between these openings may be about twelve feet.

Suitable dispositions should be made to relieve the dam of all surplus water in very wet seasons. This may be done by cutting off the sources of supply, or else by allowing the water to flow out of the reservoir, at some suitable point which will not endanger the dam.

Lift of Locks. From the preceding observations on the expenditure of water for the service of the navigation, it appears, that isolated locks are more favorable under this point of view than locks in flights. The engineer is not, however, always left free to select between the two systems, for the form of the natural surface of the ground may compel him

to adopt a flight of locks at certain points. As to the comparative expense of the two methods, a flight is in most cases cheaper than the same number of single locks, as there are certain parts of the masonry which can be suppressed. There is also an economy in the suppression of the small gates, which are not needed in flights. It is, however, more difficult to secure the foundations of flights from the effects of the water, which forces its way from the upper to the lower reach under the locks, than in isolated locks. Where an active trade is carried on, a double flight is sometimes arranged; one for the ascending, the other for the descending boats. In this case the water which fills one flight may, after the passage of the boat, be partly used for the other, by means of an arrangement of valves made in the side wall separating the locks.

The lift of locks is a subject of importance, both as regards the consumption of water for the navigation, and the economy of construction. Locks with great lifts, as may be seen from the remarks on the passage of boats, consume more water than those with small lifts. They require also more care in their construction, to preserve them from accidents, owing to the great pressure of water against their sides. The expense of construction is otherwise in their favor; that is, the expense will increase with the total number of locks, the height to be ascended being the same. The smallest lifts seldom are less than five feet, and the greatest, for ordinary canals, not over twelve; medium lifts of seven or eight feet are considered the best under every point of view. This is a point, however, which cannot be settled arbitrarily, as the nature of the foundations;—the materials used;—the embankments around the locks;—the changes in the direction of the canal, caused by varying the lifts, are so many modifying causes, which should be carefully weighed before adopting a definitive plan.

The lifts of a flight should be the same throughout; but in isolated locks the lifts may vary according to circumstances.

If the supply of water from the summit level requires to be economized with care, the lifts of locks which are furnished from it may be less than those lower down.

Reaches The position and the dimensions of the reaches must be mainly determined by the form of the natural surface. Those points are naturally chosen to pass from one reach to another, or as the positions for the locks, where there is an abrupt change in the surface.

A reach, by a suitable modification of its cross section, can be made as short as may be deemed desirable; there being but one point to be attended to in this, which is, that a boat passing between the two locks, at the ends of the reach, will have time to enter either lock before it can ground in the reach, on the supposition, that the water drawn off to fill the lower lock, whilst the boat is traversing the reach, will just reduce the depth in the reach to the draft of the boat. If this condition cannot be satisfied by giving the reach the ordinary cross section of the canal, it will then be necessary either to widen or deepen it, as may be judged best.

Locks. A lock (Fig. 118) may be divided into three disdinct parts;—1st. The part included between the two gates, which is termed the *chamber*;—2d. The part above the upper gates, termed the *fore*, or *head-bay*;—3d. The part below the lower gates, termed the *aft*, or *tail-bay*.

The lock chamber must be wide enough to allow an easy ingress and egress to the boats commonly used on the canal; a surplus width of one foot over the width of the boat across the beam is usually deemed sufficient for this purpose. The length of the chamber should be also regulated by that of the boats; it should be such, that when the boat enters the lock from the lower reach, the tail gates may be shut without requiring the boat to unship its rudder.

The plan of the chamber is usually rectangular, as this form is, in every respect, superior to all others. In the cross section of the chamber, (Fig. 119,) the sides receive generally a slight batter, as when so arranged they are found to

give greater facility to the passage of the boat than when vertical. At bottom the chamber is either flat or curved; more water will be required to fill the flat bottomed chamber than the curved, but it will require less masonry in its construction.

The chamber is terminated just within the head gates by a vertical wall, the plan of which is cylindrical. As this wall separates the upper from the lower reach, it is termed the *lift-wall;* it is usually of the same height as the lift of the reaches. The top of the lift wall is formed of cut stone, the vertical joints of which are normal to the cylindrical face of the wall; this top course projects from six to nine inches above the bottom of the upper reach, and is arranged with an angular point, so that the bottom of the head gates, when shut, may rest closely against it. This arrangement is termed the *mitre-sill.* Various degrees of opening have been given to the angle between the two branches of the mitre-sill; it is, however, generally so determined, that the perpendicular of the isosceles triangle, formed by the two branches, shall vary between one fifth and one sixth of the base.

The cross section of the chamber walls is either trapezoidal, or the backing is made with offsets from the top to the base; the former method is now mostly in use, as it presents equal stability with the latter, and admits of a more solid construction. The facing, as has just been explained, receives a slight batter. The chamber walls are exposed to two opposite efforts; the water in the lock on one side, and the embankment against the wall on the other. The pressure of the embankment is the more permanent effort of the two, but that of the water is in most cases the greater; there are, however, exceptions to this; the embankment, for example, may separate slightly from the wall, leaving a crevice, from top to bottom, into which a filament of water may be introduced, the pressure of which would be greater than that of the water in the lock, because the embankment is carried up to a level with the top, whereas the surface of the water in the cham-

ber, when the lock is full, is usually twelve or eighteen inches below the top of the wall; another exception, of a still more weighty character, may arise from the nature of the earth of which the embankment is formed; if it is of a nature to imbibe a large quantity of water, and to become in that state nearly semi-fluid, it will act precisely in the same way against the wall as a fluid of greater specific gravity than water. Having considered the probabilities of either of these cases arising, the dimensions of the wall (*Note* 6) must be regulated by the most unfavorable. The usual manner of doing this, is to make the wall four feet thick at the water line of the upper reach, to secure it against filtration; and then to determine the base of the batter, so that the mass of masonry shall present sufficient stability to counteract the tendency of the pressure, which will be either to cause the wall to yield by sliding on the bed of its foundation; or to give way by a disjunction of the masonry near the base, causing an overthrow outwards, or inwards, as the case may be. The spread, and other dimensions of the foundations will be regulated, according to the nature of the soil, in the same way as in other structures.

The bottom of the chamber, as has been stated, may be either flat or curved. The flat bottom is suitable to very firm soils, which will neither yield to the vertical pressure of the chamber wall, nor admit the water to filter from the upper reach under the bottom of the lock. In either of the contrary cases, the bottom should be made with an inverted arch; as this form will oppose greater resistance to the upward pressure, and will serve to distribute the weight of the walls over the portion of the foundation under the arch. The thickness of the masonry of the bottom will depend on the width of the chamber, and the nature of the soil. Were the soil a solid rock, no bottoming would be requisite; if it is of soft mud, a very solid bottoming, from three to six feet in thickness, might be requisite.

The principal danger to the foundations arises from the

water which may filter from the upper to the lower reach under the bottom of the lock. One preventive for this, but not an effectual one, is to drive sheeting piles across the canal at the end of the head-bay; another, which is more expensive, but more certain in its effects, consists in forming a deep trench of two or three feet in width, just under the head-bay, and filling it with beton which unites at top with the masonry of the head-bay. Similar trenches might be placed under the chamber were it considered necessary.

The lift wall usually receives the same thickness as the chamber walls; but, unless the soil is very firm, it would be more prudent to form a general mass of masonry under the entire head-bay, to a level with the base of the chamber foundations, of which mass the lift wall should form a part.

The head-bay is enclosed between two parallel walls, which form a part of the side walls of the lock. They are terminated by two wing-walls, which, it will be found most economical to run back at right angles with the side walls. A recess, termed the *gate chamber*, is made in the wall of the head-bay; the depth of this recess should be sufficient to allow the gate, when open, to fall two or three inches within the facing of the wall, so that it may be out of the way when a boat is passing; the length of the recess should be a few inches more than the width of the gate. The part of the recess where the gate turns on its pivot is termed the *hollow quoin;* it receives what is termed the *heel*, or *quoin post* of the gate, which is made of a suitable form to fit the hollow quoin. The distance between the hollow quoins and the face of the lift-wall will depend on the pressure against the mitre-sill, and the strength of the stone; eighteen inches will generally be found amply sufficient.

The side walls need not extend more than twelve inches beyond the other end of the gate chamber; and indeed it is a subject well worthy of examination, whether the greater portion of the side walls beyond the hollow quoins might not be suppressed with advantage, and their places be supplied

by a simple post for the gate to rest against when open. The wing-walls may be extended back to the total width of the canal, but it will be more economical, and they will be found equally serviceable, to narrow the canal near the lock, and to extend the wing-walls only about two feet into the banks or sides. The dimensions of the side and wing-walls of the head-bay are regulated in the same way as the chamber walls.

The bottom of the head-bay is flat, and on the same level with the reach; the exterior course of stones at the entrance to the lock should be so jointed as not to work loose.

The gate chambers for the lower gates are made in the chamber walls; and it is to be observed, that the bottom of the chamber, where the gates swing back, should be flat, or be otherwise arranged not to impede the play of the gates.

The side walls of the tail-bay are also a part of the general side walls, and their thickness is regulated as in the preceding cases. Their length will depend chiefly on the pressure which the lower gates throw against them when the lock is full, and partly on the space required by the lock-men in opening and shutting gates manœuvred by the balance beam. A calculation (*Note* 11) must be made for each particular case to ascertain the most suitable length. The side walls are also terminated by wing-walls, similarly arranged to those of the head-bay. The points of junction between the wing and side-walls should, in both cases, be arched, or the stones at the angles be simply rounded off. One or two perpendicular grooves are sometimes made in the side walls of the tail-bay, to receive pieces of scantling, termed ***stop-planks***, which are fitted into them horizontally when a temporary dam is needed, to shut off the water of the lower reach from the chamber, in case of repairs, &c. Similar arrangements might be made at the head-bay, but they are not indispensable in either case.

The stress on the walls at the hollow quoins is greater than at any other points, owing to the pressure at those points from

the gates, when they are shut, and to their effects when in motion; to counteract this and strengthen the walls, buttresses should be placed at the back of the walls, in the most favorable position behind the quoins to subserve the object in view.

The bottom of the tail-bay is arranged, in all respects, like that of the head-bay.

The top of the side-walls of the lock may be from one to two feet above the general level of the water in the upper reach, the top course of the masonry being of heavy large blocks of cut stone, although this is not an indispensable crowning for the walls, as smaller masses have been found to suit the same purpose, but are less durable. As to the masonry of the lock, in general, it will only be necessary to observe, that those parts alone need be of cut stone where there is great wear and tear from any cause, as at the angles generally; or where an accurate finish is indispensable, as at the hollow quoins. The other parts may be of brick, rubble, beton, &c., but it must be observed that all the parts must be laid in the best hydraulic mortar.

The filling and emptying the lock chamber have given rise to various discussions and experiments, all of which have been reduced to the comparative advantages of letting the water in and off by valves made in the gates themselves, or by culverts in the side-walls, which are opened and shut by valves. When the water is let in through valves in the gates, its effects on the sides and bottom of the chamber are found to be very injurious, particularly in high lift-walls, besides the inconvenience resulting from the agitation of the boat in the lock. To obviate this, in some degree, it has been proposed to give the lift-wall the form of an inclined curved surface, along which the water might descend without producing a shock on the bottom.

The side culverts are small arched conduits, of a circular, or an elliptical, cross section, which are made in the mass of masonry of the side walls, to convey the water from the up-

per reach to the chamber. These culverts, in some cases, run the entire length of the side walls, on a level with the bottom of the chamber, from the lift-wall to the end of the tail-wall, and have several outlets leading to the chamber. They are arranged with two valves, one to close the mouth of the culvert, at the upper reach, the other to close the outlet from the chamber, to the lower reach. This is, perhaps, one of the best arrangements for side culverts. They all present the same difficulty of repairs when out of order, and they are moreover very subject to accidents. They are therefore on this account inferior to valves in the gates.

It has also been proposed, to get rid of the inconveniences of culverts, and the disadvantages of lift-walls, by suppressing the latter, and, in its place, gradually increasing the depth of the upper reach, to the bottom of the chamber. This method has never been put in practice; it presents a saving in the mass of masonry, but the gates will be more expensive, as the head and tail gates must be of the same height. It would entirely do away with the objection to valves in the gates, as the current through them in this case, would not be sufficiently strong to injure the masonry.

The bottom of the canal below the lock should be protected by what is termed an *apron*, which is simply a covering of plank laid on a grillage, or else one of brush wood and dry stone. The sides should also be faced with dry stone. The length of this facing will depend on the strength of the current; generally not more than from fifteen to thirty feet from the lock will require it. The entrance to the head-bay is, in some cases, similarly protected, but this is unnecessary, as the current has but a very slight effect at that point.

Lock Gates. A lock gate (Fig. 120) is composed of two leaves, each leaf consisting of a solid frame work covered on the side towards the water with thick plank made water tight. The frame usually consists of two uprights of several horizontal cross pieces let into the uprights; and of a diagonal piece, or brace, intended to keep the frame of an in-

variable form. The upright around which the gate turns, termed the *quoin*, or *heel-post*, is rounded off on the back to fit in the hollow quoins; it is made slightly eccentric with them, so that it may turn easily without rubbing against them; its lower end rests on an iron *gudgeon*, to which it is fitted by a corresponding indentation in an iron *socket* on the end; the upper extremity is secured to the side walls by an iron *collar*, within which the post turns. The collar is so arranged that it can be easily fastened to, or loosened from two iron bars, termed *anchor-irons*, which are firmly attached by bolts, or a lead sealing, to the top course of the walls. One of the anchor-irons is placed in a line with the leaf when shut, the other in the line when open, to resist most effectually the strain in those two positions of the gate. The opposite upright, termed the *mitre-post*, has one edge bevelled off, to fit against the mitre-post of the other leaf of the gate.

A long heavy beam, termed a *balance beam*, from its partially balancing the weight of the leaf, rests on the quoin post, to which it is secured, and is mortised with the mitre post. The balance beam should be about four feet above the top of the lock, to be readily manœuvred, its principal use being to open and shut the leaf.

The top cross piece should be about on a level with the top of the lock; the bottom piece should swing clear of the bottom of the lock. A wooden beam, termed a *fender-beam*, is sometimes fastened to the mitre-sill for the bottom piece to rest against; this forms a better water-tight joint, and also preserves the mitre-sill from injury from the shock of the gate. It has also been suggested, to make the hollow quoins of wood, for the same reasons; but as this material is very perishable when exposed, as it must be, in such a situation, and as it does not unite with mortar, it is doubtful whether it would prove an advantageous substitute for stone. The arrangement of the intermediate cross pieces may be made to depend on their dimensions; if they are of the same dimensions, then they should be placed nearer together at the bottom, as the pressure of the water (*Note* 12) is there

greatest; but, by making them of unequal dimensions, they may be placed at equal distances apart; this, however, is not of much importance except in large gates, and considerable depths of water.

The plank may be arranged either parallel to the uprights, or parallel to the diagonal brace; in the latter position they will act with the brace to preserve the form of the frame.

A wide board supported on brackets, is often affixed to the gates, both for the manœuvre of the machinery of the valves, and to serve as a foot bridge across the lock. The valves are small gates which are arranged to close the openings made in the gates for letting in or drawing off the water. They are arranged to slide up and down in grooves, by the aid of a rack and pinion work; or they may be made to open or shut by turning on a vertical axis, in which case they are termed *paddle gates*. The openings in the upper gates are made between the two lowest cross pieces, and nearer to the quoin than to the mitre-post, for the double reason of fatiguing the gate less by their additional weight, and that of their machinery, and to throw the current through the openings more towards the centre of the chamber. In the lower gates the openings are placed just below the surface of the water in the reach. The width of the opening will depend on the time in which it is wished to fill the lock; it is usually between two and four feet.

Accessory Works. Under this head are classed those constructions which are not a part of the canal proper, although generally found necessary on all canals; as the culverts for conveying off the water courses which intersect the line of the canal; the inlets of feeders for the supply of a reach; aqueduct bridges, &c. &c.

Culverts. The disposition to be made of water courses intersecting the line of the canal will depend on their size, the character of their current, and the relative positions of the canal and stream.

Small brooks which lie lower than the canal are conveyed under it through an ordinary culvert (Fig. 122). If the level of

the two is nearly the same, it will then be necessary to make the culvert in the shape of an inverted syphon, and it is therefore termed a *broken-back* culvert. If the water of the brook is generally limpid, and its current gentle, it may, under the last case, be received into the canal. The point at which a brook, or a feeder, is received into the canal should be so arranged that the water may be shut off, or let in at pleasure, in any desired quantity. For this purpose a cut is made through the sides of the canal, and the sides and bottom of the cut are faced with masonry laid in hydraulic mortar. A sliding gate, fitted into two grooves made in the side walls, is manœuvred by a rack and pinion work, so as to regulate the quantity of water to be let in. The water of the feeder, or brook, should first be received in a basin, or reservoir, near the canal, where it may deposite its sediment before it is drawn off. In cases where the line of the canal is crossed by a torrent which brings down a large quantity of sand, pebbles, &c., it may be necessary to make a permanent structure over the canal forming a channel for the torrent; but if the discharge of the torrent is only periodical, a movable channel may be arranged, for the same purpose, by constructing a boat with a deck and sides to form the water-way of the torrent. The boat is kept in a recess in the canal near the point where it is used, and is floated to its position, and sunk when wanted.

Aqueducts. When the line of the canal is intersected by a wide stream, it will be necessary to construct an aqueduct over the stream for the use of the canal. These constructions have been made of masonry, of wood, and of cast iron. They consist essentially of a water-tight channel, supported on the ordinary construction for a bridge. The supports of the bridge, and its superstructure for sustaining the water-way, may be either of masonry, or of frame work. When the substructure is of masonry, (Fig. 121,) the water-way is of the same material, and consists simply of a channel wide enough for the passage of a single boat; of a tow-path of the

smallest width on one side ; and a narrow foot-path on the other. The water-way is faced with stone, or brick, and the tow and foot paths with flagging. The spaces between the facing of the water-way, the backs of the arches under it, and the head walls of the aqueduct, should be filled in with beton, or rubble, laid carefully in the best hydraulic mortar. These spaces sometimes, for economy, are filled in with a clay puddling; but the method is bad, as it is almost impracticable to render it perfectly water-tight. A wooden water-way, consists simply of a water-tight trough, made of thick plank, which are supported by a suitable frame work on the exterior. The tow-paths in this case should be separate from the water-way, to give greater security to the system. Iron water-ways are formed on similar principles to those of wood.

Canal Bridges. Bridges for roads, over a canal termed *canal bridges*, are constructed like other structures of the same kind. In planning them the engineer should endeavor to give sufficient height to the bridge to prevent those accidents, of but too frequent occurrence, from persons standing upright on the deck of the passage-boat whilst passing under a bridge.

Waste-Wier. To rid a reach of its surplus water, a construction, termed a *waste-wier*, is formed. This consists (Fig. 122) simply of a cut through the side of the canal, which is faced with masonry. The cut may be closed by a sliding gate, or by stop-plank, placed in grooves in the side walls, like the methods used for the tail-bay of a lock. The opening remains permanently closed from the bottom to a given height, and when the water in the reach rises above this point, it discharges itself over the waste-wier. By raising the gate, or taking out the stop-plank, the waste-wier operates as a drain. Waste-wiers have also been made on the principle of the syphon; but the method is bad, as they are more expensive, and less effective, than the simple means just described.

Reach Dams. In long reaches, an accident happening at

any one point might cause serious injury to the navigation, besides a great loss of water. To prevent this, in some measure, the width of the canal may be diminished, at several points of a long reach, to the width of a lock, and the sides, at these points, may be faced with masonry, arranged with grooves and stop-planks, to form a temporary dam for shutting off the water on either side.

Tide or *Guard Lock.* The point at which a canal enters a river requires to be selected with judgment. Generally speaking, a bar will be found in the principal water course at, or below, the points where it receives its affluents. When the canal therefore follows the valley of an affluent, its outlet should be placed below the bar, to render its navigation permanently secure from obstruction. A large basin is usually formed at the outlet, for the convenience of commerce; and the entrance from this basin to the canal, or from the river to the basin, is effected by means of a lock with double gates, so arranged that a boat can be passed either way, according as the level in the one is higher or lower than that in the other. A lock so arranged is termed a *tide* or *guard lock*, from its uses. The position of the tail of this lock is not indifferent in all cases where it forms the outlet to the river; for were the tail placed up stream, it would be more difficult to enter a boat, or take it out, than if it were down stream.

RIVERS.

Improvements in rivers, for the purposes of navigation, should be based on a knowledge of the phenomena presented by currents in open channels. These phenomena are best studied, by observing the changes which take place in natural watercourses, arising from any variation in the volume of the water, and the velocity and direction of the current. When the relations between the cross section of a stream, its longitudinal slope, the nature of its bed, and the volume of water remain permanently invariable, or change insensibly with time, the river is said to have acquired a fixed *regimen*.

A river with a fixed regimen is said to be *regulated* when its banks are protected either naturally, or artificially, from the erosion of the current,—the surrounding country is secured from inundations,—and the draught of water, and the banks are in a suitable state for navigation.

Changes in the bed of a river vary with the velocity of the current;—the size, form, and specific gravity of the particles which compose the soil of the bed,—the side slopes of the banks, and their direction with respect to the current.

It is generally found in nature that the longitudinal slope of rivers is greatest nearest the source, and that this slope decreases towards the mouth, at which point the cross section is generally widest, and is found to decrease in ascending towards the source. From this natural form of the bed, the velocity is also greatest towards the source, and gradually decreases towards the mouth. This diminution of the velocity causes the heaviest particles to be deposited in the upper part of the bed, where the current, from the change in its velocity, is no longer able to bear them along, and the lighter particles to be deposited lower down, according to their weight. From the successive deposites, accumulations, termed *bars*, are formed at different points in the course of a river; the one at its mouth being usually formed of the lightest particles that the current will bear along. The bars would gradually accumulate without shifting, and the river would acquire a fixed regimen, were the volume of water thrown into it uniformly the same; for the current would by degrees acquire a uniform velocity, by widening, or deepening, the cross section of the bed, where it was confined in a narrow valley, from which a uniform velocity would result; or if in a wide valley, the river would gradually take a winding course, from which the longitudinal slope would be decreased, and the same result would follow.

An abrupt change in the course of a river is termed an *elbow*. When an elbow commences to form, the tendency of the current will be to render it more prominent; for the origin

of elbows arises from some deflection in the current against one of the banks, which wears that bank away, and throws the débris on the opposite shore; and as the erosion on the one side, and the accumulation on the other, become more prominent, the deflection becomes greater, and the action of the current is thereby proportionally increased. Rivers with wide valleys take a very winding course from this action; and in those with narrow valleys the elbows shift gradually from point to point, occupying in a series of years every point of the course.

From this continued action of the current on the banks, the bed is gradually filled up, and becomes less capable of giving a free issue to the ordinary volume of water thrown into it; the consequences, found to ensue from this, are, that in seasons of freshets the banks are torn away, and frequently new arms are formed to the river. This is particularly seen to take place near the mouth, giving rise to those numerous channels which are named *delta*, after the peculiar form of that of the Nile.

From the foregoing remarks it appears, that a sudden increase in the volume of water, and in the velocity, are the principal causes in the changes of the regimen of a river; and, therefore, that in all attempts at regulating it, these causes should principally be borne in mind.

In forming a plan for a river improvement, four principal objects are to be considered by the engineer. 1st, The means to be taken to protect the banks from the action of the current. 2d, The means to prevent inundations of the surrounding country. 3d, The removal of bars, elbows, and other natural obstructions to navigation. 4th, The means to be resorted to for obtaining a suitable depth of water for boats, of a proper tonnage, for the trade on the river.

To protect the banks, some artificial means must be resorted to, which, by decreasing the velocity of the current in shore, will lessen its action on the soil; or else a facing of some material sufficiently durable to resist its action must be employed.

The former method may be used when the banks are low, and have a gentle declivity; the simplest plan consists in planting such shrubbery on the declivity as will thrive near water; or by driving down short pickets and interlacing them with twigs, forming a kind of wicker work. These constructions break the force of the current, and diminish its in shore velocity, and thus cause the water to deposite its finer particles, which gradually fill out and strengthen the banks. If the banks are high, and are subject to cave in from the action of the current on their base, they may be either cut down to a gentle declivity, as in the last case; or else they may receive a slope of nearly 45°, and be faced with dry stone, care being taken to secure the base by an enrockment, or by a facing of brush and stone laid in alternate layers.

At the points in the course of a river where inundations are to be apprehended, the water-way, if practicable, should be increased, all obstructions to the free discharge of the water below the point should be removed, and dykes of earth, usually termed *levees*, should be raised on each side of the river. By increasing the water-way a temporary improvement only will be effected; for, except in the season of freshets, the velocity of the current at this point will be so much decreased as to form deposits, which, at some future day, may prove a cause of destruction. In confining the water between levees, two methods have been tried; the one consists in leaving a water-way strictly necessary for the discharge of freshets; the other consists in giving the stream a wide berth. There is an example of the first method in the Po in Italy, the effect of which has been to raise the bed of the stream so much that in many parts the water is habitually above the natural surface of the country, leaving it exposed to serious inundation should the levees give way. The other method has been tried on the Loire in France, and observation has proved that the general level of the bed has not sensibly risen for a long series of years; but it has been found that the bars,

which are formed after each freshet, are shifted constantly by the next, so that when the waters have subsided to their ordinary state, the navigation is extremely intricate from this cause. Other means have been tried, such as opening new channels at the exposed points, or building dams above them to keep the water back; but they have all been found to afford only a temporary relief.

The progress of an elbow may be stopped, and the deposite from it be removed, by building a straight dyke, so as to shut in the concave side, and turn the current on the other. This is perhaps the best method, as the current is not abruptly deflected by it. Another method consists in building out a dike perpendicular to the shore on the concave side, or else making such an angle with it as to direct the head of the dike down stream. By this means the current will be abruptly deflected towards the opposite shore, and the dike, which is termed a *wing-dam*, will protect its own shore for a distance above it equal to about twice its length, and below it about three times its length, so that the bed, in shore, will gradually be filled up, whilst on the opposite shore it will deepen and widen.

The dikes used for this purpose may be formed entirely of dry stone; of crib-work filled with loose stone; or of alternate layers of brush and loose stone. The last method is in general use on the Rhine, and is found fully to answer both for straight and wing-dams.

As bars are formed by widening the bed, the remedy for dissipating them is in narrowing it. This is done by confining the channel between dikes, formed of alternate layers of brush and gravel, or stone, below the lowest water line; and of earth faced with dry stone, or of dry stone alone, above it. If the river divides into several channels near the bar, which will be found frequently to take place, they should all be barred except the main channel, so that by throwing all the water into one channel, the effects of the current may be more sensibly felt. If the bar only forms on one side, it may be dissipated by deflecting the current from the opposite shore upon it.

Bars may be remedied by placing a series of dams on the upper part of a river's course, to arrest the heavier particles that would otherwise be borne farther down, so that those which are formed in the lower parts, being of light particles, may be more easily dissipated. But whatever means may be used to remove this obstruction, its effects can only be temporary; for unless the deposits can be removed entirely from the river, they must accumulate at some point, and finally form an obstruction. Those which form at the mouths of rivers are peculiarly of this character, and can only be removed by machinery, or, in certain localities, where a great head of water can be obtained from high tides, by forming artificial reservoirs which are filled at high water, and discharged at low water on the bar.

When the bed is obstructed by rocks, it may be deepened by blasting the rocks, and removing the fragments with the assistance of the diving bell, and other machinery.

In some of our rivers, obstructions of a very dangerous character to boats are met with, in the trunks of large trees which are imbedded in the bottom at one end, whilst the other is near the surface, they are termed *snags*, and *sawyers* by the boatmen. These obstructions have been very successfully removed, within late years, by means of machinery, and by propelling two heavy boats, moved by steam, which are connected by a strong beam across their bows, so that the beam will strike the snag, and either break it off near the bottom, or uproot it. Other obstructions, termed *rafts*, formed by the accumulation of drift wood at points of a river's course, are also found in some of our western rivers. These are also in process of removal, by cutting through them by various means which have been found successful.

When the general depth of water in a river is insufficient for the draught of boats of the most suitable size for the trade on it, an improvement, termed *lock and dam navigation*, is resorted to. This consists in dividing the course into several suitable *reaches*, or *pools*, by forming dams to keep the water

in the pool at a constant head; and by passing from one pool to another by locks at the ends of the dams.

The dams are only made of sufficient height to give the pool the requisite depth, the surplus water being suffered to flow over the dam. This arrangement will require the surface, at least, of the dam, as well as the bed for some distance below it, to be faced with some material that will withstand the effects of the current. Various constructions have been used for this purpose according to the means at hand. Dams of alternate layers of brush and gravel, with a facing of plank, fascines, or dry stone, answer very well in gentle currents. If the dam is exposed to heavy freshets, to shocks of ice, and other heavy floating bodies, as drift wood, it would be more prudent to form it of dry stone entirely, or of crib-work filled with stone; or, if the last material cannot be obtained, of a solid crib-work alone. The bed below the dam is protected by an enrockment, or by a facing of fascines and loose stone. If the dam is to be made water-tight, sand and gravel in sufficient quantity may be thrown in against it in the upper pool. The points where the dam joins the banks, which are termed the *roots* of the dam, require particular attention to prevent the water from filtering around them. The ordinary precaution for this is to build the dam some distance back into the banks.

The ordinary lock is the safest means to pass the boats from one pool to another. It is placed at one extremity of the dam, and perpendicular to it, near the bank. The part of the dam adjoining the lock should be raised above the highest water level, to prevent a current near the lock. The head of the lock is also protected from floating bodies by a small wing-dam above it.

In place of the lock of usual form, it has been proposed to make a small basin at the end of the dam, to be entered from the upper pool by ordinary lock gates placed along the banks, with lock gates similarly placed for the passage to the lower pool. This basin might be made large enough, if it

were deemed advisable, for the passage of several boats. Its construction would be probably less expensive than that of the common lock, as its sides might be faced with dry stone alone.

A very common, but unsafe method of passing from one pool to another, is that which is termed *flashing*; it consists of a sluice in the dam, which is opened and closed by means of a gate revolving on a vertical axis, which is so arranged that it can be manœuvred with ease. One plan for this purpose is to divide the gate into two unequal surfaces by an axis, and to place a valve in the greater surface of such dimensions, that when opened the surface against which the water presses shall be less than that of the smaller surface. The play of the gate is thus rendered very simple; when the valve is shut, the pressure of water on the larger surface closes it against the sides of the sluice; when the valve is opened, the gate swings round and takes a position in the direction of the current. Various other plans for flashing on similar principles are to be met with.

SEACOAST IMPROVEMENTS.

The following subdivisions may be made of the works belonging to this class of improvements. 1st. Artificial Roadsteads. 2d. The works required for natural and artificial Harbors. 3d. The works for the protection of the seacoast against the action of the sea.

Roadsteads. This term is applied to indentations of the coast, forming arms of the sea, where vessels may ride securely at anchor whilst waiting to proceed to sea. If the indentation is covered by natural projections of the land, or *capes*, from the action of the wind and waves, it is said to be *land-locked*; in the contrary case it is termed an *open* roadstead.

Before adopting any definitive plan for the improvement of the seacoast at any point, the action of the tides, currents, and waves at that point must be ascertained.

The theory of tides is well understood; their rise and duration, caused by the attraction of the sun and moon, are also dependent on the strength and direction of the wind. Along our own sea-board, the highest tides vary greatly between the most southern and northern parts. At Eastport, Me., the highest tides, when not affected by the wind, vary between twenty-five and thirty feet above the ordinary low water. At Boston they rise from eleven to twelve feet above the same point, under similar circumstances; and from New-York, following the line of the seaboard to Florida, they seldom rise above five feet.

Currents are principally caused by the tides, assisted, in some cases, by the wind. The theory of their action is simple. From the main current, which sweeps along the coast, secondary currents proceed into the *bays*, or indentations, in a line more or less direct until they strike some point of the shore, from which they are deflected, and frequently separate into several others, the main branch following the general direction which it had when it struck the shore, and the others, not unfrequently taking an opposite direction, forming what are termed *counter currents*, and, at points where the opposite currents meet, that rotary motion of the water known as *whirlpools*. The action of the currents on the coast is to wear it away at those points against which they directly impinge, and to transport the débris to other points, thus forming, and sometimes removing, natural obstructions to navigation. These continual changes, caused by currents, make it extremely difficult to foresee their effects, and to foretell the consequences which will arise from any change in the direction, or, the intensity of a current, occasioned by artificial obstacles.

A good theory of waves, which shall satisfactorily explain all their phenomena, is still a desideratum in science. It is known that they are produced by winds acting on the surface of the sea; but how far this action extends below the surface, and what are its effects at various depths, are questions that

remain to be answered. The most commonly received theory is, that a wave is a simple oscillation of the water, in which each particle rises and falls, in a vertical line, a certain distance during each oscillation, without receiving any motion of translation in a horizontal direction; and that the effects caused by this oscillation are only felt within its limits, or between the highest and lowest points of the wave; or, if felt beneath the surface of the water, they are only so at very inconsiderable depths. The objection to this theory is, that it does not explain many phenomena which are observed in connection with waves.

In a recent French work on this subject, its author, Colonel Emy, an engineer of high standing, combats the received theory; and contends that the particles of water, receive also a motion of translation horizontally, which, with that of ascension, cause the particles to assume an orbicular motion, each particle describing an orbit, which he supposes to be elliptical. He farther contends, that in this manner the particles at the surface communicate their motion to those just below them, and these again to the next, and so on downwards, the intensity decreasing from the surface, without however becoming insensible at even very considerable depths; and that, in this way, owing to the reaction from the bottom, an immense volume of water is propelled along the bottom itself, with a motion of translation so powerful as to overthrow obstacles of the greatest strength if directly opposed to it. From this he argues that walls built to resist the shock of the waves should receive a very great batter at the base, and that this batter should be gradually increased upwards until, towards the top, the wall should project over, thus presenting a concave surface at top to throw the water back. By adopting this form he contends, that the mass of water, which is rolled forward, as it were, on the bottom, when it strikes the face of the wall, will ascend along it, and thus gradually lose its momentum. How far these views are correct, observation remains to determine. It is certain, from experiments made by

the author in question upon walls of the form here described, that they answer fully their intended purpose.

The anchorage of open roadsteads is often insecure, owing to violent winds setting into them from the sea, and occasioning high waves which are very straining to the moorings. The remedy applied in this case is to place an obstruction, near the entrance of the roadstead, to break the force of the waves from the sea. These obstructions, termed *break-waters*, are artificial islands of greater or less extent, and of variable form, according to the nature of the case, made by throwing heavy blocks of stone into the sea and allowing them to take their own bed.

The first great work of this kind undertaken in modern times, was the one at Cherbourg in France, to cover the roadstead in front of that town. Various experiments were at first tried with no good result, and the plan was finally adopted of throwing the stone in loosely, and allowing it to take the best form. From the experience gained at this work, results have been arrived at with regard to the most suitable form for the cross section of a break-water. It appears (Fig. 123) that towards the shore, the mass of stone should receive a slope not less than 45°;—that towards the sea, the part of the mass below the lowest water line should receive a slope somewhat less than one perpendicular to one base;—the portion between this point and the highest water line should receive a slope between five and six base to one perpendicular;—and the part above this a slope of two base to one perpendicular. The Cherbourg break-water is ten feet above the highest tides towards the shore, and is twenty feet wide at top, a parapet wall being built on top, towards the sea, to afford a shelter to persons on the break-water during heavy gales.

The next work of the kind was built to cover the roadstead of Plymouth in England. Its cross section was, at first, made with an interior slope of one and a half base to one perpendicular, and an exterior slope of only three base to one

perpendicular; but from the damage it sustained in the severe tempests in the winter of 1816–17, it is thought that its exterior slope was too abrupt.

A work of the same kind is still in process of construction on our coast, off the mouth of the Delaware. The same cross section has been adopted for it as in the one at Cherbourg.

All of these works were made in the same way, by vessels discharging the stone on the spot, and allowing it to take its own bed; except for the facing, where, when practicable, the blocks were carefully laid, so as to present a uniform surface to the waves. The interior of the mass, in each case, has been formed of stone in small blocks, and the facing of very large blocks. It is thought, however, that it would be more prudent to form the whole of large blocks, because, were the exterior to suffer damage, and experience shows that the heaviest blocks yet used, have at times been carried off by the shock of the waves, the interior would still present a great obstacle.

From the foregoing details, respecting the cross sections of break-waters, which have been found to answer from experiment, the proper form and dimensions of the cross section in similar cases may be arranged. As to the plan of such works, it must depend on the locality. The position of the break-water should be chosen with regard to the direction of the heaviest swells from the sea, into the roadstead,—the action of the current, and that of the waves. The part of the roadstead which it covers should afford a proper depth of water, and secure anchorage for vessels of the largest class, during the most severe storms; and vessels should be able to double the break-water under all circumstances of wind and tide. Such a position should, moreover, be chosen that there will be no liability to obstructions being formed within the roadstead, or at any of its outlets, from the change in the current which may be made by the break-water.

Harbors. The term *harbor* is applied to a secure anchor-

age of a more limited capacity than the idea attached to a roadstead, and therefore offering a safer refuge during boisterous weather. Harbors are either *natural*, or *artificial*. An artificial harbor is usually formed by enclosing a space on the coast between two arms, or dikes of stone, or of wood, termed *jetties*, which project into the sea from the shore, in such a way as to cover the harbor from the action of the wind and waves.

The plan of each jetty is curved, and the space enclosed by the two will depend on the number of vessels which it may be supposed will be in the harbor at the same time. The distance between the ends, or *heads*, of the jetties, which forms the mouth of the harbor, will also depend on local circumstances; it should seldom be less than one hundred yards, and generally need not exceed more than five hundred. There are certain winds at every point of a coast which are more unfavorable than others to vessels fetching both in and out of the harbor, and to the tranquillity of its water. One of the jetties, should, on this account, be longer than the other, and be so placed that it will both break the force of the heaviest swells from the sea into the mouth of the harbor, and facilitate the ingress and egress to vessels, by preventing them from being driven by the winds on the other jetty, just as they are entering or quitting the mouth.

The cross section, and construction of a stone jetty differ in nothing from those of a break-water, except that they are usually wider on top, thirty feet being allowed, as they serve as wharves to vessels unloading. The head of the jetty is usually made circular, and considerably broader than the other parts, as it, in some instances, receives a lighthouse, and a battery of cannon. It should be made with great care, of large blocks of stone well united by iron or copper cramps, and the exterior courses should moreover be protected by fender beams of heavy timber, to receive the shock of floating bodies.

Wooden jetties are formed of an open frame work of heavy

timber, the sides of which are covered on the interior by a strong sheeting of thick plank. Each rib of the frame (Fig. 124) consists of two inclined pieces, which form the sides,—an upright centre piece,—and horizontal clamping pieces, which are notched and bolted in pairs on the inclined and upright pieces; the inclined pieces are farther strengthened by struts, which abut against them and the upright. The ribs are connected by large string-pieces, laid horizontally, which are notched and bolted on the inclined pieces, the uprights, and the clamping pieces, at their points of junction. The foundation, on which this frame work rests, consists usually of three rows of large piles driven under the foot of the inclined pieces and the uprights. The rows of piles are firmly connected by cross and longitudinal beams notched and bolted on them; and they are, moreover, firmly united to the frame work in a similar manner. The interior sheeting does not, in all cases, extend the entire length of the sides, but open spaces, termed *clear-ways*, are often left, to give a free passage and spread to the waves confined between the jetties, for the purpose of forming smooth water in the channel. If the jetties are covered at their back with earth, the clear ways are made in the shape of inclined planes.

The foundations of the jetties require particular care, especially when the channel between them is very narrow. An enrockment around the piles is the ordinary construction used for this purpose; and, if it be deemed necessary, the bottom of the entire channel may be protected by an apron of brush and loose stone.

The top of the jetties is covered with a flooring of thick plank, which serves as a wharf. A strong hand railing should be placed on each side of the flooring as a protection against accidents. The sides of jetties have been variously inclined; the more usual inclination varies between three and four perpendicular to one base.

Jetties are sometimes built up to form a passage to a natu-

ral harbor, which is either very much exposed, or subject to bars at its mouth. By narrowing the passage to the harbor between the jetties, greater velocity is given to the current caused by the tide, and this alone will free the greater part of the channel from deposits. But at the head of the jetties a bar will, in almost every case, be found to accumulate, from the current along shore, which is broken by the jetties, and from the diminished velocity of the ebbing tides at this point. To dissipate these bars resort may be had, when practicable, to reservoirs, and sluices, arranged with turning gates, like those adverted to for river improvements. The reservoirs are formed by excavating a large basin in-shore, at some suitable point from which the collected water can be directed, with its full force, on the bar. The basin will be filled at flood tide, and when the ebb commences the sluice gates will be kept closed until dead low water, when they should all be opened at once to give a strong water chase.

In harbors, where vessels cannot be safely and conveniently moored along side of the quays, large basins, termed *wet-docks* are formed, in which the water can be kept at a constant level. A wet dock may be made either by an in-shore excavation, or by enclosing a part of the harbor with strong water-tight walls; the first is the more usual plan. The entrance to the basin may be by a simple sluice, closed by ordinary lock gates, or by means of an ordinary lock. With the first method vessels can enter the basin only at high tide; by the last they may be entered or passed out at any period of the tide. The outlet of the lock should be provided with a pair of guard gates, to be shut against very high tides, or in cases of danger from storms.

The construction of the locks for basins differs in nothing in principle, from that pursued in canal locks. The greatest care will necessarily be taken to form a strong mass free from all danger of accidents. The gates of a basin-lock are made convex towards the head of water, to give them more strength to resist the great pessure upon them. They are hung and

manœuvred differently from ordinary lock gates; the quoin-post is attached to the side walls in the usual way, but at the foot of the heel-post an iron roller is attached, which runs on an iron roller-way, and thus supports that end of the leaf, relieving the collar of the quoin-post from the strain that would be otherwise thrown on it, besides giving the leaf an easy play. A chain is attached to each leaf near the centre of pressure of the water, and the gate is opened, or closed, by the motion of a windlass to which the other end of the chain is fastened.

The quays of wet-docks are usually built of masonry. Both brick and stone have been used; the facing at least should be of dressed stone. Large fender-beams may be attached to the face of the wall, to prevent it from being brought in contact with the sides of the vessels. The cross section of quay-walls should be fixed on the same principles as that of other sustaining walls. It might be prudent to add buttresses to the back of the wall to strengthen it against the shocks of the vessels.

Ordinary quay-walls are, with us, frequently made of timber after the usual fashion of crib-work; the cribs being filled in with earth and rubbish to form the walk of the wharf. Another mode of construction, which is found to be strong and very durable, is now coming into use, particularly in our Eastern seaports. It consists in making a kind of crib-work of large blocks of granite, and filling in with earth and stone rubbish. The bottom course of the crib may be laid on the bed of the river if it is firm and horizontal; in the contrary case a strong grillage, termed a *cradle*, must be made, and be sunk to receive the stone work. The top of the cradle should be horizontal, and the bottom should receive the same slope as that of the bed, in order that when the stones are laid they may settle horizontally.

Dry docks, and marine rail-ways, although necessary establishments in harbors, for the repairs of vessels, are omitted here, as they would demand special details with regard

to their construction and uses, which would be more readily seized upon by an inspection of existing works of the kind.

Dikes and Sea-walls. To protect the lowlands bordering the ocean from inundations, dikes, constructed of ordinary earth, and faced toward the sea with some material which will resist the action of the current, are usually resorted to.

The Dutch dikes, by means of which a large extent of country has been reclaimed and protected from the sea, are the most remarkable structures of this kind in existence. The cross section of those dikes is of a trapezoidal form, the width at top averaging from four to six feet, the interior slope being the same as the natural slope of the earth, and the exterior slope varying, according to circumstances, between three and twelve base to one perpendicular. The top of the dike, for perfect safety, should be about six feet above the level of the highest spring tides, although, in many places, they are only two or three above this level.

The earth for these dykes is taken from a ditch in-shore, between which and the foot of the dyke a space, of about twenty feet, is left, which answers for a road. The exterior slope is variously faced, according to the means at hand, and the character of the current and waves at the point. In some cases, a strong straw thatch is put on, and firmly secured by pickets, or other means; in others, a layer of fascines is spread over the thatch, and is strongly picketed to it, the ends of the pickets being allowed to project out about eighteen inches, so that they can receive a wicker-work formed by interlacing them with twigs; the spaces between this wicker-work being filled with broken stone: this forms a very durable and strong facing, which resists not only the action of the current, but, by its elasticity, the shocks of the heaviest waves.

The foot of the exterior slope requires peculiar care for its protection; the shore, for this purpose, is in some places, covered with a thick apron of brush and gravel in alternate layers, to a distance of one hundred yards into the water from the foot of the slope.

To protect the perpendicular bluffs of a high coast, which yields to the action of the sea, a facing of stone, termed a *sea-wall*, must be resorted to. These walls should be made of blocks of the largest dimensions, they are laid dry with great care to form a solid mass, the foot of the wall being covered by a strong enrockment of large blocks.

SUPPLEMENT.

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NOTE I.

On the Angle of Friction.

Let AB, and BC, (Fig. A.) be two inclined planes, the respective lengths of which are represented by l', l'', their respective altitudes by h', and h'', and the angles which they make with a vertical line by a', and a''.

If a body, a wheel carriage for example, were to start from a state of rest at the point A, it would, in its descent to B, acquire a certain velocity; and if the transition from AB to BC were gradual at the point B, the body would commence its descent along BC with the same velocity at B which it had acquired in descending to that point, and would continue its motion to the point C, where it would be found to have acquired a new velocity.

The circumstances of the motion here considered, in which the body is acted on by the incessant force of gravity alone, are, by a well known theorem in Dynamics, represented by the expression

$$v^2 = 2g \cos. a\ l,$$

in which v is the velocity;—g the force of gravity;—l the total length of the plane;—and a the angle it makes with a vertical.

But in the descent of a body down an inclined plane, there exists certain causes of retardation to the motion, the principle one of which is the friction, and as this force is found not to vary with the velocity it may be regarded as a constant retarding force, which is in opposition to that of gravity along the plane; so that the motion of the body along the plane will be due to the difference of these two forces.

If then this constant retarding force be represented by f, the

forces, which cause the motion on the two planes, will be represented, respectively, by the difference between the components of gravity along the planes, and the force f, or by

$$g \cos. a' - f, \text{ and } g \cos. a'' - f;$$

and the circumstances of the motion, when the body has arrived at C for example, will, from a law of dynamics, be represented by the expression

$$v^2 = 2(g \cos. a' - f)\, l' + 2(g \cos. a'' - f)\, l'';$$

but as $\cos. a'\, l' = h'$, and $\cos. a''\, l'' = h''$, this expression, by reduction, becomes

$$v^2 = 2g(h' + h'') - 2f(l' + l'');$$

or, representing by $h = h' + h''$, the total altitude of the planes, and by $l = l' + l''$, their total length, there results,

$$v^2 = 2gh - 2fl.$$

Now, as this expression holds true whatever may be the velocity, if it be made nothing, or $v = 0$, the resulting expression will show the relations which must exist between the quantities to satisfy this condition; but the velocity can be nothing only at the commencement of the motion, that is when h and l are, respectively, equal to nothing, or when the body, by the effect of the retarding force, is brought to a state of rest, which, if it be supposed to take place at the point C, will be expressed by

$$0 = 2gh - 2fl,$$

or

$$fl = gh. \quad \ldots \ldots \quad \text{(A)}$$

If the mass of the body be represented by M, and each member of equation (A) be multiplied by it, there results

$$fMl = gMh; \quad \ldots \ldots \quad \text{(B)}$$

but as g and f are the accelerating and retarding forces acting on each element of the mass, the total retarding force, or the whole amount of friction, will be $fM = F$; and the total accelerating force will be $gM = W$, or the weight of the body. Substituting, therefore, these quantities in eq. (B) there results,

$$F = W\frac{h}{l},$$

for the relations between the quantities, when the body is brought to a state of rest; from which the inference is drawn, that if a body, whose

weight is W, comes to a state of rest at a point C, in descending under the above-mentioned circumstances, the total amount of friction will be found, by multiplying the weight by the height of descent, and dividing this product by the distance passed over. Now, as this holds true under all circumstances, it also follows, that if an inclined plane were constructed whose height was h, and length l, that the body would remain in a state of rest on that plane; or, in other words, that the friction would exactly counterbalance the component of gravity along the plane, and the angle of the plane thus found will therefore represent the angle of friction.

NOTE II.

On the methods of determining the points of Circular Arcs.

The methods in use for determining on the field the points of circular arcs, to connect any two straight portions of a road, or track, are based upon a few very simple elementary principles of Geometry.

Let AC, and BC, (Fig. B) for example, be the directions of two straight tracks, to be connected at the points A, and B, by a circular arc. If the stations A, and B, are visible from each other, and the field of vision, within the triangle ACB, is clear, two graphometers, or any other instruments for measuring horizontal angles, may be placed at the stations, and a series of equal angles $AB1 = 1B2 = 2BC$, and $CA1 = 1A2 = 2AB$, be laid out from them, the points 1, 2, &c., where the chords $A1$, and $B1$,—$A2$, and $B2$, &c., intersect each other, so as to make the angles $CA1 = AB1$,—$CA2 = AB2$, &c., will be points in the required arc, which is tangent to AC, and BC, at the points A, and B.

When the field of vision, within the triangle ABC, is obstructed, the lengths of the equal chords $A1 = 12 = 2B$ must be calculated, from their angles of deflection, $2BC = 1B2$, &c., with the tangents AC, and BC, and with each other, which angles are given. The lengths of these chords, which are termed *sub-chords*, and are usually ten feet, are generally assumed, and from them the angles of deflection are determined, by means of the radius AO, of the circle to which the arc belongs.

The following calculations will be required to determine the radius AO, and any chord. Since the directions of the tracks are given, the angle at C is known, and its supplement, which is the angle O between the two radii AO, and BO; and from an elementary principle of Geometry.

$$AB = \frac{2r \sin. \frac{1}{2}c}{R}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

in which r is the radius AO,—c the angle at C,—AB the chord,—and R the tabular radius. From this expression the diameter of the circle, or $2r$, may be found by logarithmic calculation; by adding the logarithms of the tabular radius and the chord AB, and subtracting from their sum the logarith of the sine of half the angle at C; the remainder will be the logarithm of the diameter.

Having found the diameter of the circle, the angle of deflection $CA1$, between the tangent AC, and the given sub-chord $A1$, may be readily found, from an expression of the same form as the equation (A); which is, that any chord is equal to the diameter multiplied by the sine of half the angle subtended by the chord, and this product divided by the tabular radius; but as the angle of deflection is equal to half the angle subtended by the sub-chord, the logarithmic value for the sine of this angle will be found, by adding the logarithms of the sub-chord and tabular radius, and subtracting the logarithm of the diameter. Having thus found the angle of deflection, between the sub-chord $A1$, and the tangent AC, the position of the sub-chord can be laid down, and the point 1 be marked.

To find the point 2, at the extremity of an equal sub-chord; the angle of deflection, between the sub-chord 12, and the sub-chord $A1$ prolonged, must be known; this angle, from an elementary principle of Geometry, is twice the angle of deflection, between the sub-chord $A1$, and the tangent AC. The angle of deflection between all other equal chords will be the same.

Another method, for laying out the arcs, consists in dividing the chord AB, (Fig. C) into any number of equal parts, as, for example, ten feet; making $A1=1\,2=2\,3=3\,4$, &c., and then calculating the perpendicular offsets, to the chord at the points 4, 3, 2, 1, as $a4$, $b3$, &c.

This calculation may be readily made, when the radius AO, and

the distance $O4$ of the chord from the centre are known. The radius is found, as has just been explained; and

$$O4 = \sqrt{AO^2 - (\tfrac{1}{2}AB)^2}, \quad . \quad . \quad . \quad . \quad . \quad (B)$$

Now from a property of the circle, any ordinate, as bc, is expressed as follows,

$$bc = \sqrt{(r + Oc)\,(r - Oc)}, \quad . \quad . \quad . \quad . \quad . \quad (C)$$

and consequently,

$$b3 = bc - O4, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (D)$$

is known. In the same manner $d2$ would be found, by first calculating the ordinate at the point d, and subtracting $O4$ from it.

By means of the expressions (A), (B), (C), (D), the sub-chords, angles of deflections, and the offsets, for any given chord, may be found; and the whole may be arranged in a tabular form for practical operations on the field.

If, instead of connecting the points A, and B, by one arc, it were deemed advisable to use three, or any other odd number, in order to pass from the straight to the curved position of the track, by a very small angle of deflection, the method explained in (NOTE 4) might be applied, to determine the radii, &c., of the different arcs, which should be tangent to each other at the point of passage from one to the other.

NOTE III.

On the removal of earth in forming Excavations and Embankments.

The problem of the removal of earth, from one point to another, when reduced to its most simple terms, consists in finding what is the least cost for which a given excavation, which may be denoted by M, can be made, and the earth from it be carried to another point, for the purpose of forming a given embankment M'. The problem in itself is of a very complicated character, embracing in the data for its solution the most suitable tools, and machines, to which the force of men, animals, or other motive power, can be applied, both for excavating, and for transporting the earth.

To go into a full discussion of this subject would greatly exceed the limits of this work, and the present Note will, therefore, be

confined to a consideration of some of the more usual means of removal, and to some of the more simple questions arising out of it.

The motive force most generally employed in excavations is that of men, and the tools, to which this force is applied, are the pick and the shovel.

When the earth to be excavated is of such a character that a man, with his usual efforts, can remove it with the spade and shovel, without requiring it to be broken up with the pick, it is termed *earth of one man*. If the earth is of such hardness as to require it to be broken up with a pick, before it can be removed with the shovel, it is termed *earth of two, three,* or *four men*, according as one, two, or three picks may be found necessary to break up ground enough to keep one man with a shovel constantly employed. There is also reckoned fractional parts of a man's labor; as, for example, if one pick will keep two shovels employed, the earth is said to be of $1\frac{1}{2}$ *men*; if there are two shovels and three picks, of $2\frac{1}{2}$ *men*, &c.

The quality of the earth, or the number of men which it will require to excavate, can only be determined by experiment, and this may be done as follows:—an able bodied laborer is set to work with a pick, a certain time denoted by t, having broken up the ground; another, also of average strength and skill, is employed to pitch out the earth thus broken with a shovel; denoting by t', the time which he takes, it is evident that the ratio of the picks to be employed, to that of the shovels, will be expressed by $\frac{t}{t'}$, and that the quality of the earth will be represented by $1 + \frac{t}{t'}$. In estimating the value of this expression, no fraction less than $\frac{1}{2}$ is taken into account. From a variety of experiments on the daily labor of a man in excavating earth requiring only the use of the spade, it appears that a good laborer can throw out, on an average, 526 cubic feet, nearly, in ten hours' labor. If p then denotes the price of his day's work, the cost of a cubic foot of such earth, when thrown out, will be $\frac{p}{526}$; and, for a similar reason, the cost of a cubic foot of earth, the quality of which is represented by $(1 + \frac{t}{t'})$, will be expressed by

$$\frac{p}{526}(1 + \frac{t}{t'}).$$

This therefore will be one element of the total cost of removing a cubic foot; to determine the other elements, it will be necessary to examine the means of transportation. These means consist, either in throwing the earth from one point to another with the shovel, in which case it may be thrown about 12 feet in a horizontal, or five feet in a vertical direction; or, by conveying it in wheel-barrows by men, each wheel-barrow containing a load equivalent to about a cubic foot of ordinary earth; or else horse, or other power, may be used with suitable machinery. The case to be examined here will be that of transportation by wheel-barrows.

The most economical method of conducting this kind of transportation consists, in so arranging the distance, termed a *relay*, for the first wheel-barrow to go over, that a man can go and return in the time that it takes to load an empty barrow. The relay on horizontal ground has been fixed, by various experiments, at about 120 feet; and similar experiments have shown, that on sloping ground, the inclination should not exceed twelve base to one perpendicular, in order that the effort of a man, in ascending, may not be too great; and with this inclination, the relay should be two thirds of that on horizontal ground, or 80 feet, measured on the slope of one-twelfth. When there is more than one relay between the two points, between which the earth is to be carried, a man is stationed at the end of each relay, who receives the full wheel-barrows, and returns the empty ones. The other element of the total cost will be the price of transportation for each relay, which will be paid as the load itself: that is as the price of a day's work, divided by the whole number of cubic feet carried over the relay.

Having thus established the price for the excavation of a cubic foot, and that of its transportation over one relay, it is evident that the total cost will be the least possible, only when the distance to which the earth is carried is also the least possible; or, denoting by dM, the mass of any small quantity of the total excavation, and by r the distance that it is carried, the condition to be satisfied, in effecting the transportation, is that the sum of the products of these elements, and the distances over which they pass, shall be the least possible, or, $\int rdM$, a minimum. The question, as thus stated, evidently admits of as many solutions as there can be assigned different forms to the excavation and embankment, all, therefore,

that can be proposed to be done, will be to seek some general rules, which can be applied to each particular case; and, to do this in the clearest manner, it will be necessary, in the first place, to examine the most simple cases, and to pass from them to those of a more complicated kind.

Let the transportation, in the first place, be supposed to be effected on horizontal ground; and let it be proposed to remove a certain quantity of earth from an excavation, represented by the right line *AB* (Fig. D) to an embankment represented by the right line *ab*; *AB* and *ab* lying in the same line. Designating by M the length *AB*—by M' the length *ab*, — by d the distance *Bd*, and supposing the depth and breadth of M equal to unity; the section along *ab*, will be represented by $\frac{M}{M'}$, since the excavation is equal to the embankment. Then, as $\int rdM$ becomes, in this case, $\int rdr$, its value will be expressed by

$$\int rdr = \frac{r^2}{2} + \text{C}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let this integral be first taken with respect to the point D, the middle of *Ba*; then for this point, $r = M + \frac{1}{2}d$, and the integral represented by equation (1) being taken within the limits $r = M + \frac{1}{2}d$, and $r = \frac{1}{2}d$ will give

$$\int rdr = M\left(\tfrac{1}{2}M + \tfrac{1}{2}d\right),$$

that is, the cost of the transportation of M to the point D, is represented by the product of M and the distance of its centre of gravity from the point D.

If, now, the transportation of M, from D to *ab*, be considered, it must be borne in mind, that the element of M' is no longer dr, but, is represented by $\frac{M}{M'}dr$ making, therefore, this substitution in equation (1), and taking the integral between the limits $r = M' + \frac{1}{2}d$, and $r = \frac{1}{2}d$, there results

$$\int r \frac{M}{M'} dr = \frac{M}{M'}\left(\tfrac{1}{2}M + \tfrac{1}{2}d\right) M';$$

which shows, that the cost of removing M, from D to M', will be measured by the mass M, multiplied into the distance of the point D from the centre of gravity of M'; so that the total cost of the removal of the mass M upon M', will be truly represented by

the product of M, and the distance between the centres of gravity of M and M', or representing this distance by R, the minimum cost of removal will be represented by

$$\int rdM = MR. \quad . \quad . \quad . \quad (3)$$

The problem, as here solved, for this particular case, admits of a more general solution, from the analogy between it and the theory of moments; for $\int rdM$, is the sum of the products of each element multiplied in each of their respective distances from the point D, and this, from the theory referred to, is known to be equal to the total mass M, multiplied into the distance of its centre of gravity from the same point.

When several lines of excavation lie on one side, and several lines of embankment lie on the other, so that the carriage is in the same direction, it is perfectly indifferent as to what point the removal is commenced at; but, when the lines alternate, it is no longer so, for were the removal to take place in such a way that the particles should cross each other, there would be an unnecessary expense incurred, equal to the carriage over twice the distance between the points at which the particles are deposited. This is evident from the (Fig. E;) for if the embankment AB, is to be furnished by the two excavations ab, and $a'b'$; and a particle from ab, is taken to m, and a particle from $a'b'$ to m', it is clear that the paths passed over by the two particles, in thus crossing, is greater by twice mm' than they would have been, had the particles not crossed each other.

The line AB must be then so divided that one portion shall be furnished from ab nearest to it, and the other portion from $a'b'$. The same reasoning would apply to any number of lines under the same circumstances.

Let the case be now taken, in which it is proposed to remove a plane area of excavation, M, upon an area of embankment, represented by M'. Before examining this case, it will be necessary to demonstrate a very simple, but important, principle in the removal of earth, first pointed out by Monge; which is, if there are two elements m, and m', (Fig. F) to be removed to the two positions on the embankment n, and n', that the shortest road will be that, in which the lines passed over by m, and m', to n and n', do not cross each other between those points; for suppose the lines passed over to

intersect at o, it is evident that $mn'+m'n=m'o+on+mo+on'>mn+m'n'$, which proves the principle. Therefore, in the removal of the elements m, m', m'' &c., of any area, their new positions on the embankment must be so taken, that the right lines, which these elements follow, shall not intersect each other; for, in the contrary case, the lines gone over will be longer than they might have been, and the cost of the transportation will not be as small as it might have been.

To apply this principle to the case in point, draw two tangents to the two curves which enclose the areas M, and M'; it is evident, in the first place, that the particle m, (Fig. G) at the point of contact on M, must be removed to the position n, the point of contact on M'; for were it otherwise placed, some other particle must supply the place at n, and the lines passed over by these two particles would necessarily cross, and the principle just laid down would, therefore, be contravened. Let another point a be now taken, on the curve bounding M, very near to the point m, and let a line ab be so drawn, that the small area, between it and the tangent on M, shall be equal to that cut off on M'. This being done, it is evident, that the small elementary area cut off on M, must be removed upon that on M', for, since the two are equal by construction, if a single particle of the one were transported without the other, another particle would have to be taken from M, without ab, to supply its place, and as the lines followed by these two particles would cross each other, the principle would be violated, and the transportation not be as short as it might have been. Drawing a second line $a'b'$ in the same manner as ab, to cut off equal elementary areas, the same principle may be applied; and, so on for any number of lines $a''b''$, $a'''b'''$, similarly drawn. But as these lines are drawn very near each other, they may be regarded as sensibly parallel, and, therefore, the removal of each small elementary area of M, upon that of M', may be regarded as the case first treated, or as the removal of a right line on another; and, consequently, the minimum cost of the removal of M, upon M', will be represented by the sum of the products of these elementary areas of M, and the distances of their respective centres of gravity from those of the corresponding areas on M'. The quantity which is thus found, as representing the entire cost of removal, must not be confounded with the product

MR, or the mass into the distance of the centres of gravity of M and M', for this would represent the smallest value of $\int rdM$, which, evidently, would be less than the true cost, which is truly represented by the average, or mean of the distances r.

The case just treated, supposes that there is no very sensible divergence between the lines ab, $a'b'$, &c.; were it otherwise, the question as solved would not hold true. To ascertain, therefore, what is to be done in this case of exception, another principle, first laid down by M. Dupin, must be demonstrated; which is, if there are two equal bodies m and m' (Fig. H) to be removed to two points n and n', the latter lying on the line of direction mm', then the least transportation will be that in which m' is carried to n', and m to n; for join $m'n$, then there will obtain $mn<mm'+m'n$; therefore, $mn+m'n'<mm'+m'n+m'n'$, or $mn+m'n'<mn'+m'n$, which proves the principle advanced.

To apply this principle, let there be an area M, to be removed on another M'. Draw in the first place, the two tangents, ab, cd, (Fig. I) common to the two curves;—then assume any number of points a', a'', a''' &c., and c', c'', c''' &c., very near each other, and draw the secant lines $a'b'$, $a''b''$, &c., and $c'd'$, $c''d''$ &c., to cut off equal elementary areas on M, and M'; and continue these subdivisions, until two secants are found which intersect each other at a point x on the curve which encloses M. Up to this point the circumstances of the transportation will be precisely the same as in the case last treated; but at this point, and within the angle formed by the secants xz, and xz', there are particles which like x may be transported indifferently on either side, and those particles, when found, will lie on a curve, which will divide M into two parts, such, that the particles on each side of it must not be carried across another line which divides the area M' in a similar way. This is sufficiently evident from the principle which has just been laid down, without farther demonstration.

To construct the curve of separation on M, let it be supposed found, and let a second point x' be taken infinitely nigh to x; then from the nature of this curve, and the positions of x and x', it will be indifferent whether x is carried to z, and x' to z'; or x' to z, and x to z'; therefore, there will obtain, $xz+x'z'=xz'+x'z$; or $xz-x'z=xz'-x'z'$; but, if from x' two perpendiculars be demitted

on xz, and xz', respectively, the distances xp, and xp' will, evidently, be equal to the two differences just found, consequently, the element of the curve xx' bisects the angle zxz'. If then the angle zxz' be bisected, and a small element xx', be taken on the bisecting line, it will be an element of the curve sought; then from x' drawing two other secants $x'z''$, and $x'z'''$, to cut off equal areas on M and M', the next element $x'x''$ can be found by bisecting the angle $z''x'z'''$, and so on. A very few points, approximately determined, will be sufficient for all practical purposes.

Thus far, the space between the two areas M and M', has been considered perfectly free, so that the elementary areas could be transported by the most direct line to their positions on M'; but this is not often the case, and the mass will, on the contrary, be frequently obliged to pass wholly through one, or more points, determined either by some accident, as a bridge across a brook separating the areas, &c. Let there be, for example, the area M, (Fig. *K*) to be removed to M', subjected to pass through the point D. By dividing M into a great number of elementary areas, by lines drawn from D, the limiting lines of these elements may be considered, without any very sensible error, as parallel; and the cost of removal of each element m to D, will be represented by the product of m and the distance of its centre of gravity from D, or by mr; in the same manner the cost of removal of m' will be $m'r'$, &c.; by adding together these different products, there will result, for the sum or $\int r dM$ a quantity represented by MR, in which R is the average, or mean distance of r, r', r'', &c., from D. By a similar process, there will be obtained, for the cost of removal of M, from D upon M', a quantity MR', in which R' is the mean distance of the centres of gravity of the elementary areas of M' from D. Consequently, the total cost of removal from M to M', will be represented by $M(R + R')$.

If the point D could be arbitrarily taken, then its position should be so fixed, that the distance $R + R'$ should be the least possible. To find this minimum, would lead to a problem of a very complicated character. The shortest method, in practice, would be to assume several positions for D, and ascertain the best, by a series of trials. If two points D, and D', (Fig. L) were given, there would necessarily be found on the area M, a line xy such, that the

area on one side of it must pass through the point D, whilst that on the other side must pass through D'. A curve zW of a similar nature must also be found on the area M'. If these curves are supposed to be found, it is evident that any particle, x', on the one, may be removed to z', on the other, indifferently through D, or D'; there will, therefore, obtain $Dx'+Dz'=D'x'+D'z'$, or $Dz'-D'z'=D'x'-Dx'$, a property which belongs to the hyperbola, whose foci are the points D, and D', and whose axes will be determined by the condition, that the elementary areas on M and M' intercepted by the lines drawn through D and D', shall be respectively equal to each other. If there are a greater number of points than two, they must be considered two and two, and the different curves of separation, corresponding to them, be drawn. If the number of points is infinite the problem falls back into the first case of the transportation of areas.

Having examined the most ordinary cases of removal on horizontal ground, the next case to be considered will be that of removal on sloping ground, the inclination of which is less than one-twelfth; that is, the horizontal distance is greater than twelve times the perpendicular distance between any two particles of the areas M and M'.

Let M (Fig. M) be given, to be removed on M', and let any particle m, be supposed to be deposited at m'. In making this ascent the amount paid is estimated, by a rule adopted in such cases, to be the same as if the particle m (Fig. N.) was removed horizontally to the point m'', at the foot of a ramp $m'm''$, whose slope is 12 base to one perpendicular, and then carried up this ramp to m'. This being premised, let $m'a$ be the difference of altitude between the two particles, and ma be the horizontal distance between them, then the path passed over by m to m' will be represented by $mm''+\frac{3}{2}m''a$, since the transportation on the ramp $m'm''$ is reckoned equivalent to once and a half the transportation horizontally over its base $m''a$; but $mm''+\frac{3}{2}m''a=mm''+m''a+\frac{1}{2}m''a$, or $ma+6m'a$, since $m''a=12m'a$. That is, the cost of removal of the particle m to m' will be represented by the product formed by multiplying the particle by the sum of the horizontal path and six times the vertical distance between the points of departure and arrival. Designating by a the horizontal distance, and h the vertical distance between any

two particles, and by H the vertical distance between the centres of gravity of the two areas, the expression for the cost of removal of any element dM, will be represented by

$$\int rdM = \int adM + 6\int hdM$$

but, as $\int hdM = HM$, it follows that $\int rdM$, will be the least possible, only when $\int adM$ is also the least possible; and as $\int adM$ represents the cost of removing the area M, projected on a horizontal surface, upon the area M' projected on the same, it also follows, that to obtain the minimum cost in the case in point, the projections M and M' must be divided into elementary areas as on horizontal ground, and the removal be conducted exactly in the same way as in that case.

Having examined the case for the removal on a surface less inclined than one-twelfth, there remains to be considered the one, in which the earth is carried over one more inclined. In the first place, it may be remarked, that however great the inclination of the surface may be, a line can be laid out upon it, having a constant inclination of one-twelfth, or twelve base to one perpendicular; and, moreover, that, in starting from any point to reach another point of a higher level, this line may be either straight, or curved, or be composed of broken lines forming a zigzag path, since the inclination being the same at every point, each of these lines between the two levels will, from a property of lines on the same surface with a constant slope, be of the same length; so that whatever path it may be thought best for the particle to follow, the cost of removal will be the same in each case.

If the distance in projection of the point, from which the particle is to be removed, to that where it is to be deposited, is less than 12 times the difference of level, or of height between them, there will still be an infinite number of combinations of paths, having an inclination of one-twelfth, which can be laid out between the two points; and, moreover, if there are several points of departure, and several points of arrival, it will be perfectly indifferent in what order these points may be joined; and whether the paths joining them cross each other or not, provided, that the line followed by any point shall not correspond to a base greater than 12 times the altitude, for were this the case, the cost of removal would include a certain horizontal path which would be a fractional part of a relay to be added to the vertical relays, which would be so much pure loss.

But it may happen, from the order in which the points of departure and arrival present themselves, that a certain portion of horizontal relays must be paid for; and the question then presents itself, as to what way the lines must be disposed, to arrive at a minimum cost in this case. To examine this case, let m, and m' (Fig. O) be two points of departure, and n, and n', two points of arrival, such, that the inclination of the line joining m' and n' shall be less than one-twelfth, and that joining $m'n$ shall be greater: then the least cost for horizontal removal, supposing the particles to be removed partly horizontally, and partly on lines with an inclination of one-twelfth, will be, when the particle m is carried to n, and the particle m' to n'; for let the two lines dn, and $d'n'$ be drawn, with an inclination of one-twelfth, then to remove m to n', the horizontal path md', would have to be paid for, besides the carriage on $d'n'$, whereas by removing m to n, by md, and dn, and m' to n', by $m'd'$, and $d'n'$, there will only be a horizontal carriage equal to $md+m'd'<md'$ to be paid for.

Again, if instead of following only the horizontal paths, and those inclined one-twelfth, the particles m, and m', (Fig. P) could be carried to n and n', by the diagonal paths mn', less inclined than one-twelfth, and $m'n$ more inclined, still the least cost would result from depositing m at n, and m' at n'; for, join mn, and $m'n'$, and set off $no'=no$, then as o, and o', by this construction, will be on the same level, and as mn' is less inclined than mn, it follows, that $mo>mo'$; therefore, $mo+on>mn$; and, by a similar method, it may be shown, that $m'o+on'>m'n'$, consequently, that $mo+on+m'o+on'=mn'+m'n>mn+m'n'$. It, therefore, appears, that to obtain the minimum cost of removal, no path less inclined than one-twelfth must cross another of greater inclination.

To apply these principles to the removal of M to M', (Fig. Q) on a surface whose slope is greater than one-twelfth, draw, in the first place, on the surface, which for more simplicity, will be considered as plane, a line AB of one-twelfth; then see whether M and M', can be divided by a line ab parallel to AB into parts, such, that the part $x=z$ and $y=W$; that being the case, no particle from x, can be carried to W, for its place on z would have to be taken up by a particle from y, in the removing of which, a path, less inclined than one-twelfth, would necessarily cross one more inclined, and the cost

of removal would therefore be greater than it might have been. Drawing next the two tangents *cd*, and *ef*, if the first is less inclined than one-twelfth, the removal of x to z will be solved by the case already treated, for removal on ground less inclined than one-twelfth; and if the last is more inclined than one-twelfth, then it will be indifferent what system of paths are established for the removal of y to W; unless it should happen that there are some $a'b'$, parallel to *ab*, which will cut off equal areas on y and W.

Having thus terminated the examination of the most important cases of removal on horizontal and inclined ground, the next point for consideration is that in which the excavation and embankment are removed along paths considered in a vertical profile. In forming the embankment, by removing the earth in wheelbarrows, it is usual to carry up the work by depositing the earth in successive horizontal layers, from nine to twelve inches thick; but in excavating, the earth need not be removed in horizontal layers, although it should, in all cases, be removed in parallel layers, which may have any inclination that may present the greatest facilities to the laborers.

To examine the case proposed, let *abcd*, (Fig. R,) be the cross section of an excavation, and *efgh* that of a corresponding embankment, for which it is required to find the best method of removing the earth, so that the cost shall be the least possible. From the nature of the case, it is evident, that no particle can follow the most direct path, as on horizontal ground, and, as the point d remains invariable during the whole time of the work, all the paths, followed by the different particles, must pass through it. Through this point, therefore, let a line *dk*, with an inclination of one-twelfth, be drawn; the portion of the section above *dk* can be removed direct through the point d, without deviating from the plane of the cross section, either by a horizontal path, or by one of one-twelfth; and by following out this mode of removal, it is evident that the least expense will be incurred; for, suppose that a particle, instead of following the horizontal path $a'd'$ to the inclined path of one-twelfth, and then proceeding along it to d, was pushed forward to d''; having arrived there, it would be necessary to make an inclined path of one-twelfth from d'' to d, which would, necessarily, be out of the plane of the cross section, and as d'' and d' are on the

same level, this new path would be just as long as $d'd$, so that the particle if pushed forward to d'', would have to go over the horizontal path $a'd'' > a'd'$, and the two equal inclined paths, and by so doing, would cost more than in following the path $a'd'd$.

Having thus determined the most advantageous paths for the particles of akd as far as the point d, the next step will be to determine how they are to reach their positions on the embankment. This will be effected by making a path of one-twelfth along the inclined surface ef of the embankment, which surface is supposed steeper than one-twelfth, and having gained the top of any stratum, to follow then a horizontal path $e'h'$.

To find, in the next place, the cost of removing the portion akd, in this manner: Let its area be represented by z, m being one of its particles, d the horizontal path followed by this particle, along the excavation, d' its horizontal path along the embankment, and h the total height to which it has been raised; the cost of removing the particle m will then be expressed by $m(d + d' + 18h)$, since the base of the inclined path is twelve times the height, and the relay on it is paid as if it had passed over once and a half the base; as the same will hold true of all the other particles, and as their horizontal paths are parallel, it follows that, if D represents the horizontal distance of the centre of gravity of z to the point d, and D' the horizontal distance of the centre of gravity of the corresponding portion W of the embankment to the same point d, and H the vertical height between the centres of gravity of z and W, the total cost c' for removing z upon W will be expressed by

$$c' = z(D + D' + 18H').$$

There now remains the portion of the excavation below dk, to be removed. As no direct path of one-twelfth, below dk, passes through the point d, it will be necessary, in order not to add any useless cost for horizontal relays, that the particles shall all follow paths parallel to dk until they arrive at cd, and then ascend to d along paths of one-twelfth, placed along the surface cd; from the point d the particles must follow the same paths as those taken by the particles of z. Designating therefore the area $kbcd$ by z', —the horizontal distance of the centre of gravity of its corresponding portion of the embankment to the plan ef, added to the constant distance de by D'', —by H'' the vertical height between

the centres of gravity of the two portions in question, then the cost c'' of removing z' will be expressed by

$$c'' = z'(D'' + 18H'').$$

The total cost, therefore, or $c' + c''$ will be expressed by

$$c = z(D + D') + 18zH' + 18z'H'' + z'D'',$$

but designating the total area by $M = z + z'$, — by D''' the horizontal distance between the centre of gravity of the embankment $efgh$, and a line df' parallel to ef, — and by H the vertical distance between the centres of gravity of the areas of the excavation and embankment, then there will obtain

$$zH' + z'H'' = MH, \text{ and } zD' + z'D'' = MD'''$$

from which, by substituting these values in the expression for c, there results

$$c = zD + M(18H + D''').$$

Taking the foot as the unit of measure, the total number of relays will be expressed by dividing this expression by 120, or the number of feet in a relay; and to obtain the total cost, this quotient must be multiplied by c, or the cost of removing a cubic foot the distance of one relay. The above expression thus becomes

$$c = \frac{czD}{120} + cM\left(\frac{18H + D'''}{120}\right).$$

The foregoing is the simplest case of removal in a vertical direction. Let there now be considered two embankments, formed from an excavation between them. Designate by M, (Fig. 8) the area of the excavation, and by h the distance of its centre of gravity below the line ad, which line is supposed horizontal. It is evident, in the first place, that in whatever way the removal may be made, the cost of raising M, to the level ad, will be the same, and, from what has been shown, will be proportional to Mh, so that the only economy that can be effected will be, in so regulating the removal, that the horizontal removal shall be the least possible.

Having drawn the two lines of one-twelfth, am and dm, through the points a, and d, it will be shown, in the first place, that the area which they cut off is the only portion of M for which an expense for horizontal removal will be made. This portion cannot be limited at will by a line dm', less than one-twelfth, nor by one dm'', greater than one-twelfth, so as to avoid this expense; for, whether the portion $am'd$ is carried through a, or d, there will still be an

expense for horizontal carriage; and with respect to the portion *an''d*, the part of it *mn''d*, below *md*, which should only incur an expense for vertical carriage, would in this case, have to incur an expense for horizontal carriage to the point *a*. This point being settled, the next question is, as to the division of the portion *amd*, between the two embankments. Without going into the proof of this, it will be simply necessary to state, that it must, in all cases, be divided into two parts, by a line *mn* bisecting the angle *amd;* one part being carried through the point *a*, the other through *d*.

With respect to the remaining portion of *M*, below *amd*, it must be so divided by a line *mp* that the portion *apn* shall be equal to the area *efgh*, and the remaining portion equal to that *iklo*.

This case admits of an exception, where the line of division *mp*, (Fig. T) is less inclined than one-twelfth; for, by prolonging the line *dm* to *e*, it is evident, that an expense for horizontal carriage will be incurred for the triangle *mpe*, which must pass through the point *d*. An inspection of the Figure will show, that the least cost will not be obtained by carrying *mpe* through *d*, but, that to effect this, it will be necessary to cut off an area *nmgo*, equal to the triangle *mpe*, by a line *og*, which, from the nature of the problem, will be nearly parallel to *mn*, and to carry the area *dogebc* through the point *d*. This manner of dividing the area *abcd*, can evidently only be practically done, by excavating the part *aego*, before the excavation is pushed beneath the line *de*.

Let there now be taken the case where an embankment is formed from two excavations on each side of it. From what has already been laid down nothing more remains to be said respecting the division and removal of the excavations. But for the embankment, there must be found a line of separation such, that the portions into which the embankment is divided by this line, shall be respectively equal to the excavations next to them, and, moreover, that the expense of removal shall be the least possible.

Let *abcde*, (Fig. U) be the form of the embankment, and let the line of separation *pg* required be supposed found. It is then evident, since this line belongs equally to the two excavations, that if any two particles *k*, and *l*, be taken, it ought to be indifferent whether *k* followed the horizontal path *fk*, and *l* that *il;* or *k* followed the path *kh* and *l* that *gl*. There obtains, therefore, $k + il =$

$gl+hk$ or $il-hk=gl-fk$. If then, km be drawn parallel to ab, and ln parallel to de, $kn=il-hk$; and $lm=gl-fk$; and, in order that these lengths should always be equal, for all distances between the horizontal lines fh, and gi, the line kl must be a right line. The direction of this line can be readily found, as follows; draw any parallel nl, to de, and bisect it at o,—through o draw mk parallel to ab, and join the points l, and k, intercepted between any two horizontals, and kl will be the required direction.

The direction of this line will, of course, vary with that of the slopes of the embankment, being straight, broken, or curved, as these surfaces are uniform, irregular, or curved.

In the cases last considered, and in those which present but a multiplication of these same cases, as, for example, where there are several excavations and embankments so interlocked, that each embankment is formed from the two excavations nearest to it, as in the last cases, it is perfectly indifferent as to the time of making the different parts of the excavations and embankments, provided, however, the embankments be carried up throughout in regular horizontal layers; but, in the cases which are now to be considered, where several excavations furnish earth for the same embankment, or where there are several of the two kinds so interlocked, that the embankments are not formed from the excavations next to them, the time at which the different parts should be made is no longer a matter of indifference, for the paths followed by the particles must not only not cross each other, by proceeding along horizontal paths towards each other, as was noticed, when treating of the removal of lines; but they must also follow the shortest paths in a vertical direction to do which they should never descend from one level to rise to it again, nor the reverse.

To explain what is to be done in these cases, let M', M'', M''', (Fig. V), &c., be embankments to be formed from the excavations N', N'', N''', &c., and suppose they are so interlocked, that each embankment cannot be formed from the two excavations next to it, but that earth must be brought from N''', for example, for M', or M'', and so on. Since N' will not furnish enough for M', earth must be brought either from N'', or M''' to supply the deficiency; and the question then arises from which of them must it be taken. A moment's consideration will show, that it must not be taken from N''

unless it can furnish, not only what is wanted to supply the deficiency of N', but also, to furnish enough for M'', for otherwise, it would be necessary in forming M'' to take the earth for it from N''', either before, or after the excavation of N'', if this is done before the earth from N''', for M' would have to pass over that deposited at M'', which would cause the particles to ascend from one level to descend again to it; or if it is done after, the earth form N''', for M''', would have to descend from a level to ascend to it again; in both of which cases an increase of expense for vertical carriage will be incurred. The deficiency of M' must, therefore, be first supplied from N''', by a suitable line of separation, drawn in its section, before either of the excavations N', or N'' are commenced. From this it also follows, that if an embankment is to be furnished by several excavations lying on the same side of it, the removal must be commenced from the one farthest from the embankment.

As to the calculation of the cost in these last cases, it will be done as has been already explained, by calculating, in the first place, the horizontal and vertical paths, and multiplying their sum by the cost of a cubic foot removed to one relay's distance.

The above discussion comprehends the most general cases for this subject; but it remains quite incomplete, since many interesting particular cases of a complicated character still remain to be considered; besides the disposition of the inclined and horizontal paths, and that of the relays of laborers to carry forward the work with the most convenience and despatch. Enough, however, has been said to call attention to the works of M. M. Monge and Dupin, where it will be found more fully treated, and from what is here laid down, some useful hints may be drawn by those who are called upon to apply the general rules to particular cases.

NOTE IV.

Methods of describing Oval or Basket-handle Curves composed of Circular Arcs.

The span and rise of the intrados of an arch being given, in which the latter is some fractional part less than one-half of the former, an infinite number of curves, formed of arcs of circles, can be made to pass through the points of the springing lines, and the crown of the intrados, so as to satisfy the conditions of having their common tangent at the crown horizontal, and those at the springing lines vertical. To give the problem therefore a determinate character, some other conditions must be imposed, in addition to those just mentioned.

When the rise is not less than one-third of the span, a curve, composed of three circular arcs, or, as it is termed, an *oval of three centres*, is found to give a very suitable water-way, and, at the same time, a form which is pleasing the eye. This curve may be traced in various ways. The two following methods are in most frequent use:

Let AB be the half span, (Fig. W,) and AC the rise; with the radius AB describe the quadrant Ba,—take the arc $Bb = 60°$,—draw ab, bB, and Ab,—draw Cc parallel to ab, and from c, the intersection of Cc and Bb, draw cO parallel to Ab;—the points P and O will be the required centres, and PB and OC the required radii. The angle subtended by each of the three arcs will be $60°$, which is the other condition imposed by the construction.

Another method is as follows: Join D and C, (Fig. W,) make $Cd = Ca$ equal to the difference between the half span and rise,—bisect the distance Dd by a perpendicular, and the points R and S, where it intersects the span and rise produced, will be required centres. This construction results from the imposed condition that the ratio $\frac{R}{r}$ of the least and greatest radii, shall be the least possible; or from the principle of maxima and minima, that $\frac{d(\frac{R}{r})}{dr} = 0$.

The analysis, from which the result is obtained, is of a very simple character; for designating by $R = SC$ the greater radius;

by $r = RD$ the lesser,—by $a = AD$ the half span, and by $b = AC$ the rise, there results, from the right angle triangle SAR,

$$\overline{SR}^2 = \overline{AS}^2 + \overline{AR}^2,$$

or

$$(R - r)^2 = (R - b)^2 + (a - r)^2,$$

from which is obtained

$$\frac{R}{r} = \frac{a^2 + b^2 - 2ar}{(2b - 2r)r}.$$

Now by differentiating this expression, and placing its differential co-efficient equal to zero, or making $\frac{d(\frac{R}{r})}{dr} = 0$, there results, after the terms are reduced,

$$r = \frac{a^2 + b^2 - (a - b)\sqrt{a^2 + b^2}}{2a} = \frac{\sqrt{a^2 + b^2}}{a} \cdot \left(\frac{\sqrt{a^2 + b^2} - (a - b)}{2}\right),$$

but $\sqrt{a^2 + b^2} = DC$, and $\sqrt{a^2 + b^2} - (a - b) = Dd$, hence the given construction for the centres required.

By comparing the two methods just explained, for the same span and rise, it will be seen that the former gives a curve in which the lengths of the arcs differ less than in the latter, and which is therefore more agreeable to the eye.

When the rise is less than one-third, and greater than one-fourth of the span, a three centre oval does not give a curve of an agreeable form, and five, or a greater number of centres, should therefore be taken, as circumstances may seem to demand. The objects to keep in view, being to form a continuous curve without abrupt transitions, so as to please the eye; to obtain the most simple arrangement for the voussoirs, and, if necessary, the greatest practicable water-way. As these are not all compatible conditions, circumstances must decide which to reject. The most pleasing form of curve will result from the greatest number of centres; but the simplest arrangement of the voussoirs, and the largest water-way, will result from the smallest number of centres. The water-way may, moreover, be increased by making the smallest radius as great as practicable, but not to exceed the rise, and making the arc which belongs to this radius also as great as practicable, keeping it, however, within a limit which will make the largest radius, or that of the crown, not longer than once and a half the span.

To give the problem, with the above conditions, a determinate character, other conditions may be imposed, as that the arcs shall receive the same decreasing ratio of curvature from the one at the springing line to that of the crown, in which case their radii will have the same increasing geometrical ratio,—and that the arcs shall be of equal length, in order to produce an agreeable effect; or, if this cannot be done without making the radius of the crown too great, then the arcs may be made unequal, but increasing by a constant ratio from the springing line to the crown, which ratio may be as nearly that of equality as may be desired.

The above conditions may be expressed analytically for any number of centres; but it has been deemed best in this place to give a solution of the problem for a particular case, which will show the course to be pursued in all others.

Let it be required to ascertain the relations of the span, rise, the radii, and the number of degrees in each arc, in an oval of five centres, where the arcs receive the same decreasing ratio of curvature from the springing line to the crown, and a given increasing ratio in length. Designate the half span AB by p, (Fig. X,) the rise by q,—the ratio of the radii by m,—the ratio of the arcs by n, and the number of degrees in the arc at the springing line by a. Suppose the centres O, P and Q found, and draw PS perpendicular to AB,—and PR perpendicular to BC produced.

The radii OA, PE and QD will be represented respectively by r, rm, and rm^2,—and the angles AOE, EPD, and DQC, between them by

$$a,\ a\frac{n}{m},\ \text{and}\ a\frac{n^2}{m^2};\text{—}$$

now, from the properties of the figure, the following equations are obtained

$$a + a\frac{n}{m} + a\frac{n^2}{m^2} = 90^\circ \quad . \quad . \quad . \quad : \quad . \quad . \quad . \quad (A)$$

$$AB = p = r + OS + PR, \quad . \quad . \quad . \quad . \quad . \quad (B)$$

$$BC = q = rm^2 - (PS + QR), \quad . \quad . \quad . \quad . \quad (C)$$

From the right angle triangle OPS, and PQR, there results,

$$OS = OP\cos.\ a = (rm - r)\cos.\ a;$$

$$PS = OP\sin.\ a = (rm - r)\sin.\ a;$$

$$PR = PQ\cos.\left(a + a\frac{n}{m}\right) = (rm^2 - rm)\cos.\left(a + a\frac{n}{m}\right);$$

$$QR = PQ \sin.\left(a + a\frac{n}{m}\right) = (rm^2 - rm)\sin.\left(a + a\frac{n}{m}\right);$$

by substituting these values in equations (B) and (C), there results,

$$p = r\left\{1 + (m-1)\cos. a + (m^2 - m)\cos.\left(\frac{m+n}{m}\right)a\right\}; \qquad \text{(E)}$$

$$q = r\left\{m^2 - (m-1)\sin. a - (m^2 - m)\sin.\left(\frac{m+n}{m}\right)a\right\}; \qquad \text{(F)}$$

and by reduction, equation (A) becomes,

$$a = \frac{(m-n)\,m^2}{m^3 - n^3}90^\circ; \quad \ldots\ldots\ldots\ldots \quad \text{(G)}$$

The equations (E), (F) and (G) express, therefore, the relations which subsist between the six quantities p, q, r, a, m and n when the imposed conditions are satisfied. Let three of these quantities as m, n and r be assumed, the others will be found from the three equations in question; that is the span, rise, and number of degrees in the arc at the springing line, which correspond to the given values.

From the solution here given, the ratio of p to q, or $\frac{p}{q}$ is found; but as the rise and span are usually a part of the data, this ratio $\frac{p}{q}$ may be different from that $\frac{b}{c}$ of the given half span b, and rise c; in which case, it will be necessary to assume new values for the quantities m, n, and r and find the corresponding values of p, q and a, until the ratio $\frac{p}{q}$ is equal to, or nearly the same as $\frac{b}{c}$. When a suitable approximation has been obtained, it will be easy to find a curve which shall differ but little from the required one, and whose half span and rise shall have the required ratio $\frac{b}{c}$.

To effect this, let x be the quantity which must be added to p and q, respectively, to make their ratio the same as that of b to c; this condition will be expressed by the equation,

$$\frac{b}{c} = \frac{p+x}{q+x},$$

from which there results

$$x = \frac{pc - qb}{b - c}; \quad \ldots\ldots\ldots\ldots \quad \text{(H)}$$

if now this quantity be set off from A to M, (Fig. X) and from C, to N, and a new curve AN be described from the same centres O, P and Q, it will be parallel to the curve AC, whose half span and rise are p and q, and the half span BM, and rise BN, will have the same ratio as b to c. To pass from this curve to a similar one, described on the given half span b, and rise c, it will be only necessary to multiply each line of the figure $QPOMN$ by the ratio

$$\frac{b}{p+x};$$

or, substituting for x its value, as determined in equation (H), by

$$\frac{b-c}{p-q},$$

since the figures being similar, their homologous lines are proportional, or, for example,

$$p+x : b :: OM : \frac{b}{p+x} OM;$$

which will give the line, corresponding to OM, in the figure of which b is the half span, and c the rise.

The method here explained may be applied to any number of centres, but where the rise is not less than one-fourth of the span, an oval of five centres will be found to answer fully all the required conditions.

There are other methods of describing the oval of five, or, a greater number of centres, which are rather more simple for calculation than the general method just given.

By assuming, for example, the greatest and smallest radii within suitable limits, the intermediate radius may receive the condition, of being a mean proportional between these two; or designating it by x, there will result $x=\sqrt{R \times r}$;—R being the greatest radius, and r the least. Having found x, the position of the intermediate centre P is found, by describing an arc from Q with a radius $R-x$, and another from O with a radius $x-r$, and taking their point of intersection P.

A similar process might be followed for an oval of seven centres, by finding the two intermediate terms of a geometrical progression, of which r and R are the two extremes.

To apply this Note to the objects of (Note I.), it will be simply necessary to find the angle of deflection between the chords of the

different arcs at their points of contact;—for example, the angle of deflection bcC, between the chord Bc (Fig. W) produced, and the chord cC. By drawing the common tangent to the two arcs, at the point c, a very simple calculation will show, that the angle bcC is the sum of the angles of deflection of the two chords Bc, and cC, and the common tangent at the point c; and as these two last angles can be readily calculated, by the formula in (Note I.), the required angle can be obtained. With regard to the intermediate points, they will be found by the methods explained in (Note I.)

NOTE V.

Theory of the Equilibrium of Arches.

The mathematical conditions of the equilibrium of arches early attracted the attention of mathematicians, and various solutions of the problem were given, but as they all reposed on hypotheses, which were not in accordance with the results of observation, but little progress was made in the theory, and that little rather of a speculative than of a practical character. The subject remained in this state until Coulomb, guided by the observation of the effects which actually are seen to take place in the construction of an arch, placed it under a new and more accurate point of view. It is upon the suggestion of this able *savan*, whose hints ever have been pregnant with important results to the cause of physico-mathematical science, that later writers have built up a theory, and arrived at results, which are more nearly in accordance with the true state of the question, as derived from observation and experiments.

The method of treating the question, as proposed by Coulomb, may be briefly stated as follows: He takes any semi-arch, as $AMNB$, (Fig. Y) which he supposes to be kept in equilibrium by the action of a force Q, applied horizontally at some point of the joint M, and he proposes to find the limits of this force by which the arch will be prevented from yielding, either by sliding along any one of its joints as mn, for example, in the direction mn, or that nm, or by

a motion of rotation inwards, around the point m of the joint, or outwards around the point n. Having obtained the expression for the force Q, he finds the maximum value of it, which will prevent the arch from sliding in the direction nm, and the minimum value which will cause it to slide in the direction mn, and states it as evident, that the limit of Q, or the force which will be just sufficient to maintain the equilibrium, must be greater than the maximum, and less than the minimum values thus obtained. He next examines the conditions of equilibrium with regard to rotation around the points m and n, and finds, in a similar manner, the maximum value of Q, which will prevent the arch from yielding at any joint, by rotation around the point m, and the minimum value that will cause it to yield by rotation around n, and shows, what is evident, that to maintain the equilibrium, the force must be greater than this maximum, and less than the minimum. By this process, two superior, and two inferior limits are obtained for the force Q, between which it must be found, in order that the arch shall neither yield by sliding, nor by rotation at any joint.

The question, thus presented, admits of a natural division into two parts; 1st, the mathematical development of the theory, and the consequences resulting from it; 2d, the application of the theory to the determination of the eqilibrium of given arches.

Let there be an arch $ABMN$, (Fig. Z,) the lower extremity AB of which rests against an immovable plane, whilst the voussoirs above AB are retained in a state of equilibrium by a horizontal force Q, applied at any point of the joint MN; it is required, in the first place, to ascertain the limits of the force Q, such that the portion of the arch above any joint, as mn, shall be prevented from separating, from the part below the joint, by sliding in either of the directions nm or mn.

Designate by W the weight of the portion $mMNn$, above the joint mn; and suppose a force equal to W to act vertically, in a line passing through the centre of gravity of $mMNn$. Let θ represent the angle which the joint mn, makes with the vertical line ON, and designate by z the length of the joint,—by c the cohesion on the unit of length along the joint,—and by f the coefficient of friction, which is supposed proportional to the pressure.

The force Q, which may be represented by ac, taken on its line of direction, can be decomposed into two others, one $bc = Q \sin. \theta$,

parallel to the direction of the joint; and the other $ab = Q \cos. \theta$ perpendicular to it. By a similar decomposition of the force W, represented by df, taken on its line of direction, there will be obtained $ef = W \cos. \theta$, for the force parallel to mn, and $ed = W \sin. \theta$ for the perpendicular component.

The forces which tend, therefore, to prevent the sliding on the direction mn, are $Q \sin. \theta$ and the friction and cohesion on the joint mn; the friction being proportional to the perpendicular pressure on the joint will be represented by

$$f(Q \cos. \theta + W \sin. \theta),$$

and the cohesion, being proportional to the length of the joint, will be represented by

$$cz,$$

but the force which tends to produce the motion in question is

$$W \cos. \theta,$$

Now in order that a state of strict equilibrium may ensue, this last force must be equal to the sum of the other three, or

$$Q \sin. \theta = W \cos. \theta - f(Q \cos. \theta + W \sin. \theta) - cz; \quad . \quad (1)$$

but as the force Q, in this expression, varies with the angle θ, its limit will be found from that value of θ which gives a maximum; as it is clear that any value of Q less than this, if applied at the joint MN, would not counteract the tendency of the arch to give way by sliding.

As any value of Q, greater than this maximum, will prevent the downward tendency to sliding, such a value, applied at MN, will maintain the equilibrium; but by increasing this value, it might become, at length, so great as to cause the arch to yield, by sliding upwards, in the direction mn. Now since this motion would be caused by,

$$Q \sin. \theta,$$

and would be counteracted by the force

$$W \cos. \theta,$$

by the friction

$$f(Q \cos. \theta + W \sin. \theta),$$

and by the cohesion

$$cz,$$

it is necessary in order that a state of strict equilibrium shall ensue, that the first must be equal to the sum of the three last, or,

$$Q \sin.\theta = W\cos.\theta + f(Q\cos.\theta + W\sin.\theta) + cz, \quad . \quad . \quad (2)$$

In this case the force varies as θ, and its limit will be found from that value of θ which makes it a minimum; for it is evident, that were a force greater than this, applied at MN, it would cause an upward sliding along mn, and the arch would yield, from the upward portion being pushed upwards by the action of force Q greater than the minimum.

The two limits of the force Q are thus clearly defined; and, it is evident, that if any force less than the maximum be applied to MN, that rupture will ensue, by a downward sliding; and if it be greater than the minimum, by an upward sliding, and, therefore, the force that will maintain the arch in a state of stable equilibrium, must lie between these two limits. In establishing the conditions of equilibrium, in the circumstances just treated, it is immaterial at what point along MN (Fig. AA) the force Q is applied; but, in the case of rupture by the portion above any joint mn turning around either point m, or n, the value of Q will vary with the position of the point of application along MN. In the cases about to be discussed, the point of application will be taken at N for the case in which the rupture ensues from the portion $mMNn$ turning inwards, around the point m, as it is the one most unfavorable to the motion taking place; whereas it will be taken at M for the same reason, when rupture ensues from the upper portion turning outwards around the point n.

Designate, as before, the horizontal force, applied at N, or M, by Q; and the weight of the portion $mMNn$, which acts in a vertical through its centre of gravity, by W. Let x represent the length of the perpendicular drawn from the point m to the vertical along which W acts, and y the perpendicular from the same point to the line of direction of the force Q, applied at N. Represent by a the cohesion on a unit of length of the joint mn, and by z the length of the joint.

When the arch tends to yield, by turning inwards around the point m, this tendency will be measured by the moment of the force W taken with respect to this point, or by

$$Wx;$$

this tendency will be counteracted, in the first place, by the moment of the force Q, taken with respect to the same point, or by

$$Qy,$$

and by the moment of the force of cohesion along mn, taken with regard to the same point. To ascertain the value of this last moment, it will be necessary to adopt some hypothesis with respect to the action of the resistance offered by the force of cohesion to the particular kind of rupture in question. Nothing perfectly satisfactory is known with regard to the law of variation of this force; but it seems natural to suppose, that it varies for each point as the distance of the point from m. If then an element dv of the joint mn, whose distance from m is v, be taken, the resistance of this element will be represented by

$$\frac{v}{z}\,a\,dv,$$

and its moment with respect to the point m by

$$\frac{v}{z}\,a\,dv \times v,$$

the integral therefore of this expression, or the sum of the moments of the elementary resistances, which is represented by

$$\int \frac{v}{z}\,a\,dv \times v = \frac{a}{z}\int v^2\,dv,$$

will give the total moment required; that is

$$\frac{a}{z}\int_o^z v^2\,dv = \tfrac{1}{3}az^2,$$

by taking the integral between the limits $v = o$ and $v = z$.

The equation of strict equilibrium will therefore be expressed by

$$Qy = Wx - \tfrac{1}{3}az^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

But as x and y vary with θ, or the inclination of the joint mn, Q will also vary with the same quantities, and that value of the force, which will maintain the equilibrium will evidently be the maximum value of Q. Reasoning, as in the case of rupture from sliding, it will appear that no value of Q less than this will satisfy the required conditions; but by increasing Q it may, at length, become so great as to push the upper portion outwards by turning around the point n, and, as in this case, it would have to overcome both the moment of W, and the moment of cohesion, with respect to that point, the equation of equilibrium would be expressed by

$$Qy' = Wx' + \tfrac{1}{3}az^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4),$$

y' and x' representing the respective perpendiculars from n to the

lines of directions of the forces. Now the maximum value of Q, which is furnished by equation (4), is the greatest force which can be applied at N, without the circumstances of rupture, here considered, taking place; and as the maximum value furnished by equation (3), is the least that will keep the upper part of the arch from turning inwards, it follows that a stable equilibrium can be maintained only by a force whose value lies between the two just found.

If the point of application of the force be taken at M, a similar system of equations will be found to express the conditions of equilibrium; and a similar course of reasoning will be followed to define the superior and inferior limits of the force Q.

Before proceeding to notice the inferences drawn from the developments just given, it will be well to examine the nature and value of the pressure on the surface of the joints arising from the forces Q and W. This value is immediately derived from the first part of this discussion; for the components of Q and W, perpendicular to any joint mn, were there found to be $Q \cos. \theta$, and $W \sin. \theta$, and their sum will be the total amount of pressure, or designating it by P,

$$P = Q \cos. \theta + W \sin. \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (R).$$

For the horizontal joint AB, $\theta = 90°$, and the pressure becomes

$$P = W;$$

and for the vertical joint MN, $\theta = 0$, gives

$$P = Q.$$

To return to the equations (1), (2), (3) and (4), it will be found, by neglecting the values for friction and cohesion, that equations (1) and (2) are reduced to one and the same expression, which is

$$Q \sin. \theta = W \cos. \theta,$$

or

$$\tan. \theta = \frac{W}{Q} \quad . \quad . \quad . \quad . \quad (5);$$

a relation which shows that the direction of the resultant of the two forces Q and W must be perpendicular to the joint nm in order that sliding shall not take place under the circumstances here considered.

The equations (3) and (4), the first of which is derived from the condition that Q is applied at the point N, and the second or (4) at the point M, are reduced, by neglecting friction and cohesion, to

$$Qy = Wx, \text{ and } Qy' = Wx' \quad . \quad . \quad . \quad (6);$$

or $$Qy - Wx = 0 \text{ and } Qy' - Wx' = 0;$$

which express the conditions that the resultant must pass through the points m or n in the case of a strict equilibrium, or in the case of a stable equilibrium, which is expressed by

$$Qy > Wx, \text{ and } -Qy' < Wx',$$

that the resultant must pass between the points m and n.

Having derived these consequences, let it be proposed to find the conditions of a strict equilibrium in the case of a *plate-bande,* or flat arch.

Let $AMNB$, (Fig. BB,) be the half of a plate-bande, of which the half span $AM = a$, and the thickness $MN = b$, are given. Let θ' designate the angle which the extreme joint AB makes with the vertical ON; and θ the angle of any joint as mn, — let x represent the distance mM,—finally, let w designate the weight of an unit of volume of the plate-bande.

The area of the trapezoid $mMNn$ is equal to the areas of the rectangle Mc, and the triangle mcn, or equal to

$$bx + \tfrac{1}{2}b^2 \tan. \theta;$$

and the weight of the portion of the plate-bande $mMNn$, corresponding to a unity of length, will be

$$w(bx + \tfrac{1}{2}b^2 \tan. \theta) = W;$$

by substituting, therefore, this value of W in equation (6), there results

$$\tan. \theta = \frac{w(bx + \tfrac{1}{2}b^2 \tan. \theta)}{Q};$$

from which, by reduction, there is obtained

$$\tan \theta = \frac{2wbx}{2Q - b^2 w}.$$

By the same process, there will be obtained

$$\tan. \theta' = \frac{2abw}{2Q - b^2 w},$$

by dividing, therefore, the first of these expressions by the second, there results

$$\frac{\tan. \theta}{\tan. \theta'} = \frac{x}{a} = \frac{mM}{AM};$$

an expression which shows that the two joints AB and mn prolonged, must pass through the same point O of the vertical; and as the joint

mn is taken at will, the condition of equilibrium, expressed by equation (5), will be satisfied, when all the joints of the plate-bande pass through the point O.

From the expression for tan. θ there results, by reduction,

$$Q = w\frac{2ab + b^2 \tan. \theta'}{2 \tan. \theta'};$$

so that Q, or the horizontal thrust, is easily found when the angle θ' of the first joint is known.

To ascertain the limit of this angle, the condition expressed in equation (6) must be examined. These equations express that the moment of the force Q, applied either at N or M, must be equal to the moment of the trapezoid $ABNM$, taken with respect to the same points. But, as the moment of the trapezoid is equal to the sum of the moments of the rectangle MC, and the triangle ABC, when referred to the point B, and to the difference of their moments when referred to the point A, it will be expressed in the latter case by

$$wab \times \tfrac{1}{2}b - \tfrac{1}{2}wb^2 \tan. \theta' \times \tfrac{1}{3}b \tan. \theta',$$

or,

$$\tfrac{1}{2}wab^2 - \tfrac{1}{6}wb^3 \tan.^2 \theta';$$

and in the former by

$$wab\,(\tfrac{1}{2}b + b \tan. \theta') + \tfrac{1}{2}wb^2 \tan. \theta' \times \tfrac{2}{3}b \tan. \theta',$$

or

$$\tfrac{1}{2}wab(b + 2b \tan. \theta') + \tfrac{1}{3}wb^3 \tan.^2 \theta';$$

and the equations of condition (6) will become, by substituting these values, and placing them equal to the moment of the force Q,

$$Qb = \tfrac{1}{2}wab^2 - \tfrac{1}{6}wb^3 \tan.^2 \theta';$$

and

$$-Qb = \tfrac{1}{2}wab\,(b + 2b \tan. \theta') + \tfrac{1}{3}wb^3 \tan.^2 \theta'.$$

The last of these equations, which expresses the condition of strict equilibrium, becomes

$$-Qb < \tfrac{1}{2}wab\,(b + 2b \tan. \theta') + \tfrac{1}{3}wb^3 \tan.^2 \theta',$$

for a state of stable equilibrium; and as the condition will be satisfied by any positive value of Q, it follows, that it will be impossible for the plate-bande to turn outwards around the point B, by the application of any force Q at M.

With regard to the first of these expressions, the value of the angle θ', which will satisfy the required conditions, can be deduced

from it by substituting for Q, its value already found in terms of θ'. This substitution being made, there results

$$\tan.^3\theta' - 3\left(\frac{a^2}{b^2} - 1\right)\tan.\theta' + \frac{6a}{b} = 0;$$

an equation from which θ' may be determined.

The limit of θ', found from this equation, may be determined by the following construction which results from the conditions expressed in the preceding equations. Through the centre of gravity o of the trapezoid $AMNB$, let a vertical be drawn to intersect the direction of the force Q, applied at N, at the point d. Let the force Q be compounded with the weight W of the trapezoid; from the preceding conditions their resultant must be normal to the joint AB, and must not pass below the point A. If then this resultant be drawn, and through the point A where it intersects the lower line of the plate-bande, a line AO be drawn perpendicular to it, the construction will give the limit of the angle θ', which the first joint AB must make with the vertical.

Before proceeding to the second part of this question, or the application of the theory to determining the dimensions of the parts of an arch of a given form, it will be well to notice the experiments which have been made to determine the manner in which rupture usually takes place in arches; and the position of what are termed the *joints of rupture*, or the joints where the arch most usually separates.

The following general results on this point were obtained partly by special experiments, and partly from observations made on the changes which were found to take place in the arches of large bridges. From the experiments, it was observed, that any motion which took place in the different parts of the arch, arose, invariably, from a tendency to rotation around either the upper or lower edges of the joints, according to the value of the pressure;—that the arch, when rupture ensued, usually separated into four parts, presenting a joint of rupture at the key-stone, one between it and the joints, at, or near the springing lines; and one at or near the springing lines; the two upper portions of the arch falling inwards, by turning around the exterior edge of the joint at the key-stone, and the interior edges of the next two lower joints of rupture, whilst the lower portions were thrust out-

wards, turning around the exterior edges of the joints of rupture near the springing lines; — the joints of rupture below the key-stone, in full centre arches, were found by the experiments to be near the joint whose angle with the horizon was about 30°; and in the oval of three centres, of which the rise was between one-third and one-fourth of the span, the joint of rupture was usually observed to show itself between the 45° and 55° of the small arcs, estimating from the springing line; in segment arches, when the segment did not exceed 120°, the joint of rupture was found at the springing line.

The observations made on large bridges accorded nearly with the results of the experiments.

In treating the second part of the question, the following data are assumed, and the conditions of equilibrium are determined from them, viz:—The span and rise,—the curve of the intrados,—the height of the abutments or piers,—the distribution of the weights to be sustained by the arch,—finally, the thickness of the arch at the key-stone, this thickness being, generally, regulated by existing arches which most nearly approach the one in question. All the voussoirs receive the same thickness, or else their thickness is gradually increased from the key-stone to the springing line.

Having in this manner regulated the principle dimensions, the next point is to examine whether the arch thus projected will possess the necessary stability. To ascertain this, recourse must be had to the equations (1), (2), (3) and (4), which establish the relations that must exist between the forces which tend to cause rupture, and those which tend to prevent it.

The equation (1), when reduced, may be placed under this form,

$$Q = \frac{W(\cos.\ \theta - f \sin.\ \theta) - cz}{\sin.\ \theta + f \cos.\ \theta} \quad . \quad . \quad (7);$$

from which Q may be found; but as its value varies with the angle θ, a separate calculation must be made for each joint, and that one must be taken for the effective pressure at the key-stone which is the greatest. This greatest value, which is in fact the pressure sustained at the joint of the key-stone, arising from the weight of the two semi-arches, is termed the *horizontal thrust* of the arch; and it is the least force, which applied at any point of the joint

MN, will prevent the upper portion of the arch from sliding inwards along any joint as nm.

The equation (2), by similar reduction, gives

$$Q = \frac{W(\cos.\ \theta + f \sin.\ \theta) + cx}{\sin.\ \theta - f \cos.\ \theta} \quad . \quad . \quad (8),$$

in which the value of Q represents the force which applied at the key-stone would cause the upper portion of the arch to slide outwards along mn. Having, in a similar manner, calculated each value of Q for all the joints, its least value must be less than the horizontal thrust; otherwise the arch will be thrust outwards by sliding along mn. Or, in other words, the equilibrium requires that the minimum of equation (8), must be less than the maximum of equation (7).

From the form usually given to arches, the joint of rupture, which results from the maximum of equation (7), will be found (Fig. CC) between the key-stone and the springing line at some point as m; whilst the joint of rupture, for the minimum of equation (8) will be found at the springing line, in which case the tendency of the arch to give way will arise from the upper portions sliding inwards along nm, thrusting the lower portions outwards along the springing line.

If, however, it were found that the joint, which corresponds to the maximum of equation (7), is at the springing line, whilst that corresponding to the minimum of equation (8), lies between the key-stone and the springing line, then the tendency to rupture would arise from the lower portions sliding inwards (Fig. DD) along the joint of rupture near the springing lines, thereby forcing the upper portions upwards along the other joints of rupture.

Considering in the next case, the equations (3) and (4), there results, by reduction,

$$Q = \frac{Wx - \frac{1}{3}az^2}{y}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and,

$$Q = \frac{Wx' + \frac{1}{3}az^2}{y'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

for the values of the limits of Q, when it is applied to the point N.

The value of Q equation (9), varies also with the angle θ, of

the different joints, and its value must be calculated for every joint, and the maximum of those values be applied at N, to prevent the upper portion of the arch from falling inwards around the point n.

By a similar calculation the minimum value of equation (10) must be ascertained; and the force, which is applied at N to keep the arch in equilibrium, must be greater than the maximum of equation (9), and less than the minimum of equation (10).

These conditions suppose that the maximum of equation (9) is given by a joint of rupture between the key-stone and the springing line, whilst the minimum of equation (10) arises from a joint near the springing line, in which case, the tendency to rupture will arise from the upper portions of the arch falling inwards (Fig. EE) around the point m, thereby thrusting the lower portions outwards around the exterior edges of the joints near the springing lines.

If the force Q were applied at M, the tendency of the arch to rupture would arise from the lower portions falling inwards, (Fig. FF,) and thereby forcing the upward portions outwards.

The conditions of equilibrium in this case would be of the same form as in equations (9) and (10), the quantities x, y, x', and y', being estimated with respect to the new directions of the forces Q and W.

The equation of equilibrium with respect to the point m, will give,

$$Q = \frac{Wx - \frac{1}{3}az^2}{y}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and for the point n

$$Q = \frac{Wx' + y'\frac{1}{3}az^2}{y'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The maximum value of Q in equation (11), found, as has already been pointed out, will be the force, which applied at M, will prevent the upper portion of the arch from turning outwards around the point m. And the least value of Q equation (12), will be the force, which applied at the same point, would cause the upper portion to turn outwards around the point n. The equilibrium, therefore, requires, that the maximum of equation (11), must be less than the minimum of equation (12).

In the preceding discussions it has been supposed, that rupture will ensue either by sliding simply, or by a rotation of the parts;

but it might take place partly by sliding, and partly by rotation. To be certain that it will not take place under any circumstances, the greatest values of Q in equations (7) and (9), must be less than the least values in equations (8) and (10); also, that the values of Q in equations (7) and (11), must be less than the least values furnished by equations (8) and (12).

The series of operations, therefore, which must be gone through with, to ascertain whether a given arch is in a state of stable equilibrium will be simply the following:—to calculate, in the first place, the greatest value of Q furnished by equation (9), and to take this as the horizontal thrust of the arch;—then, in the second place, to examine whether this horizontal thrust will be less than the force necessary to cause the arch to turn outwards, around the exterior edge of any of its joints, as furnished by equation (10);—finally to ascertain whether this horizontal thrust will be insufficient to cause the arch to slide outwards along one of its joints, that is less than the least value of Q furnished by equation (8).

The joints of rupture, furnished by the equations (10) and (8), will usually be near the springing line, as it is evident, from inspection, that the lower portions of the arch will yield more readily at these points, either to sliding or rotation outwards, than along joints whose inclination to the horizon is greater. In seeking the joints which correspond to the maximum of equation (9), the results of the experiments on arches and large bridges may be resorted to, for the purpose of abridging the calculations.

There still remains to be considered the manner of calculating the quantities W, x, x', y and y', all of which are functions of the angle θ; and of deducing the maximum value of Q, in equation (9) from that of θ. The spirit of this calculation may be gathered, from the following applications to particular cases, without entering into all the details that the calculation necessarily requires. Let $AMNB$ (Fig. GG) be the half of a full centre arch, which rests on abutments $ADEF$, whose height is AD; and let it be required to find the horizontal thrust of this arch; the circumstances of its own strength to counteract this thrust; and the thickness to be given to the abutment to prevent the thrust from turning it around its exterior edge E.

From an inspection of equation (9), it appears that the value of Q,

for any point mn, will be found by multiplying the weight of the portion $mMNn$, which is represented by W, into the perpendicular distance from the point m, represented by x, to the vertical passing through the centre of gravity of $mMNn$, and dividing this product by the perpendicular from the point m, represented by y to the line of direction of the horizontal force Q, applied at n, the resistance due to cohesion, being neglected, which neglect is in favor of the stability.

Now, as the area in question, as well as the quantities x and y, vary with the position of the joint mn, or with the angle θ, it follows, that these quantities are functions of θ, and that Q, therefore, is a function of θ, and will be generally expressed by the relations

$$Q = \mathrm{F}(\theta)$$

To find then the value of θ, which corresponds to the maximum of Q, the first differential coefficient of this expression must be placed equal to zero, or

$$\frac{dQ}{d\theta} = O,$$

and the value of θ which results from it, being substituted in equation (9), will give the required maximum of the force Q.

As the operations here indicated to obtain Q, will generally be tedious, and somewhat complicated, either of the following methods may be resorted to in its place, and the same result be arrived at. Firstly, since the general value of Q equation (9), is of the form,

$$\frac{X}{Y},$$

and as,

$$\frac{X}{Y} = \frac{dX}{dY},$$

it follows that the value of θ may be substituted in either of these fractions; and the resulting value of Q will be the one sought. A simple inspection will show which of these two fractions will give the simplest expression. Secondly, as the value of θ varies with the joint mn, its values for each joint mn, $m'n'$, &c., may be substituted successively in equation (9), and the one which gives the maximum of Q be taken as the joint of rupture. In making

this calculation, by successive substitutions, the labor may be abridged, by taking advantage of the results of the experiments on the rupture of arches, substituting at once for θ the angles there given, and comparing the resulting value of Q with those obtained for the joints just above and below the one assumed.

Having, by either of these methods, calculated the horizontal thrust, it must be compared with the least values of Q furnished by equations (8) and (10), and if it be less than both of them, the arch will be in a state of stable equilibrium.

Let the joint $m'n'$, now be supposed as the one which corresponds to this maximum of Q, and let this maximum be designated by Q'. Now, since the arch itself is in a state of stable equilibrium, the only manner in which rupture can take place will be either by the force Q' causing the whole of the semi-arch, and its abutment, to turn outwards around the point E; or else to slide along the base ED. As the tendency to the last kind of motion can be easily prevented, by the methods of construction, the first kind need alone be considered. The conditions of strict equilibrium, in this case, require, that the moment of the force Q', with respect to the point E, shall be equal to the moment of the weight of the semi-arch, and its abutment, with respect to the same joint.

Designating then by W', the weight of the semi-arch $ABNM$,—by $d'=Ed$ the length of the perpendicular, from E, to the vertical passing through its centre of gravity O',—by W'', the weight of the abutment $DEFA$,—by $d''=Ec$ the perpendicular from E to the vertical through its centre of gravity,—finally by $h=Ef$ the perpendicular from E to the direction of Q', the equation of equilibrium will be,

$$W''d'' + W'd' = Q'h.$$

To find from this expression the thickness of the abutment, when its altitude is given, the following method is taken: Designate the the altitude AD by a,—the thickness ED by x,—and the weight of the unit of volume by w,—then the area $AFED=ax$ and the weight W'', corresponding to the unit of length of the abutment, will be $W''=wax$. The foregoing expression will therefore become, by substituting these values for d'', d', and W'',

$$\tfrac{1}{2}wax^2 + W'(x + Dd) = Q'h,$$

from which x can be readily obtained.

Having thus completed the solution of the problem of equilibrium for cylindrical arches in general, a few words remain to be said respecting the calculations which are required in finding the different values of Q equation (9). These calculations simply consist in finding the values for different areas, and the distances of their centres of gravity from a given line. As the arch is bounded by curved and right lines, these areas, as well as the distances of their centres of gravity from any vertical line, can be found by the usual problems for finding these different values. If the area $ABMN$, (Fig. HH,) for example, is considered, its area will be found by an expression of the form

$$\int_o^X ydx\,;$$

and the distance of its centre of gravity from the vertical AL, will be found from an expression of the form

$$\frac{\int_o^X xydx}{\int_o^X ydx}\,;$$

expressions in which $x = Ap$, $y = mn$, and the integral is taken between the limits $x = o$, and $X = AP$. But as this method of calculating these values may, in many cases, be very long, it will, generally, be better to use some of the methods of approximation which will give results less accurate, it is true, but also requiring less labor. The following methods, given by Legendre for computing quadratures by approximation, may be used in the cases referred to. Let $ABMN$ be a portion of a circular ring, or of any other figure bounded by the two curves AM and BN, of which the equations are known, and let AL and AP be two rectangular axes to which these curves are referred. Divide the abscissa $AP = X$ into any number of equal parts $Ap = pp' = p'p''$, &c., and designate each of these equal parts by a;—designate by c, c', c'', &c., the ordinates of the curve AM which correspond to the abscissas $\frac{1}{2}a$, $\frac{3}{2}a$, &c. Then if the distances a are taken very small, the portions of the curve between any two consecutive ordinates c, and c',—c', and c'', &c., may be regarded as belonging to an arc of a parabola, and the small area, included between them as a portion of the parabolic

area. Each of the partial areas Apm, $mpp'm'$, &c., will be equal respectively to

$$\frac{a}{6}(o+4c+c'),\ \frac{a}{6}(c'+4c''+c'''),\ \frac{a}{6}(c'''+4c^{\text{iv}}+c^{\text{v}})\ \&c.,$$

and by adding these partial areas, their sum, or the whole area $APM=\int_o^X dxF(x) = A$, will be

$$= A = \frac{a}{6}(4c+2c'+4c''+2c'''+\ldots\ldots c^{2n}),$$

in which n is the whole number of parts into which the length $AP = ma$ is divided. This formula admits of another form which would give a nearer approximation, but it will give a sufficiently accurate result in its present form, if the length a is taken very small.

Another method is as follows: Divide the abscissa, as before, into any number of small equal parts each equal to $\frac{1}{2}a$, and construct the ordinates corresponding to $\frac{1}{2}a$, $\frac{3}{2}a$, $\frac{5}{2}a$, &c., which designate by c, c', c'', &c.; then the area will be expressed by the formula

$$A = a(c+c'+c''+c'''\ldots\ldots c^n) - \frac{a^2}{24}\left(\frac{dF(X)}{dx} - \frac{dF(o)}{dx}\right),$$

in which $\frac{dF(X)}{dx}$, and $\frac{dF(o)}{dx}$, are the values which these two quantities take, when $x = X = AP$, and $x = o$. This formula gives a nearer approximation than the former, and requires a calculation of only half as many terms.

The area ABN may be divided in a similar manner, and the area $ABNM$, which is the difference between the two, be as easily found.

Having determined the limits of the horizontal thrust, there remains to be considered the means which must be resorted to in order to render the structure stable, when this force is so great as to threaten rupture in the given arch. These means consist either in increasing the weight of the parts which are opposed to the horizontal thrust, in making them thicker, or in adopting such a method of construction as will answer the end in view. When it is found that there is danger from a motion of rotation around either of the points B or E, it may be counteracted, for the first, by increasing the thickness of the arch; and, for the last, by increasing

the thickness of the abutment; or, in either case, by increasing the weight of the parts which tend to counteract the horizontal thrust. If there is danger from rupture by sliding along AB, it may be counteracted by inclining the joint inwards, or by making a very firm bond between the top course of the abutment and the bottom course of the arch by iron cramps, or by letting the stones into each other. A tendency to slide on the base of the abutment must be counteracted in a similar manner.

In making the preceding calculations, the value of f should be taken as that which is found for the tangent of the angle of friction between cut-stone laid dry. For stone smoothly chiselled, f may be taken at 0,58; and for dressed hammered stone at 0,78. The letters c and a, which represent, respectively, the adhesion between the stone and mortar, and the cohesion of the mortar itself have different values, according to the quality of the mortar and the state in which it is when the centre of the arch is struck. From some experiments made to ascertain the value of c, it appears that it may be estimated at about 2 lbs. on the square inch for green mortar, and about 10 lbs. on the square inch for the best mortar when fully set. The value of a may be found from the experiments of Treussart detailed in the Subject of Mortar.

If the cohesion is neglected in making the calculation for the thickness of the abutment, then the rupture should be considered as taking place somewhere along a line drawn in the direction AE, in which case the portion of the abutment below this line should be disregarded, as it will have no effect towards counteracting the tendency of Q.

When the foundations of the abutment are laid on a compressible soil, the spread ab and the depth Db of the foundation must be so determined, that the resultant of the horizontal thrust and the weight of the semi-arch, with its abutment and foundation, shall pass through the middle point of ab. The reasons for this will be given in NOTE VI.

In the discussion of the circumstances of rupture occasioned by rotation, the parts were supposed to give way by turning around an interior or exterior edge of the joint of rupture; but this supposition is evidently incorrect, except where the voussoirs are perfectly hard and inelastic bodies. But in admitting the reverse of this

supposition, the question presents itself as to what law of variation the pressure at different points of the joint follows. It seems natural to admit that so soon as any joint commences to open, the pressure is nothing at the edge where the opening begins, and is greatest at the opposite edge,—that, owing to the elastic nature of the material, the pressures increase uniformly from one edge to the other,—and that the compressions, therefore, which are due to these pressures on the different elements of the joints, offer resistances which are proportional to the pressures.

If the joint mn of rupture be considered, its length being designated by z,—an element of this length, at the distance r from the point m, where the pressure is greatest, by dv,—and the greatest pressure on the unit of length at m, by p'; then the pressure on the element dv, according to the preceding suppositions, will be represented by

$$\frac{v}{z}\,pdv;$$

but the expression for the total pressure on mn, equation (R), as found in the preceding part of this Note, is

$$W \sin.\ \theta + Q \cos.\ \theta,$$

and as this must be equal to the sum of the elementary pressures, there results

$$\frac{p}{z}\int_0^z vdv = W \sin.\ \theta + Q \cos.\ \theta,$$

from which, by taking the integral indicated,

$$\tfrac{1}{2}pz = W \sin.\ \theta + Q \cos.\ \theta,$$

or, for the maximum pressure on the unity of length,

$$p = \frac{2(W \sin.\ \theta + Q \cos.\ \theta)}{z} \quad . \quad . \quad . \quad (S).$$

If, instead of adopting the hypotheses here admitted, the pressures were supposed uniformly distributed over the joint m, that on a unit of length, which may be designated by p', would be

$$p' = \frac{W \sin.\ \theta + Q \cos.\ \theta}{z},$$

which shows, that in the suppositions adopted $p = 2p'$.

Adopting the same hypotheses and notation for the joint at the key-stone, and designating by s the versed-sine of the arc mM, the

pressure on any element dv, at the distance v from M, will be represented by

$$\frac{v}{z}pdv,$$

and since the sum of the moments of these elementary pressures taken with respect to the point m, must be equal to the moment of the weight of the portion $mMNn$, in the case of equilibrium, there will result to express the conditions of this equilibrium,

$$\frac{p}{z}\int_0^z vdv.\,(s+v) = Wx\ ;$$

or, taking the integral indicated,

$$p(\tfrac{1}{2}zs+\tfrac{1}{3}z^2) = Wx,$$

from which

$$p=\frac{6Wx}{3zs+2z^2},$$

Having thus obtained the maximum pressure for the unit of length on the joints MN and mn; and knowing the strength of the stone as given in the Subject on the Strength of Materials, it can be readily seen, whether the strength of the stone, at the above mentioned joints, will be sufficient to resist this pressure.

In adopting the value of p, as found by the preceding suppositions, it is evident that any error committed will be on the safe side; for as the arch will have an excess of stability, when its equilibrium is stable, the joints, so far from opening, as has been supposed, will remain very nearly, if not entirely, in contact throughout their length, so as to distribute the pressure over the whole of their surfaces, making it, however, rather less at the edge, where the opening would begin, than at the other.

Equilibrium of Groined and Cloistered Arches. The preceding discussions on cylindrical arches proper, may be readily applied to the circumstances of equilibrium of groined or cloistered arches, as they are composed of cylindrical arches.

Let $ABCD$ (Fig. II.) be the space covered by a cloistered arch, composed of two cylindrical arches, of which the portions BOC, and AOD, belonging to the same arch, are sustained by the abutments and $aADd$, $bBCc$, and the other two portions AOB, and DOC, belonging to the same cylindrical arch, are sustained by the abutments $cCDd$, and $aABb$. To establish the conditions of equili-

brium relative to the abutments, and to find their thickness, the two portions *BOC*, and *AOD*, with their corresponding abutments, should be considered independently of the other portions, and their abutments as if there were no connection between them, along the lines *ac*, and *bd*. By this means the equation expressing the conditions of equilibrium, and from which the thickness of the abutments is found, will give an excess of stability, arising both from the adhesion of the mortar along the lines *ac* and *bd*, and from arches of this kind being generally so arranged, that the stones along these lines shall form a portion of each of the cylindrical arches of which these lines are the intersections.

In the groined arch, the portions of the arch *aAOBb*, and *dDOCc*, (Fig. KK) belong to the same cylindrical arch; and the same is the case with the portions *b'BOCc'*, and *a'AODd'*; so that the points of support of the whole arch are the four pillars *aAa'a''*, *bBb'b''*, &c., which perform the same functions as the abutments in the cloistered arch. The equations of equilibrium are found in the same manner, as in the preceding case, except that the portions *aAOBb*, and *dDOCd*, for example, which tend to turn the pillars over, around their edges, *a'a''*, *b'b''*, &c., are opposed, not only by the moments of the weights of the four pillars, taken with respect to those edges, but also by the moments of the weights of the portions *b'BOCc'*, and *a'AODd'*, taken with reference to the same points; so that although the upper portions of the arches, in this case, are heavier, and their centres of gravity fall farther from the exterior edges of the points of support, than in the cloistered arch, they still present more stability, and require a smaller mass to sustain their thrust, owing to the weight and the thrust of the opposite parts counteracting each other, whilst in the cloistered arch they act entirely independently.

In the calculations for cylindrical arches, the operations are reduced to finding the expressions for the areas, and their centres of gravity, since the portion of the arch considered, is supposed to correspond to a unit of length. But in the arches now in question, calculations of solids and the position of their centres of gravity, must be made, since the portions of the arches considered are not of uniform length at top and bottom. These calculations present no difficulties, as they amount only to simple cubatures, which

may be rigorously effected by well known geometrical methods, or by approximations of a similar nature to those already laid down for quadratures.

Uses of Iron Ties for strengthening arches. Cases may arise, where cylindrical or other arches may be requisite, in which, from some causes, it may not be practicable to give sufficient thickness, or weight, to the abutments, to insure a stable equilibrium. A remedy, in such cases, may be found in the use of iron rods, or *ties*, to connect more firmly those parts which are most liable to yield, from the effects of the horizontal thrust.

In arranging a tie, for the purpose in view, three points must be considered, 1st. The best position for the tie—2d. The tension brought upon it from the thrust in this position,—3d. The additional tension, arising from the contraction of the metal. The cross section of the bar must be of sufficient dimensions to resist these different forces.

Let *ABNM*, (Fig. LL) be a semi-arch and its abutment, and let *mn* be the joint of rupture.

Now, when the arch yields, by the action of the portion *nMNn* falling inwards, the point *m*, will be thrust outwards, and will describe a certain path in a horizontal direction, at the commencement of the motion, which will be greater than that of any other point; but as this motion might be counteracted by a horizontal force, applied in a contrary direction to the force that produces the motion, and, as, by the principle of virtual velocities, the moment of this force will be equal to its product by the horizontal path described in the small derangement of the parts, which takes place at the commencement of the motion, it follows, that as the moment is the same, the force will be the smaller as the path described is the greater; therefore, the point *m* will require the least force, and a tie placed at, or just beneath it, will be best placed, since the tension on it from the horizontal thrust will be the least also.

The tension due to the horizontal thrust is determined by the consideration, that its moment, taken with reference to the point *B*, must be equal to the difference of the moments of the horizontal thrust, and the weight of the semi-arch and its abutment *ABNM*, taken with respect to the same point.

To determine, in the third place, the total tension, arising from

a change of temperature, let t designate the tension, just found, at the highest presumed degree of temperature,—a the area of the cross section of the tie,—l its length,—l' the absolute variation of this length for a given decrease of temperature,—R' the greatest tension on a unity of surface to which forged iron can safely be submitted,—and E the co-efficient of elasticity for forged iron. Then, as the tie has become shortened by the quantity l', owing to the decrease of temperature, it will be in the same state as if it were submitted to a strain which would elongate it by the same quantity, the measure of which strain is expressed by,

$$Ea\frac{l'}{l},$$

therefore, the total tension on the tie has become, owing to the decrease of temperature,

$$t + Ea\frac{l'}{l}.$$

The area of the cross section then should be so determined that $R'a$, which is the greatest tension to which the tie can be submitted, shall be equal to that just found, or

$$R'a = t + Ea\frac{l'}{l},$$

from which,

$$a = \frac{t}{R' - E\frac{l'}{l}}.$$

As the value of a becomes infinite, when the denominator of this fraction becomes equal to zero, or when $R'l = El'$, it follows that it is impossible to fulfil the required condition of not exceeding a certain tension R', if the decrease of temperature is such as to satisfy this equation, or if

$$\frac{l'}{l} = \frac{R'}{E}.$$

NOTE VI.

Theory of the Equilibrium of Sustaining Walls for Earth and Water.

The method to be pursued in treating this subject was also pointed out first by Coulomb. The question presents two distinct problems, the first being to ascertain the value of the pressure exerted against a plane surface, by either earth or water; and the second, being to ascertain the dimensions of a wall of a given form which will be capable of sustaining the pressure.

Observation having shown when an embankment of earth *NMBA* (Fig. MM) is sustained on any one of its faces *AB*, by a wall, or any other obstacle, as a rigid plane, for example, that if this plane be removed, a portion of the embankment will come away with it, separating from the main body along a line *AD* drawn through *A*, which is sensibly a right line, and that when the cohesion of the particles of earth was entirely destroyed, the part of the embankment sustained by *AB* would finally take a slope *AC*, which is termed the *natural slope*. Coulomb observed that since the line of separation *AD* might occupy any position between *AC* and *AB*, and that to each of these portions, there would correspond a portion of the embankment *DAB*, whose pressure against *AB* would vary with the position of *AD*, or the size of the portion which came away with the plane, that the effective pressure on *AB*, should be measured by that position of *AD* which would make it greatest; or, in other words, that the resistance offered by *AB* should be equal to the greatest pressure against it, arising from any of the portions *ADB*, formed by giving different inclinations to the line *AD*; for if the resistance offered by *AB* were less than the pressure thus found, it would yield to it; and if it were equal or greater, there would be no yielding, since the pressure is the greatest possible. The portion thus defined is denominated the *prism of maximum pressure*.

Profiting by these remarks of Coulomb, succeeding engineers have given a complete solution of the problem for finding the prism of maximum pressure by assuming data which agree very sensibly with the results of observation. These data are: 1st, that

the line of separation AD is sensibly straight;—2d, that the density and cohesion of the earth are uniform throughout the mass;—3d, that the friction of the particles along AD is proportional to the perpendicular pressure on the same sine;—4th, that the adhesion and friction of the earth along the plane AB may be neglected, as by so doing it will favor the stability.

These data being admitted, let AB (Fig. NN) be the position of the plane, AC the line of the natural slope, AD the line of separation of any prism DAB, and AF a vertical line, which is equal to the height of the embankment. Designate by φ the angle CAF, which is the complement of the angle of the natural slope,—by θ the angle DAB of the prism,—and by ε the angle BAF of the plane with the vertical.

The weight of the prism which will be designated by W, may be represented by the right line ab, taken on the vertical passing through its centre of gravity; and the resistance offered by the plane AB, or the pressure of W on it, which will be designated by P, may be represented by a line df perpendicular to AB. Now as the tendency of W is to cause the prism to slide along the line of disjunction AD, whilst that of P is to prevent this motion, it will be necessary to find the components of W and P which favor this tendency, and those which oppose it, and in the case of equilibrium their difference must be equal to zero.

The components of $W = ab$, which are perpendicular and parallel, respectively to AD, are represented by

$$bc = W \sin. (\theta - \varepsilon), \text{ and } ac = W \cos. (\theta - \varepsilon).$$

The like components of $P = df$, will be expressed by

$$ef = P \cos. \theta, \text{ and } ed = P \sin. \theta.$$

But since the component of W parallel to AD, is the only one of these forces whose action would tend to make W slide in the direction DA, and as it is opposed to the component of P parallel to DA, which acts from A towards D, as well as to the friction along AD, arising from the pressure caused by the components of W and P perpendicular to AD, and also to the cohesion along AD; the equation of equilibrium will be expressed by

$$W \cos. (\theta - \varepsilon) = P \sin. \theta + fP \cos. \theta + fW \sin. (\theta - \varepsilon) + C \quad (1);$$

in which f is the ratio of friction to the pressure, and C the cohesion along AD. From this equation there obtains

$$P=\frac{W\{\cos.(\theta-\varepsilon)-f\sin.(\theta-\varepsilon)\}-C}{\sin.\theta+f\cos.\theta}\ .\ (2);$$

which expresses the value of the pressure on AB, caused by any prism DAB.

Designating the altitude AF by h,—the base of the triangle DAB will be expressed by

$$BD=\frac{h\sin\theta}{\cos.\varepsilon\cos.(\theta-\varepsilon)},$$

and the side AD by

$$AD=\frac{h}{\cos.(\theta-\varepsilon)}.$$

If w be taken to represent the weight of the unit of volume of the earth; the weight of the prism will be expressed by the area $DAB+w=W$; or, finding the value of the area in terms of the base and altitude by,

$$W=\frac{wh^2\sin.\theta}{2\cos.\varepsilon\cos.(\theta-\varepsilon)}.$$

As the quantity f represents the ratio of the friction to the pressure, and as the value of this quantity is expressed by the tangent of the angle made by the line AD when the prism would remain at rest owing to the friction alone; and as this angle moreover is the same as that of the natural slope, since the particles remain at rest on this slope, it follows, that

$$f=\cot.\varphi=\frac{\cos.\varphi}{\sin.\varphi}.$$

To determine C it will be necessary to know the value of the cohesion for the unit of surface. Let this value be supposed found, and designate it by c, then the total value of the cohesion on AD will be represented by

$$C=c\frac{h}{\cos.(\theta-\varepsilon)}.$$

By substituting for W, f, and C, their respective values in equation (2), there results

$$P=\frac{\frac{wh^2}{2\cos.\varepsilon}\sin.\theta\left\{\cos.(\theta-\varepsilon)-\frac{\cos.\varphi}{\sin.\varphi}\sin.(\theta-\varepsilon)\right\}-\frac{ch}{\cos.(\theta-\varepsilon)}}{\sin.\theta+\frac{\cos.\varphi}{\sin.\varphi}\cos.\theta},$$

which, by reduction, becomes

$$P=\frac{\frac{wh^2}{2\cos.\varepsilon}\sin.\theta\{\sin.\varphi\cos.(\theta-\varepsilon)-\cos.\varphi\sin.(\theta-\varepsilon)\}-ch\sin.\varphi}{(\sin.\varphi\sin.\theta+\cos.\varphi\cos.\theta)\cos.(\theta-\varepsilon)};$$

but, from the expression for the sine of the difference, and the cosines of the difference of two angles, this last expression reduces to

$$P=\frac{\frac{wh^2}{\cos\varepsilon.}\sin.\theta\sin.(\varphi+\varepsilon-\theta)-2ch\sin\varphi}{2\cos.(\varphi-\theta)\cos.(\theta-\varepsilon)};$$

and this finally becomes, by substituting for the products of the sines and cosines, their values in terms of the cosines of the sum and difference of the angles,

$$P=\frac{\frac{wh^2}{2\cos.\varepsilon}\cos.(\varphi+\varepsilon-2\theta)-\left\{\frac{wh^2}{2\cos.\varepsilon}\cos.(\varphi+\varepsilon)+2ch\sin.\varphi\right\}}{\cos.(\varphi+\varepsilon-2\theta)+\cos.(\varphi-\varepsilon)} \quad (3);$$

which represents the value of the pressure of any prism, whose angle DAB is θ, against the plane AB.

But to obtain the effective pressure against the plane AB, this value must be taken the greatest possible, and as it depends on θ alone, which is the only variable quantity in the expression, the condition to be satisfied, in order to obtain the maximum of P, is $\frac{dP}{d\theta}=0$. If then the equation (3) is differentiated, and its first differential coefficient placed equal to zero, the value of θ determined from the resulting equation, will be the one which corresponds to the maximum of P.

When this differentiation is effected, it will be found, that the numerator of the resulting equation, which is equal to $\frac{dP}{d\theta}$, contains a factor $\sin.(\varphi+\varepsilon-2\theta)$ common to all the terms, and as this factor must become equal to zero, when $\frac{dP}{d\theta}=0$, it follows, that the condition of P being a maximum, gives

$$\sin.(\varphi+\varepsilon-2\theta)=0,$$

and as the angle becomes zero when its sine is zero, there results,

$$\varphi+\varepsilon-2\theta=0, \text{ or } \theta=\tfrac{1}{2}(\varphi+\varepsilon);$$

which shows, that the angle of the prism of maximum pressure is one half of that included between the line AB of the plane, and

that AC of the natural slope; or, in other words, that this angle is bisected by the line of separation AD.

Having thus obtained the value of θ, corresponding to the prism of maximum pressure, the value of that pressure will be found, by substituting the value of θ in equation (3). By making this substitution, and recalling, since $\varphi+\varepsilon-2\theta=0$, that cos. $(\varphi+\varepsilon-2\theta)=1$, there results,

$$P=\frac{\frac{wh^2}{2\cos.\varepsilon}\{1-\cos.(\varphi+\varepsilon)\}-2ch\sin.\varphi}{1+\cos.(\varphi-\varepsilon)};$$

but since

$$1-\cos.(\varphi+\varepsilon)=2\sin.^2\tfrac{1}{2}(\varphi+\varepsilon),$$
$$1+\cos.(\varphi-\varepsilon)=2\cos.^2\tfrac{1}{2}(\varphi-\varepsilon),$$

this expression becomes,

$$P=\frac{\frac{wh^2}{\cos.\varepsilon}\sin.^2\tfrac{1}{2}(\varphi+\varepsilon)-2ch\sin.\varphi}{2\cos.^2\tfrac{1}{2}(\varphi-\varepsilon)}\quad\ldots\ldots\ldots\quad(4)$$

If the value of P, equation (4), be placed equal to zero, and the value of h, corresponding to $P=0$, be designated by h', the equation, when reduced, will give

$$h'=\frac{2c\sin.\varphi\cos.\varepsilon}{w\sin.^2\tfrac{1}{2}(\varphi+\varepsilon)};\quad\ldots\ldots\ldots\quad(a)$$

from this expression the value of $2c$ sin. φ can be obtained; by substituting which in equation (4), and placing,

$$\frac{\sin.^2\tfrac{1}{2}(\varphi+\varepsilon)}{\cos.^2\varepsilon\cos.^2\tfrac{1}{2}(\varphi-\varepsilon)}=t^2\quad\ldots\ldots\ldots\quad(b)$$

in order to abridge the expression, there will, finally, result

$$P=\tfrac{1}{2}wh(h-h')t^2\cos.\varepsilon.\quad\ldots\ldots\ldots\quad(A)$$

Before interpreting this value of P in words, it will be necessary to ascertain what the quantity t, which enters into it, represents. For this, the expression for the base BD of the prism must be resumed. This expression becomes, when the value of $\theta=\frac{1}{2}(\varphi+\varepsilon)$ is substituted in it,

$$BD=\frac{h\sin.\tfrac{1}{2}(\varphi+\varepsilon)}{\cos.\varepsilon\cos.\tfrac{1}{2}(\varphi-\varepsilon)},$$

which being divided by h gives

$$\frac{BD}{h}=\frac{\sin.\tfrac{1}{2}(\varphi+\varepsilon)}{\cos.\varepsilon\cos.\tfrac{1}{2}(\varphi-\varepsilon)}=t,$$

from which it appears that the quantity t^2 equation (a), is the square of the ratio between the base and altitude of the triangle DAB. Having thus obtained the value of t, the expression (A) can be easily interpreted in words; since w is the weight of the unit of volume of the earth, h the total altitude of the prism, h' the altitude of a prism of earth when the pressure is zero, t the ratio of the base and altitude of the triangle DAB, and s the angle between AB and a vertical line.

In the equation (A) if s is made equal to zero, the cos. s becomes equal to unity; and when the slope AB is assumed in the direction AB', or the plane is supposed to slope towards the embankment, the angle s must be regarded as negative.

Having, in this manner, obtained the value of the maximum pressure, the next step in the investigation will be to ascertain its effects on the plane AB; and for this purpose, the point of application of the force P must be found. This point corresponds to the one, which is termed the *centre of pressure* in fluids.

Resuming equation (A) it will take the form of

$$P = \frac{w}{2} z\,(z - h')\, t^2 \cos.\ s,$$

for any distance $z = h$, below the horizontal BC. By differentiating this expression, there results,

$$dP = wt^2 \cos.\ s\ (z - \tfrac{1}{2}h')\, dz\,;$$

an expression which represents the pressure dP on an element of AB corresponding to dz.

Let O be the point of application of this elementary pressure, then its distance from A will be expressed by,

$$AO = \frac{h - z}{\cos.\ s};$$

the moment of dP therefore, taken with respect to the point A, which moment, equal to $dP \times AO$, will be expressed by

$$wt^2\,(h - z)\,(z - \tfrac{1}{2}h')\, dz, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

and the integral of this expression, or the sum of the moments of the elementary pressures dP will give the moment of the total pressure P with respect to the point A. This integral being taken between the limits of $z = 0$ and $z = h$, there results for the moment,

$$\tfrac{1}{2}\, wh^2\, (\tfrac{1}{3}h - \tfrac{1}{2}h')t^2\,;$$

and by dividing this moment of the total pressure by the pressure P, equation (A), there will be obtained the value of the distance of the point of application of P from A, which expression becomes

$$\frac{h(\frac{1}{2}h - \frac{1}{3}h')}{(h-h')\cos. s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B).$$

There is one point in this investigation which requires elucidation, and that is respecting the limits of z, between which the integral of the equation (c) should be taken. Since the pressure is zero for a height $h = h'$, it would, at first, seem that this integral ought to be taken between the limits of $z = h'$ and $z = h$; but, if it be considered that, although the pressure is nothing on the plane AB, when the prism has the altitude h', it does not follow that there will be no pressure from this prism when it forms a part of any other prism whose altitude is h, but, on the contrary, that the pressure due to this prism is in part owing to the prism, it will readily appear that the limits taken for z are the correct ones.

To apply the equations (A) and (B) to the case of water, it will be simply necessary to make $h' = 0$, since the cohesion is zero in this case and $\varphi = 90°$; then there will obtain for the value of t the expression

$$t = \tan. (45° \mp \tfrac{1}{2}\varepsilon) \pm \tan. s \quad . \quad . \quad . \quad . \quad (c);$$

in which s must be made positive, zero, or negative, according as the plane AB slopes from the water, is vertical, or slopes towards it. On the supposition of $s = 0$, those equations become

$$P = \frac{wh^2}{2}, \text{ and } AO = \tfrac{1}{3}h.$$

The values of the greatest pressure and the distance of its point of application from the foot A of the plane being thus determined, the second part of the question can now be entered upon, which is, to determine the dimensions of a wall of a given form which shall offer the same resistance to the pressure as the immovable plane AB.

Let $ABEF$, (Fig. OO,) be the cross section of a wall, the interior slope or batter of which is given, and equal to the angle ε, the exterior batter being represented by m, which is the ratio of the base Ee to the altitude Fe. Let O be the point of application of the pressure P as already determined. Designate by $x = AF$ the base of the wall,—by $h = Fe$ its altitude,—by w' the weight of the unit of its volume,—and by a the value of the cohesion of the ma-

sonry of the wall on the unit of surface, as given in NOTE V. The results of experience go to prove, that a wall sustaining a pressure of earth or water, will yield in one of the following ways, either the entire wall will be forced from its position by sliding along its base AF; or a disjunction will take place along some line as SF, by the upper portion $BEFS$ sliding along SF; or the whole wall will be thrown over by turning around the point F; or, finally, a disjunction will take place along SF by the upper portion turning around the point F. But, as the tendency to sliding can always be prevented by a suitable arrangement of the masonry, it will be only necessary to consider the cases where the entire wall, or only a portion it, yields by a motion of rotation around the point F.

Let the case of the rotation of the entire wall be first considered. By drawing through the point O the line OG perpendicular to AB, it will be the direction of the pressure P, and its moment with respect to the point F will be $P \times FG$; in which FG is the perpendicular from F upon the line of direction of P. Drawing from F a parallel to OG, the line $fO = AO - Af$ will be found equal to FG; but AO from equation (B), is expressed by

$$AO = \frac{h(\frac{1}{3}h - \frac{1}{2}h')}{(h - h') \cos. \varepsilon};$$

and from the triangle AFf, which is equi-angular with ABb, there results

$$Af = x \sin. \varepsilon;$$

therefore,

$$fO = FG = \frac{h(\frac{1}{3}h - \frac{1}{2}h')}{(h - h') \cos. \varepsilon} - x \sin. \varepsilon,$$

and the expression for $P \times FG$, becomes

$$\tfrac{1}{2}wt^2 h(h - h') \cos. \varepsilon \left\{ \frac{h(\frac{1}{3}h - \frac{1}{2}h')}{(h - h') \cos. \varepsilon} - x \sin. \varepsilon \right\} \quad . \text{(C)}.$$

But in order that an equilibrium between the pressure, and the weight of the wall may take place, the moment of the weight of the wall must be equal to that of the pressure. To determine this last moment, the simplest plan will be to subtract from the moment of the rectangle $AbeF$, the respective moments of the two triangles AbB and FeE. The moment of the rectangle will be expressed by its area multiplied into the weight w' of its unit of volume, and this product by the perpendicular from the point F to the vertical through its centre of gravity, or by

$$w'hx. \tfrac{1}{2}x = \tfrac{1}{2}w'hx^2.$$

The respective moments of the triangles will be found in a similar way, that of AbB will be

$$\tfrac{1}{2}w'h^2 \tan. \varepsilon \,(x - \tfrac{1}{3}h \tan. \varepsilon);$$

and that of FeE will be

$$\tfrac{1}{2}w'mh^2 = \tfrac{1}{3}mh = \tfrac{1}{6}w^2m^2h^3.$$

By adding, therefore, these two last expressions, and placing the difference between this sum and the moment of the rectangle, equal *to* the expression (C), there will result an equation which will satisfy the conditions of equilibrium required. By solving this equation with respect to x, there results

$$x = \pm\tfrac{1}{2}\left\{h \tan. \varepsilon - \frac{w}{w'}t^2 (h - h') \sin. \varepsilon \cos. \varepsilon\right\} + \sqrt{\tfrac{1}{4}\left\{h \tan. \varepsilon - \frac{w}{w'}t^2 (h - h') \sin. \varepsilon \cos. \varepsilon\right\}^2 + \frac{w}{w'}t^2 h(\tfrac{1}{3}h - \tfrac{1}{2}h') - \tfrac{1}{3}h^2(\tan.^2 \varepsilon - m^2)};$$

for the breadth AF of the base of the wall, the thickness of which at top will of course be determined from the exterior and interior slopes.

This expression will apply to a wall of any form, by making suitable substitutions in the value for x. For example, when the back AB is vertical, ε becomes zero, in which case those terms in which sin. ε and tan. ε enter, also reduce to zero. In the same way, if the face EF is vertical, it will be necessary to make $m = 0$. If the cohesion is to be disregarded then $h' = 0$. Finally, in the case of water it will be necessary to substitute the value of t found in the expression c, and to make $h' = 0$.

When the wall yields to the pressure by disjunction along any line SF, and a rotation around the point F, the value of the moment of the pressure along the line BS will be expressed by substituting z for h in the expression (C); z designating the altitude corresponding to BS. In order that an equilibrium shall take place this moment of the pressure along BS must be equal to the moment of the entire wall, diminished by the moment of the portion AFS, to which difference must be added the moment of the cohesion of the masonry along SF taken with reference to the point F. From what has just been shown, the expression for the moment of the

entire wall may be readily found: that of the triangle ASF will be equal to its area multiplied into w' and this product multiplied by the perpendicular from F on the vertical through its centre of gravity. But as it will be more simple to divide the triangle ASF into two right angled triangles, by the perpendicular RS, and to take the sum of their moments instead of that of ASF, there will result

$$\tfrac{1}{3}w'(h-z)\{x-(h-z)\tan. s\}^2$$

for the moment of the weight of FSR, and

$$\tfrac{1}{2}w'(h-z)^2\tan. s\{x-\tfrac{2}{3}(h-z)\tan. s\},$$

for that of the weight of ASR.

The moment of the cohesion along SF, adopting the same hypothesis as in NOTE V, will be represented by $\tfrac{1}{2}a\overline{SF}^2$, a representing the cohesion of the unit of surface, which becomes by determining the value of $\overline{SF}^2$ from the triangle FSR,

$$\tfrac{1}{2}a\,\{(h-z)^2+[x-(h-z)\tan. s]^2\}.$$

Having established the equation of equilibrium, by placing the moment of pressure along BS equal to the algebraic sum of the moments just found, the value of x can be determined as in the case where the entire wall yields by turning around the point F. The value of x being given in terms of z, it will be easy to determine z so that it shall satisfy the condition of rendering x a maximum; to do which it will be simply necessary to differentiate the equation of equilibrium with reference to x and z, and to make $\frac{dx}{dz}=0$, which is the condition of the maximum of x; the resulting equation will give the value of z sought. This value of z, which corresponds to the maximum of x, being substituted in the general equation which gives the value of x, there will result from it the maximum value of x, or the least thickness that the wall can receive at its base AF to prevent rupture from taking place along the line SF corresponding to this value of z.

The calculations which are here only indicated offer no difficulty. If the cohesion of the earth is neglected, it will be necessary to make $h'=0$; if that of the masonry is also neglected, a must be placed equal to zero; finally the signs of s will vary, as s is reckoned positive, nothing, or negative. In the foregoing expressions s is assumed as positive.

In the cases thus far considered the altitudes of the wall and embankment are supposed the same. When that of the latter is greatest, it will be necessary to make suitable modifications in the expressions (A) and (B) to meet the case; which may be done as follows. The line BV, the base of the prism BAV of maximum pressure, will be considered as submitted to the pressure of a medium of uniform density of such a nature that when the earth yields, by sliding along any line as AV or ST, this medium will separate along a vertical line through V or T, and the part resting on BV, or BT as the case may be, will come away with it. This hypothesis supposes that the pressure, arising from this medium of uniform density, is equally diffused over BT, or BV; as, for example, if a mass of bricks, whose cross section is the rectangle $BVmp$, were piled up on the surface BV, in such a way, that a disjunction could take place in the mass along any vertical line Tk.

To modify then the expressions (A) and (B) to suit this hypothesis, let those expressions be taken when $\varepsilon=0$, as this will not change the circumstances of the case, and will make the expressions more simple, then

$$P = \tfrac{1}{2}wh^2t^2 - \tfrac{1}{2}whh't^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A')$$

for equation (A), and for the expression (B)

$$\frac{h\,(\frac{1}{3}h - \frac{1}{2}h')}{h - h'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B')$$

But as the value of h' in equation (a), when $\varepsilon=0$ becomes,

$$h' = \frac{4c}{wt}$$

equation (A′) will become by substituting in it this value of h',

$$P = \tfrac{1}{2}wh^2t^2 - 2cht, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A'')$$

Now if w'' is taken to represent the weight of the unity of volume of the medium which is uniformly diffused over BV, it is evident, from the hypothesis assumed, that to obtain the value of P in equation (A″), for in the case here considered it will only be necessary to substitute, instead of $\frac{1}{2}wh^2$, the quantity $w''h+\frac{1}{2}wh^2$; for the pressure on a unit of surface of the wall, arising from the weight of the prism of maximum pressure, varies as $\frac{1}{2}wh^2$, whilst that which is due to the weight uniformly diffused over the base of this prism, or over BV, will evidently vary with $w''h$; making, therefore, this substitution, there results

$$P = (w''h + \tfrac{1}{2}wh^2)\,t^2 - 2cht\,;$$

but when $P=0$, this expression becomes,

$$0 = (w''h + \tfrac{1}{2}wh^2)\,t^2 - 2cht,$$

and representing by h'', what h becomes in this supposition, there results

$$h'' = \frac{4c}{wt} - \frac{2w''}{w} = h' - \frac{2w''}{w},$$

since $h' = \frac{4c}{wt}$ as has just been shown when the angle ε is zero.

A very simple calculation will now serve to show, that the expressions (A) and (B) can be converted into others to suit the hypothesis here adopted, by simply changing the letter h', in those expressions, into $h' - \frac{2w''}{w}$.

Nothing more remains to be said on the value of the pressures, or the dimensions and form of walls to resist them in the supposition of a strict equilibrium, as the preceding discussions cover the whole ground completely, for every possible case that the subject admits of; but there is still to be examined the case in which an excess of stability should be given to the wall, for the purpose of guarding against all possible accidents.

The equation of strict equilibrium was determined by placing the moment of the maximum pressure equal to that of the weight of the wall; but it is evident that any increase of the moment of the pressure, arising, for example, from an increase of weight of the earth, as when it is surcharged by a temporary weight, or has imbibed water,—or when its bulk is increased from frost, or rain, which will act as though its weight were increased,—would destroy this equilibrium. It will, therefore, be necessary to increase the moment of the pressure so as to give the wall an excess of stability. The question then arises as to the manner of determining this excess of resistance. The method proposed, and which has been generally followed, consists in making the excess of the moment of resistance proportional to that of the pressure; that is, if M represents the moment of pressure, and M' the moment of the weight of the wall, or that of the resistance, in the case of a strict equilibrium $M = M'$, but from the method proposed, the equation should be writen

$$M = M' + \tfrac{1}{n}M',$$

or

$$nM = nM' + M',$$

which amounts, therefore, to multiplying the moment of the pressure by a constant quantity n. To determine this quantity n, recourse must be had to experience; for this purpose walls which have stood for a long period of years, exposed, under the most unfavorable circumstances, to the various accidents of time, must be submitted to calculation, in order to determine their moment of resistance; having found this moment, it must be compared with that of a wall of the same form, determined according to the conditions of a strict equilibrium, and the excess of the former over the latter will give the constant number sought. From a comparison thus instituted, it appears that in sustaining walls, for ordinary cases, the excess of the moment of resistance is one-fourth greater than that of the pressure, or

$$M = M' + \tfrac{1}{4}M' = \tfrac{5}{4}M';$$

but as $M = M'$, in the case of a strict equilibrium, it follows, that for the case in point, the constant quantity n will be represented by $\frac{5}{4}$; or, in other words, the moment of the pressure must be multiplied by $\frac{5}{4}$ before it is placed equal to that of the weight of the wall; or, what will amount to the same thing, $\frac{5}{4}w'$ must be written for w', in the equation which gives the thickness of the wall at its base. This excess of the moment of resistance has received the name of the *Moment of Stability.*

Considerations of the same kind should enter into the subject of arches, in determining the thickness of the abutments of an arch at their base. From various examples, determined in the same way as for revetments, it appears that the moment of stability for arches should be $\frac{4}{3}$, that is, the moment of the horizontal thrust should be multiplied by $\frac{4}{3}$ before it is placed equal to the moment of the resisting parts.

There is another question of importance, in a practical point of view, which may be here introduced. Its object is to ascertain among the different forms which the cross sections of sustaining walls may receive, the one which offers the greatest resistance with the smallest area. To effect this comparison between any two profiles, the following process must be gone through with.

Let S (Fig. VV) represent the area of the cross section $ABCD$,

and M the moment of its weight; S', (Fig. WW), the area *abcd*, and M' the moment of its weight, and suppose the altitude h to be the same in each. The moment of S from what precedes, is represented by

$$M' = \tfrac{1}{2}h\{(x - \tfrac{1}{2}h \tan. s)^2 + \tfrac{1}{3}h^2(\tfrac{1}{2}\tan.^2 s - m^2)\};$$

and the area of S is represented by

$$S = h(x - \tfrac{1}{2}h \tan. s - \tfrac{1}{2}hm).$$

In the same manner the moment of S' will be

$$M' = \tfrac{1}{2}h\{(x' - \tfrac{1}{2}h \tan. s)^2 + \tfrac{1}{3}h^2(\tfrac{1}{2}\tan.^2 s - n^2)\};$$

and its area

$$S' = h(x' - \tfrac{1}{2}\tan. s - \tfrac{1}{2}hn.)$$

In order that these two areas shall present the same resistance, they must satisfy the condition $M = M'$. If then the quantities h, x', m, n and s are given, s being the same in both, but the exterior slopes m and n being different,—the remaining quantity x can be determined from the equation $M = M'$; and this value of x being substituted in the value of S, will determine its area such as to present the same stability as S'.

The cross section of sustaining walls may take one of the six forms as shown in Figs. (PP), (QQ), (RR), (SS), (TT), (UU). From a comparison instituted between these six forms, under the same circumstances, it appears that the form in which the exterior slope is vertical, and the interior inclined, is less advantageous than 6, where both the face and back are vertical;—that the forms 4 and 5 offer more advantages than 6;—that 4 is more advantageous than 5 when tan. s is equal to or less than $\frac{1}{4}$, the maximum of advantage resulting from tan. $s = \frac{1}{6}$.

As the remaining forms are compounded of the 2, 4 and 5, it was to have been expected that they would have participated of their qualities, and this was found to be the case, for 3 being composed of the two most advantageous forms 4 and 5, presents the greatest advantages of any of the forms, while 1, which is composed of the forms 2 and 5, loses a part of the advantages of the latter by the bad point of the former.

It therefore appears that of all the forms of cross section, the one which presents the maximum of advantage with the minimum of area is that with a counterslope of one sixth on the interior and the greatest exterior slope.

There is yet a modification of form 2, (Fig. XX,) which remains to be examined, in which offsets are substituted for the interior slope; a slight examination will show that it is inferior to 6, and in some cases to 2.

Dimensions of Foundations. When the foundations are laid on a perfectly incompressible soil, the only conditions to be satisfied are, that the foundations shall not yield by sliding along their base, nor by a motion of rotation around their exterior edge; the first condition can always be satisfied, by a suitable construction of the masonry; the last will be satisfied, provided the resultant of the pressure and the weights of the wall and its foundation, when produced, passes within the base of the foundations; and this will be the case, almost without an exception, since the resultant of the pressure and the weight of the wall alone, must pass within the base of the wall, and when this resultant is compounded with the weight of the foundation, their common resultant will approach still more nearly a vertical line.

But when the soil is compressible, it is not alone necessary that the resultant should pass within the base; but the equilibrium also requires that it should pass through the middle point of it, so that it may have no tendency to throw the wall either outwards or inwards, by causing a greater pressure on one point than on the other, from which unequal settling might result.

To satisfy this condition, the breadth or spread of the foundation, must be so determined, the thickness being given, that the resultant of the forces in question shall pass through the middle point of the base. It would be very easy to express this condition analytically, and from the equation of equilibrium, thus established, to find the spread of the foundations; but, instead of this process, the following simple geometrical solutions will answer the same purpose, and will, moreover, show more plainly the action of the forces than done by the analysis.

Let *ABCD* (Fig. YY) be the cross section of a wall resting on the foundation *abcd*, the thickness of which *ad* is known; and of which it is required to find the spread such that the resultant of the forces represented by the pressure of the earth, the weight of the wall, and that of the foundation shall pass through *q* the middle of *ab*. Through the point of application of the pressure draw *on* per-

pendicular to AB; and through the centre of gravity of $ABCD$, a vertical line intersecting on at o; from o set off $on = P$, and $om = W$ the weight of $ABCD$; construct the parallelogram of forces on these two lines, and draw the resultant op, which produce to intersect ab at q; set off $qb = aq$ and complete the rectangle $abcd$, this rectangle will be the required area of the cross section of the foundations; for, the resultant of P and W passes through q, the middle of ab, and as the vertical through the centre of gravity $abcd$ must, from the construction, pass through the same point, it follows, that the resultant of the three forces passes through q, the middle of the base ab, therefore ab is the required spread.

It was stated in the Subject of Masonry that the rectangular form of foundations was less advantageous for walls sustaining a lateral pressure than the trapezoidal; and a comparison of the two forms, made by the assistance of analysis, confirms this remark, and, moreover, points out another remarkable circumstance, which is, that, under the same circumstances of weight of wall and pressure, the spread of the rectangular foundation increases continually with the depth, whereas in the trapezoidal form the spread soon reaches a limit where it remains sensibly the same, even for an infinite depth.

The case of the trapezoidal form also admits of a geometrical solution, which is rather more complicated than in the last case. Let $ABCD$ (Fig. ZZ) be the cross section of the wall, and having set off the equal offsets $aD = Bc$ on the top of the foundation, and set off its depth ad, draw the indefinite base aX; then from c draw any inclined line cb' and suppose $ab'cd$ to be the required trapezoid. Having, as in the last case, constructed the resultant of P and W, draw a line throug o', the centre of gravity of $ab'cd$, and produce it to meet the resultant just found, and then find the common resultant of the weight W' of the trapezoid and the known resultant. Having constructed this line, produce it to intersect aX at q'; then if q' were the middle point of ab', the trapezoid $ab'cd$ would satisfy the required conditions; but if this is not so, then suppose aq' greater than $b'q'$, and having found their difference $aq' - b'q'$, set it off on a perpendicular $b'm'$ drawn through b' to aX. Assume then a second trapezoid $ab''cd$, and find, by the same process, the point q'' through which the common resultant passes; then if

$aq'' > b''q''$, set off their difference above aX, on a perpendicular to it. Having in this way found several points, both above and below aX, let the curve $m'bm''$, &c., be drawn through them; and the point b, where it cuts aX, will give the required spread ab, such that the resultant of P, W and the weight of $abcd$, will pass through the middle of ab; for the ordinate of the curve being zero for this point, it follows that the difference $aq - bq$, which corresponds to it, must also be zero or $aq = bq$.

Counterforts and *Relieving Arches.* The *counterfort* or *buttress* is seldom used except for sustaining walls for military works; its object, in those cases, being principally to limit the effect of the shock of military projectiles thrown against the wall, by preventing the whole wall from being overthrown, as might take place where it constructed without buttresses. With respect to the additional stability which they give to the wall in resisting the pressure of the earth, it is not very important, unless the buttress is placed on the exterior of the wall, and that the union between the two is so perfect that rupture can ensue only by the overthrow of the whole mass around the exterior edge of the buttress. The horizontal section of the buttress may be either rectangular or trapezoidal. If it is placed within the wall, the rectangular form will give the most stability, the volume of the buttress and its length being the same;—if placed without, the trapezoidal form, under the same circumstances, will be the best.

Relieving arches are formed by building, in connection with the wall, arches which rest on piers built out perpendicularly from the wall. These piers are nothing more than rectangular counterforts. The operation of these arches, (Fig a,) from which their name is derived, consists in relieving the wall of a portion of the pressure of the earth; and when the union between the wall, the piers, and the arches, is good, the stability is greatly increased; and the total mass of masonry may be considerably less than in the case of a simple sustaining wall. Relieving arches may be built in one or several tiers, according to circumstances. In all cases the length of their piers, and the height of the crown of the arch should be so regulated that the earth behind the wall shall not come into contact with the back of it, within the extent from the crown of the arch to the bottom of the pier.

Data for Calculation. In applying the preceding formula to practical examples the values of the quantities represented by w, w', f and c must be obtained from experiments. The following results have been found in this way:

The specific gravities of soils of different kinds are represented very nearly by the following table:

Vegetable mould or garden earth, ...	1,4.
Common earth, and earthy clay and sand, . .	1,5 to 1,7.
Pure clay, or sand, ...	1,9.

The value of w, therefore, will be found by multiplying each of those numbers by 62½ lbs., or the weight of a cubic foot of water, the linear foot being taken in the calculation as the unit of measure.

The specific gravities of different kinds of masonry are nearly as follows:

Brick, and rubble of calcareous or siliceous stone from, ...	1,7 to 2,3.
Rubble work of basalt, ...	2,5.

The value of w' will be calculated from these data in the same way as w.

To determine the values to be assigned to f, it will be necessary to ascertain the value of the angle φ which is the complement of that of the natural slope, f being the natural tangent of this last angle. The following table exhibits the values of f, as found by experiment:

Ordinary earth, when perfectly dry and pulverulent, takes a natural slope, of about 43°, 10′ whence f=0.94.

The natural slope of same slightly moistened, or in its natural state is 54°, whence f=1.38.

The natural slope of fine dry sand varies between 30°, and 40°, whence f is between 0.6, and 0.8.

To determine the value of c, or the cohesion on the unit of surface, it will be necessary to resume the expression for c in the case where the back of the wall is vertical, which is

$$c = \tfrac{1}{4} w h' \tan. \tfrac{1}{2}\varphi\,;$$

in which h' is the height at which earth, in its natural state, will stand without caving in, when the side of the excavation is vertical, and φ is the complement of the angle of the natural slope.

It has been found by experiment, that common earth, in its natural state, the specific gravity of which is 1.5, and whose natural slope is 54°, may be excavated vertically to the depth of three feet without caving in. By substituting for w, h' and tan. $\frac{1}{2}\varphi$, these values there results $c=20$ lbs. for the cohesion on a square foot for ordinary earth. For clay h' may be taken 12 feet, and $\varphi=36°$, which values will give for c about 106 lbs. This quantity, therefore, for the different kinds of earth in their natural state, or when well rammed in an embankment, may be taken within these two limits.

Effect of Water on Soils. The action of water is very different in different soils. It does not appear to affect the volume of sandy or earthy soils; but the clayey soils increase considerably in bulk by imbibing water; and those soils, which are termed soapy or marly, are changed by it into a semi-fluid state. In each of these last cases the action of the soil should be assimilated to that of a fluid of the same specific gravity as the soil itself.

In case the strata of earth to be sustained are of different densities, then it will be erring on the safe side to consider the specific gravity of the whole mass the same as that of the stratum of greatest density.

NOTE VII.

Theory of the pressure of voussoirs on Centres.

The arrangement of the frame work of centres should be based upon the manner in which the voussoirs act upon it, in order that each part of the rib may receive dimensions proportioned to the pressure thrown upon it, arising from the action of the voussoirs.

The point at which the voussoirs commence to bear upon the centre will be at that joint the natural tangent of which corresponds to the coefficient f, as given in Note V; this angle may, on an average, be taken at 32°, in which case $f=0.28$; from this point, upwards, each voussoir will bear on the centre with a force which is proportional to part of its weight, according as the angle between

its lower joint and a vertical line decreases, until this angle becoming zero the whole weight of the voussoir will be thrown on the centre. The object of the discussion about to be entered upon is, to find, not only the pressure which is thrown on the centre by each voussoir separately, but also the amount of the total pressure at each point when all the voussoirs are laid. Let *PQNM* (Fig. b) be a semi-arch, divided into voussoirs as *ABCD*, and let it be required to find the pressure on the centre at the point *B* which arises from the action of the voussoirs above the joint *AB*.

Designate by w, w', w'', &c., the weights of the respective voussoirs *ABCD*, &c.—by p, p', p'', &c., the pressures perpendicular to the respective joints *AB*, *DC*, &c.,—by $a=BOM-COM$ the angle between the two joints *AB* and *DC*,—by b the angle *BOM*,—by z the length *AB* of the joints, which for simplicity will here be supposed the same for each,—by f the coefficient of friction,—and by c the cohesion on the unit of surface along any joint *AB*.

Now the pressure on the centre, in the direction *AB*, results from the action of four distinct forces; first from the weight w of the voussoir *ABCD*; second from the pressure p exerted perpendicularly on the joint *DC*, arising from the weights of the respective voussoirs w', w'', &c., which pressure tends to maintain *ABCD* in its place, or rather to prevent its motion in the direction *AB*;—third from the friction and cohesion along the joint *DC*, which, in this case, act as a force to drag *ABCD* in the direction *AB*, since they maintain the voussoirs above *ABCD* in their place on the joint *DC*;—fourth from the cohesion along *AB*. The nature of these forces being well understood, and the direction and intensity of their action being known, it will be very easy to find their action on the centre. For this purpose, let each of the forces be decomposed into two components, one parallel to *AB* and the other perpendicular to it; the algebraic sum of the parallel components will, evidently, tend to cause motion in the direction *AB*, and this tendency will, in part, be counteracted by the friction along *AB* due to the algebraic sum of the perpendicular components, and also by the cohesion along *AB*; the pressure on the centre, therefore, will be the difference between these forces; that is, representing it by *P*, it will be equal to the sum of the components parallel

to AB, diminished by the amount of friction along AB, which is due to the sum of the perpendicular components, and also by the cohesion along AB.

To express these conditions analytically, let ab, taken on a vertical passing through the centre of gravity of $ABCD$, be assumed equal to w, then $ac = w \cos. b$ will be the parallel, and $bc = w \sin. b$ the perpendicular components with respect to AB.

The pressure p', perpendicular to DC, taken equal to de on its line of direction, being decomposed in a similar way, there results for the parallel and perpendicular components

$$ef = -p' \sin. a \text{ and } df = p \cos. a;$$

the component ec receiving the negative sign, since it evidently acts from B towards A.

The friction of CD will be represented by fp', and the cohesion by cz, their sum, therefore, will be $fp' + cz = gh$, acting from D towards C, which being decomposed with respect to AB, as above, there results

$$gi = (fp' + cz) \sin. a, \text{ and } hi = (fp' + cz) \cos. a.$$

Finally, the cohesion along AB is also cz. The equation of equilibrium, therefore, will be,

$$P = ac - ef + gi - f(bc + df + hi) - cz;$$

or, substituting the values of these quantities,

$$P = w \cos. b - p' \sin. a + (fp' + cz) \sin. a$$
$$f\,\{w \sin. b + p' \cos. a + (fp' + cz) \cos. a\} - cz. \quad . \quad . \quad (A)$$

for the value of the pressure at the point B.

The equation (A) gives rise to a remark which observation confirms, that the value of P diminishes at any one point in proportion as the voussoirs above that point are respectively placed on the centre, so that it may at length become zero or even negative, and this has been found actually to be the case in practice, the voussoirs at certain points being observed to detach themselves from the centre after a certain number of courses have been laid on it above them.

The pressure on the centre by any voussoir $ABCD$, when first laid, will be represented by

$$P = w(\cos. b - f \sin. b) - cz; \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

which is the greatest pressure perpendicular to the rib, that can take place at any point; so that the total pressure can never exceed

that which would arise by adding together the values of P found by substituting in equation (B) the successive value of the angle b from 32° to 90°.

NOTE VIII.

On the distribution of the weight sustained by a bridge over the Frame and Points of Support.

The pressures which are thrown on the different parts of the frame work of a bridge, and the points of support, may arise, in part, from the constant weight of the road-way and other parts of the superstructure, and of a variable weight, which may be equally diffused over the road-way, as, for example, when it is crowded with persons, or else when a weight acts at a particular point, as when a wheel vehicle is passing the bridge. The greatest weight which can be equally distributed over the road-way will arise from a crowd of persons on the bridge, and this weight has been estimated at   lbs. on a square foot.

To ascertain the action of these different weights on the different parts of the frame, it will be necessary, in the first place, to find the value of their components which act on the parts. Let there be considered a frame (Fig. c) composed of sleepers, supported by struts alone; and let the weight be supposed equally distributed over the road-way. Designate by l the length BD,—by $\frac{1}{2}l'$ the length $CD = CD'$,—by a the angle DAB,—and by w the weight on the unit of surface. Now since the struts are connected with the sleepers by joints at D and D', the sleeper should be regarded as composed of the three parts BD, DD' and $D'B'$. The weight, therefore, which is equally distributed over the sleeper, may be represented by wl for BD, by wl' for DD', and wl for $D'B'$; so that at the points D and D', there will act vertically a weight represented by $w(\frac{1}{2}l + \frac{1}{2}l')$. But this weight, from the connection between the struts and sleeper, will be sustained partly by the strut, and partly by the part of the sleeper DD' which, owing to this connection, acts as a straining beam. The component in the direction AD is represented by

$$\frac{w(\frac{1}{2}l+\frac{1}{2}l')}{\cos. a},$$

and that in the direction DD, by

$$w(\tfrac{1}{2}l + \tfrac{1}{2}l')\ \tan.\ a,$$

which component, with the opposite and equal component from D' toward D, will tend to compress the part DD'; but this part is also acted on by the weight wl', equally distributed over it, which weight, acting downwards, tends to bend the beam, and therefore adds to the compression on the upper fibres. This part therefore may be considered in the same state as if it were confined at C, and each half $CD' = CD$ submitted to the two forces, $w(\frac{1}{2}l + \frac{1}{2}l')$ tan. a, acting from D towards C, and $\frac{1}{2}wl'$ acting at D vertically upwards. From an investigation of the resistance which the part DD' should offer to these forces, so that the structure shall be secure, it appears that the following relations must exist,

$$R' = \frac{w}{db}\left\{(\tfrac{1}{2}l + \tfrac{1}{2}l')\tan.\ a + \frac{3l'^2}{4d}\right\} \quad . \quad . \quad (A);$$

in which d represents the depth, and b the breadth of the cross section, supposing the beam rectangular; and R' the greatest weight which can be laid with safety on the unit of surface of the cross section, as given in the subject of Strength of Materials.

The resistance which the strut AD should offer, will be calculated in the same way as that of a beam placed vertically and confined at its lower end, which sustains a weight equal to $\dfrac{w(\frac{1}{2}l + \frac{1}{2}l')}{\cos.\ a}$.

If the frame is supposed to be acted on by a weight W at the point C, for example, besides the weight w uniformly distributed over each unit of surface, the foregoing components will become

$$\frac{\frac{1}{2}W + w(\frac{1}{2}l + \frac{1}{2}l')}{\cos.\ a},$$

and $\frac{1}{2}W + w(\frac{1}{2}l + \frac{1}{2}l')$ tan. a.

The value of R', from which the cross section of DD' is found, will become

$$R' = \frac{\{W + w(l + l')\}\tan.\ a}{2db} + \frac{3(W + \frac{1}{2}wl')\frac{1}{2}l'}{bd^2} \quad . \quad . \quad (B).$$

If there be considered (Fig. e) the case, in which the frame consists of a sleeper sustained by two struts and a straining beam, the entire pressure, arising from the horizontal component of the weight will be sustained by the straining beam alone; but the resistance

of the part DD' to flexure, from the downward tendency of the weight, will be due both to the sleeper and straining beam, and the equations (A) and (B) must be modified to suit this new circumstance. These modifications will give very nearly

$$R' = \frac{w(\frac{1}{2}l + \frac{1}{2}l')\ \text{tan.}\ a}{bd} + \frac{3l'^2}{4bd'^2},$$

and

$$R' = \frac{\{W + w(l + l')\}\ \text{tan.}\ a}{2bd} + \frac{3(W + \frac{1}{2}wl')\frac{1}{2}l'}{bd'^2};$$

in which d' is the sum of the depths of the sleeper and straining beam, their breadth being considered uniform, and the other letters designate the same parts as in the foregoing case.

When several struts and straining beams are used, the resistance which each should offer may be readily calculated by the aid of the cases just treated.

To calculate the effect of these various forces on the abutment $AbBd$, it must be observed, that at the point A there will be a downward pressure equal to the vertical weight acting at the point D, whilst at B there will also be a downward pressure, due to the weight distributed over the half of BD, the sum of these pressures act sensibly along the vertical Bb, their moment, therefore, added to that of the weight of the abutment $abBd$ taken with respect to the point a, will give the resistance of the abutment to a force which would tend to thrust it outwards by turning around the point a. The only force which will have any tendency to throw the abutment over will be the horizontal component of the pressure at D, which component acts horizontally on the abutment at A; its moment, therefore, in the case of a strict equilibrium must be equal to the sum of the moments just spoken of.

NOTE IX.

Method of determining the elevation of the exterior rail of a Curved Track.

A curved track of a rail-way presents several causes of retardation to the velocity of the cars; the first arises from the successive

shocks of the flange of the exterior wheel against the exterior rail, caused by the tendency of the vehicle to continue its motion in a right line;—the second arises from the friction of the flange against the same rail, occasioned by the action of the centrifugal force, which tends to throw the vehicle outwards from the track;—and the third is occasioned by the unequal distances which the interior and exterior wheels must pass over in the same time, which causes the exterior wheel to be dragged over a portion of the exterior rail, equal to the difference of length beween the two rails, owing to the wheels being solidly connected with the axle-tree.

A remedy has been sought for these causes of retardation, first, in so arranging the wheels, that they may revolve on unequal diameters, by which means the exterior wheel, the diameter of which is greatest, will pass over a greater distance than the interior one, in the same number of revolutions, whilst it also produces the effect of giving the vehicle a curvilinear, instead of a direct motion; and second, in elevating the exterior rail so as to counteract the tendency of the centrifugal force.

The manner of arranging the wheels to remedy the first and third causes of retardation does not properly come within the limits of this course; but, from its connection with the means used to counteract the centrifugal force, it will be necessary to enter into an explanation of the principle of the arrangement. Let AB, and CD (Fig. f) be the arcs of two rails of a curved track, of which O is the centre. Designate by R the radius OA, and by $a=AC$ the width of the track,—by r the semi-diameter of the wheel on the interior rail, and by r' that of the exterior wheel.

Now, in order that the exterior wheel shall not drag on any part of its rail, it must make the same number of revolutions on CD that the interior wheel makes on the similar arc AB; therefore, the circumferences of the wheels must be to each other as these arcs; but the circumferences of the wheels are to each other as their semi-diameters, and the arcs are as their radii, consequently the ratio of the semi-diameters is equal to that of the radii, or

$$\frac{r'}{r}=\frac{R+a}{R},$$

from which by reduction, and the subtraction of r from both members of the equation, there results

$$r' - r = \frac{2ar}{R} = d,$$

for the difference d of the semi-diameters of the wheels which will satisfy the required condition. But were the wheels to receive this difference of semi-diameter throughout, they could not run on a straight track, to combine then both advantages, the tire of the wheel is made slightly conical, the diameter within the track being greater than that without it. Let A (Fig. g) be the interior, and B the exterior wheel, of which the conical form is given by the slope ab, the ratio ac of the base of which and the perpendicular bc is given, that is

$$\frac{bc}{ac} = \frac{1}{e}.$$

Now, if the wheels are running on a straight track, on the semi-diameters $o'm' = r'$, and it be wished to make them run on a curved track, which requires a difference of semi-diameters $o'm' - om = r' - r$, it will be necessary to displace the wheels laterally, by a certain quantity which must be found by calculation. To find this quantity, suppose the wheels displaced; it is evident that the point m' of A will be lowered a certain quantity, whilst the point m' of B will be raised exactly the same quantity by this displacement; therefore, if $m'n$ be equal to $r' - r$, the line mn, parallel to oo', will be equal to double the lateral displacement, but

$$\frac{mn}{m'n} = \frac{ac}{bc} = e,$$

or,

$$mn = e.\ m'n = e\,(r' - r),$$

therefore,

$$\tfrac{1}{2}mn = \tfrac{1}{2}e\,(r' - r) = d'\,;$$

and substituting for $r' - r$ the value just found, there results

$$\tfrac{1}{2}mn = \frac{ear}{R},$$

for the displacement which will make the difference of the semi-diameters equal to that which will cause the wheels to make the same number of revolutions in passing over the two curves of the track.

But since the action of the centrifugal force tends to effect this displacement, the next point to be arranged is to regulate this ac-

tion in such a way that the displacement shall not exceed that which is strictly required, as just found.

Let f designate the centrifugal force,—M the mass of the vehicle,—v its velocity, and R' the arc described by centre of gravity of the vehicle during the motion on the curved track,—then by a law of dynamics,

$$f = M\frac{v^2}{R'},$$

but $M = \frac{W}{g}$ in which W is the total weight of M, and g the accelerating force of gravity, therefore there results

$$f = \frac{W}{g}\cdot\frac{v^2}{R'}$$

for the value of the centrifugal force of a vehicle, whose weight is W, when moving with a velocity v on a curve, whose radius is R.

As the force f acts outwards from the track, the force by which it is to be counteracted must act in a contrary direction to it. This counteracting force is that of gravity, which, when the exterior rail is elevated, tends to cause the vehicle to slide inwards, in a direction contrary to that of the centrifugal force. The problem, therefore, to be solved, is to find the quantity by which the exterior rail must be elevated in order that the component of gravity along the inclined plane, crosswise the track, which is due to this elevation, and to the displacement of the wheels, shall exactly counterbalance the effect of the centrifugal force.

Let the two wheels A, and B, (Fig. h) be supposed on a level track, and resting on the equal semi-diameters $o'm'$ and $o''m''$, then, if they are displaced laterally by the quantity d', the point o' will descend to the point p, whilst the point o' on B will be raised by exactly the same quantity, each of which is equal to d the difference of the semi-diameters on which the wheels rest after their displacement. The wheels, therefore, will be in the same state as if they were on an inclined plane whose length is $a—d'$, the difference between the width of the track and the displacement d', and altitude is $2d$, or twice the difference of the semi-diameters. Let the rail on which B rests be supposed now to be raised a quantity represented by y, the altitude of the inclined plane will then become $y+2d$, but since the component of gravity along an inclined plane

Plate 2.

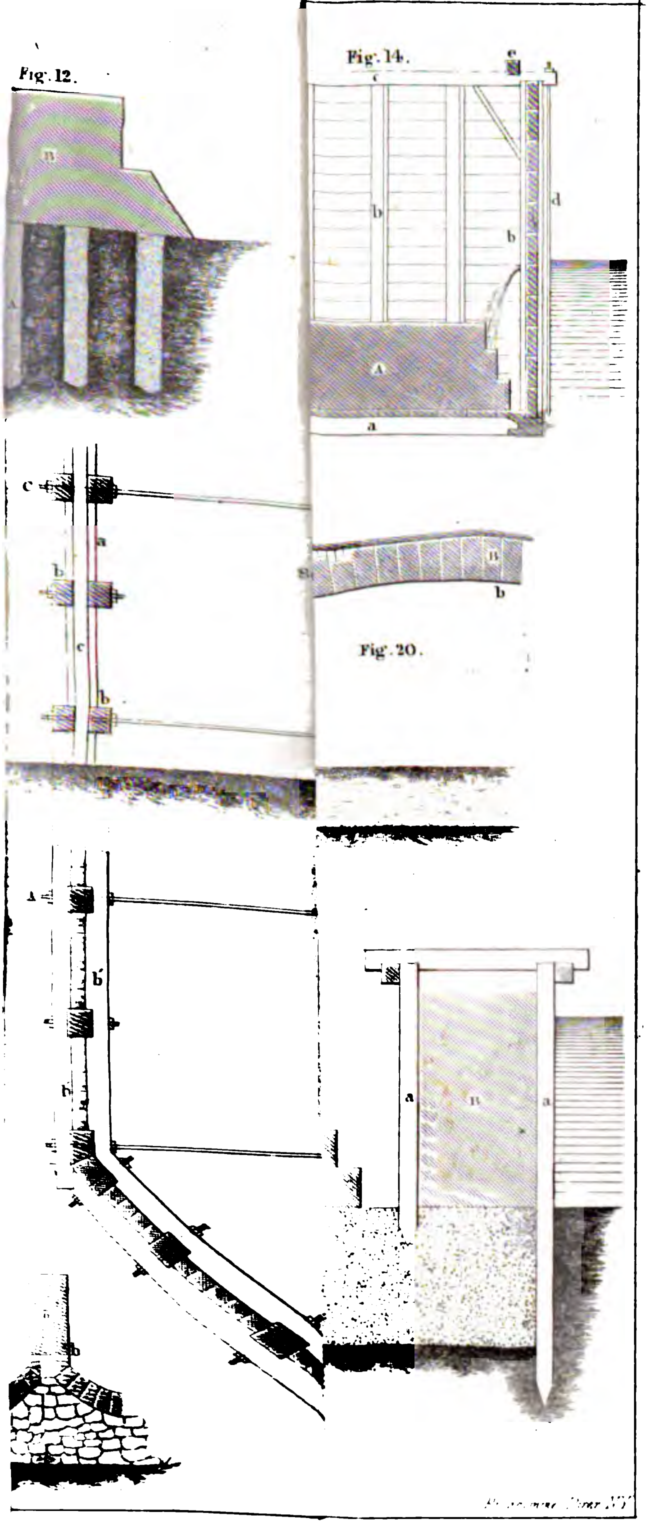


Plate 3.

Fig. 25.

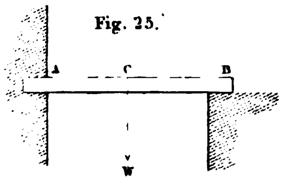


Fig. 30.

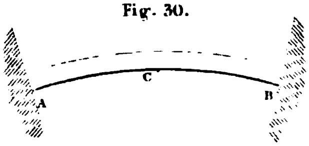
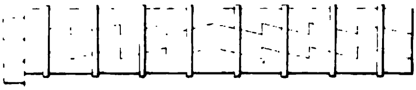


Fig. 34.



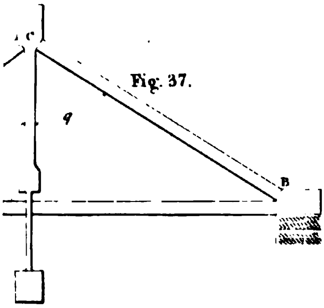
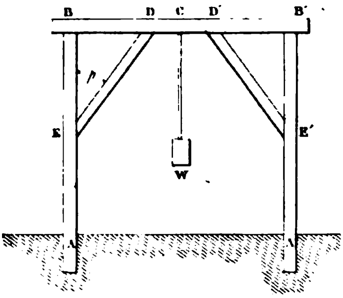


Fig. 37.

Fig. 43.



s expressed by the weight of the body multiplied by the ratio of he altitude to the length of the plane, this component, in the case in point, will be

$$W \times \frac{y + 2d}{a - d'}$$

Now the conditions of equilibrium between this component and the centrifugal force require that,

$$W.\frac{y + 2d}{a - d'} = W.\frac{v^2}{gR'},$$

rom which there results,

$$y = (a - d')\frac{v^2}{gR'} - 2d$$

or the required elevation. By substituting in this expression the values of d and d', as already found, the value of y will be known or any velocity v on a curved track, the radius of the middle point of which is R'.

NOTE X.

Methods of Gauging Water Courses and determining the relations between the slope and cross section of Canal Feeders.

The ordinary method of guaging small water courses, consists in erecting a dam at some suitable point across the bed of the stream n which a sluice or orifice of a rectangular form is made, through which the water is allowed to run, until the level of the pool above he dam remains sensibly the same, in which state it is evident that he quantity of water conveyed into the pool by the stream must be equal to that discharged from the orifice, and this last, therefore will truly represent the supply from the stream at the time the observation is made.

It is essential for the accuracy of this method, that the velocity at the orifice should remain uniform during the time of the observation; to effect which, the head of water in the pool, or the height above the orifice, should be so great as to prevent any convergency of the fluid particles towards the orifice, except at a very short distance from it; the water in the pool must remain stagnant, and

there must be no back water nor other obstruction to the free efflux of the water through the orifice.

The quantity discharged through the orifice, in a given unit of time, is calculated by a well known theorem of the discharge of fluids, which is

$$Q = ma\sqrt{2gh},$$

in which a represents the area of the orifice; $\sqrt{2gh}$ the velocity due to the head of water h above the orifice, and m is a constant quantity, by which the other two terms must be multiplied to obtain the true quantity discharged, since this quantity is less than that given by theory, which would be simply $a\sqrt{2gh}$, owing to the phenomenon known as the contraction of the fluid vein.

The value of m, as found from experiment, is stated differently by different writers on this subject. It appears, from the most recent experiments, that its value is not effected by the width of the orifice, but depends entirely on its height and the head of water. For a head of water of five feet above the orifice, and a height of orifice varying between half an inch and two inches, m is represented very nearly by the fraction 0,617; for a height of orifice of four inches and the same head of water $m = 0{,}610$, and for eight inches $m = 0{,}602$.

There are other formula for the same purpose which are based on the same principle, this is however the most simple, and when m is determined to suit it, is therefore the best. In all cases the sides of the orifice must be very thin, such, for example, as a sheet of tin, or a thin board would furnish.

Mr. De Prony has given a method, in his writings on this subject, by which the indetermination introduced by using the constant m may be avoided, which is characterized by all that elegance and ingenuity in the solution for which this great writer on physico-mathematics is distinguished; but it requires in practice a preparation and nicety of observation which are seldom at the disposal of the engineer.

The most accurate method, when it can be resorted to, consists in receiving the water discharged in a vessel of known dimensions and noting the time it takes to fill the vessel.

When it is not practicable to dam back the water, it will be necessary to ascertain the area of the cross section of the stream, and

s mean velocity for a given unit of time, at some particular point; nd the product of these two elements will give the quantity which ows through the cross section in a given time.

To find the mean velocity, it is first necessary to ascertain the reatest velocity of the stream at the surface, which usually will the velocity of the middle thread of the surface. This may be one, in a very simple way, by placing a small float of some marial, whose specific gravity is nearly the same as that of water, for example, pure camphor or white wax, in the current, and oting the time it takes it to pass between two fixed points; havg done this, if this velocity per second in feet be denoted by V, id the mean velocity by v, there will obtain

$$v = \frac{V(V+7.78188)}{V+10.34508};$$

hich expression gives the measure of the mean velocity in feet.

Theory shows that there are certain relations between the form the cross section of the bed of a stream, its slope, and the mean locity of its current, from which two of these quantities being ven, the other can be found. These relations are expressed by a ell known theorem, which is

$$\beta v^2 + \alpha v = \frac{a}{p} \cdot \frac{h}{l}.$$

which β and α are constant quantities, determined from experient,—a the area of the crose section,—p the portion of the periter of the cross section in contact with the fluid,—h the difference level between any two points of the bed, and l the horizontal disice between those points. The quantity $\frac{a}{p}$ is termed the *mean dius*, and may be designated by R; the quantity $\frac{h}{l}$ is the slope of the d, per unit of measure, and may be designated by i; introducing erefore these values in the preceding expression, and also for β d α their values, as found by experiment, there results

$$0{,}0001114\, v^2 + 0{,}0000242651\, v = Ri,$$

m which either of the quantities v, R or i may be found, when others are known.

This theorem, therefore, will serve to ascertain the supply nished by a feeder, or a river, if the quantities in it can be ar-

rived at for any part where the cross section and slope may be considered as uniform for some distance along the line of the bed.

NOTE XI.

Methods of determining the length of the Tail-wall of a Lock.

Let ACB (Fig. i) be the plan of the mitre-sill of a lock. Designate by h, the head of water in the lock which presses against the tail gates, by $b=AC$ the breadth of one of the leaves of the gate,—by a the angle BAC, which is half the supplement of the angle C of the mitre-sill.

The action of the water against the two leaves AC and BC of the gate, tends to cause a rupture along some line as Aa, or Ac of the tail-wall, and to overturn the wall around its edge ab. The problem, therefore, consists in finding what length Ab the wall must receive to counteract this tendency.

The specific gravity of water being designated by unity, the pressure of the water on the leaf AC will, from what was shown in NOTE VI, be represented by

$$\tfrac{1}{2}h^2 b\ ;$$

but as this pressure acts perpendicularly to AC it can be decomposed into two others, one parallel to Ab, which alone tends to overturn the wall, and the other parallel to AB, which is destroyed by the opposite equal component of the leaf B. The component parallel to Ab is

$$\tfrac{1}{2}h^2 b \sin. a.$$

But as this force acts at a height above the bottom of the tail wall equal to $\frac{1}{3}h$, its moment, with respect to the exterior edge ab, will be represented by

$$\tfrac{1}{2}h^2 b \sin. a \times \tfrac{1}{3}h = \tfrac{1}{6}h^3 b \sin. a.$$

As the thickness of the tail-wall ab, is known, let it be designated by a, by x the required length Ab, and by w the weight of the unit of volume of the masonry. The moment of the prism whose base is Aba, and altitude h will be represented by

$$\tfrac{1}{2}waxh \times \tfrac{1}{3}x = \tfrac{1}{6}wahx^2,$$

the equation of equilibrium between the pressure and the resistance will be

$$\tfrac{1}{6}wahx^3 = \tfrac{2}{3}bh^3 \sin. a,$$

from which the value of x is easily found.

When the rupture takes place along the line Ac, not passing through a, it will be necessary to find beforehand the most probable position of the line Ac. Experiments have been made to ascertain this point, from which it seems that the position of Ac is generally found such that $Ad = 2cd$. This being admitted, it will be necessary to find what length db of wall must be added to preserve the equilibrium. Designating this length db by x, the sum of the moments of the prism Adc, and the parallellopiped $cabd$ will be expressed by

$$wa^2h\left(\tfrac{2}{3}a + x\right) + waxh \times \tfrac{1}{2}x,$$

and the equation of equilibrium, from which x can be determined, will be

$$\tfrac{1}{2}wahx^2 + wa^2h\left(\tfrac{2}{3}a + a\right) = \tfrac{2}{3}h^3b \sin. a.$$

NOTE XII.

Manner of estimating the strain on the parts of the frame of a Lock-Gate, arising from the weight of the Gate, and the pressure of the water against it.

Any change of form in the frame of an ordinary lock-gate is prevented, either by a brace of wood AC, (Fig. k) or by a tie of wrought iron BD, in which case the entire weight of the gate is borne by the brace or tie, and the upper or lower cross pieces, thus relieving the intermediate cross pieces from any other strain but that arising from the pressure of the water.

To estimate the portions of the weight borne by the brace AC, and the top piece BC, designate by W a weight, which, suspended at the point C, will produce the same effect as the weight of the gate itself acting at its centre of gravity; and by c the angle BAC between the brace and the quoin-post; then the component of W in the direction AC, which compresses the brace, will be represented by

$$\frac{W}{\cos. c}$$

and the tension on the piece BC will be

$$W \tan. c.$$

To estimate the strain on any one of the cross pieces as EF, it may be regarded as supporting the pressure of water on half the intervals between it and the cross pieces just above and below it. Designating therefore the pressure on a unit of length of E by p, and the length of EF by $2l$, the total pressure, uniformly distributed over it, will be represented by $2pl$. But when the two leaves (Fig. 1) are closed, the one throws a pressure on the other which tends to compress the cross pieces. To find this strain, let a designate the angle DAA', which is half the supplement of that between the leaves, or of D, then the pressure $2pl$, which is uniformly distributed over AD and acts perpendicularly to it, may be regarded as decomposed into its two parallel components, one acting at A and the other at D, each of which will be represented by pl. But the component at A is destroyed by the resistance of the quoins, whilst the one at D may be decomposed into two others, one parallel to AA', which will be destroyed by the equal and opposite component of the other leaf, and the other in the direction DA, which tends to compress the cross piece, and is represented by $\frac{pl}{\tan. a}$. The cross piece EF may, therefore, be considered as a piece confined at its middle point, and each half of it submitted to a strain pl, uniformly diffused over it, and to a compression $\frac{pl}{\tan. a}$.

To estimate then the relations between the cross section of this piece, and the greatest strain that should be borne by a unit of its surface, there will obtain

$$R' = \frac{pl}{db \tan. a} + \frac{3pl^2}{bd^2},$$

in which b represents the vertical, and d the horizontal dimension of the cross section, its form being a rectangle.

is expressed by the weight of the body multiplied by the ratio of the altitude to the length of the plane, this component, in the case in point, will be

$$W \times \frac{y + 2d}{a - d'}$$

Now the conditions of equilibrium between this component and the centrifugal force require that,

$$W.\frac{y + 2d}{a - d'} = W.\frac{v^2}{gR'},$$

from which there results,

$$y = (a - d')\frac{v^2}{gR'} - 2d$$

for the required elevation. By substituting in this expression the values of d and d', as already found, the value of y will be known for any velocity v on a curved track, the radius of the middle point of which is R'.

NOTE X.

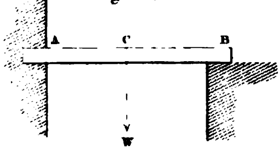
Methods of Gauging Water Courses and determining the relations between the slope and cross section of Canal Feeders.

The ordinary method of guaging small water courses, consists in erecting a dam at some suitable point across the bed of the stream in which a sluice or orifice of a rectangular form is made, through which the water is allowed to run, until the level of the pool above the dam remains sensibly the same, in which state it is evident that the quantity of water conveyed into the pool by the stream must be equal to that discharged from the orifice, and this last, therefore will truly represent the supply from the stream at the time the observation is made.

It is essential for the accuracy of this method, that the velocity at the orifice should remain uniform during the time of the observation; to effect which, the head of water in the pool, or the height above the orifice, should be so great as to prevent any convergency of the fluid particles towards the orifice, except at a very short distance from it; the water in the pool must remain stagnant, and

Plate 3.

Fig. 25.



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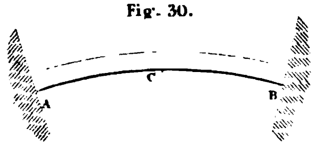

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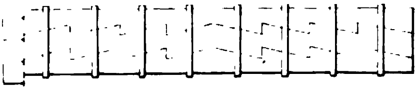

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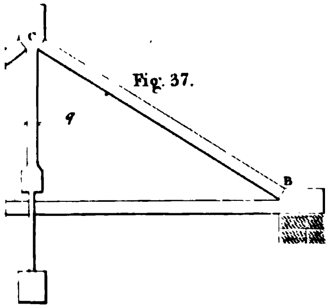
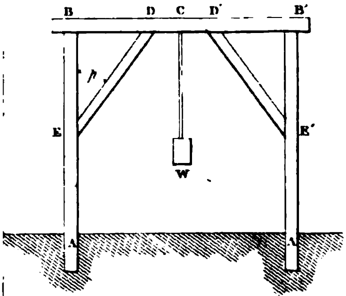


Fig. 39.

Fig. 43.



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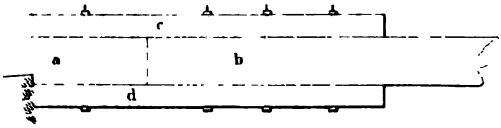
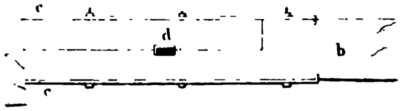
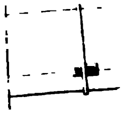


Fig. 49





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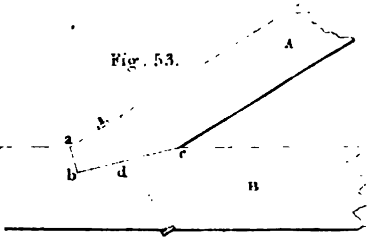

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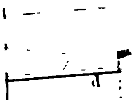
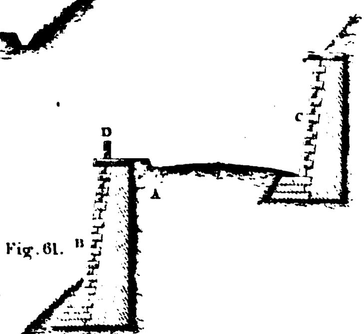


g. 60.

Fig. 61.



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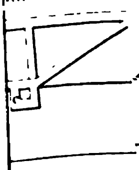

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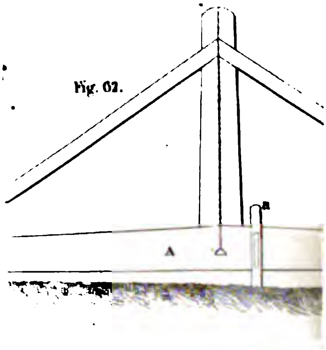

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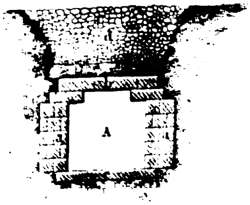
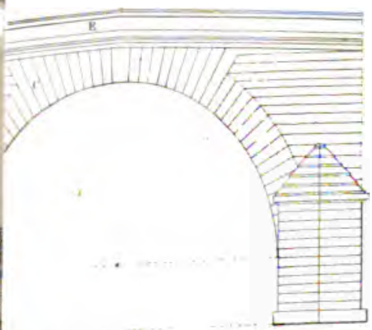


Fig. 64.

Fig. 65.

Fig. 63.



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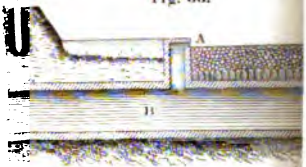
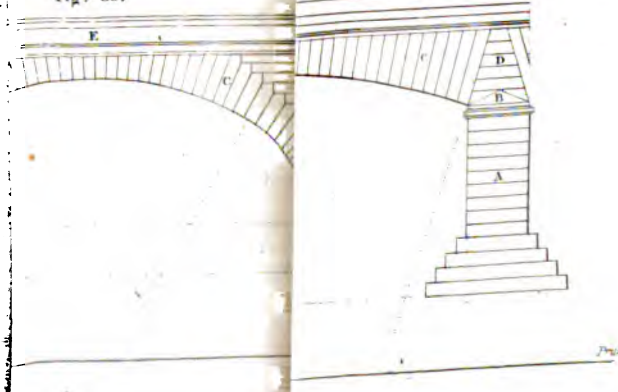


Fig. 66.

Fig. 69.



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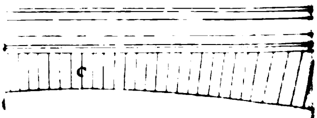

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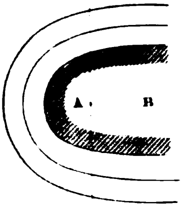

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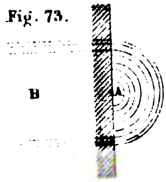

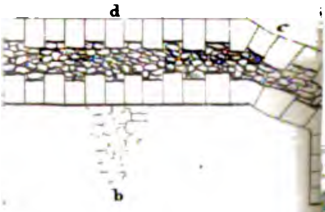

78.

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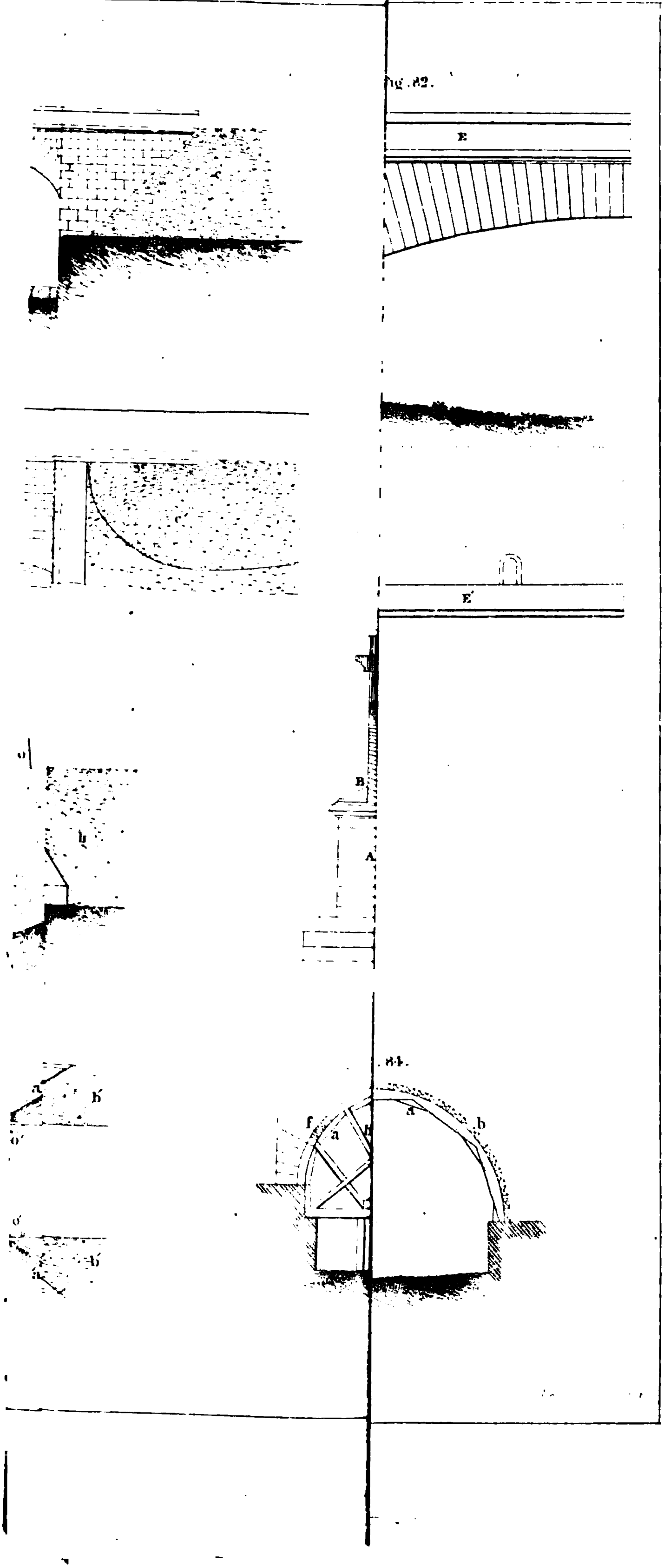
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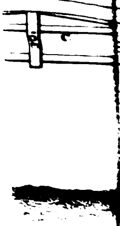

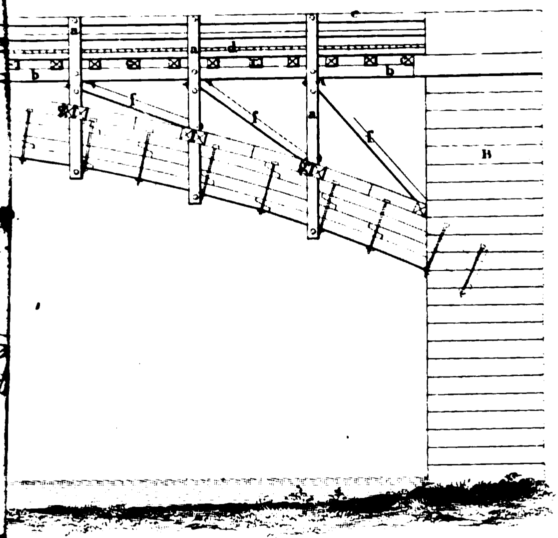

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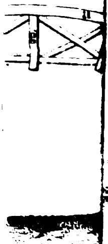

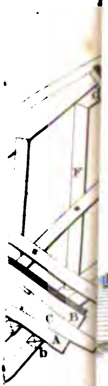

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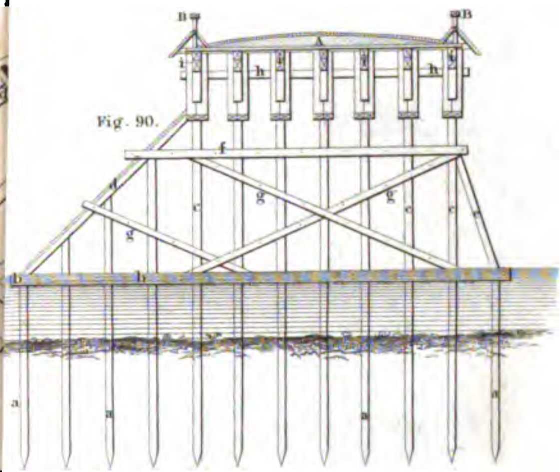

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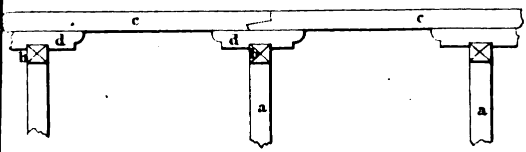
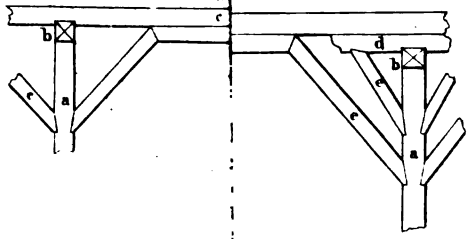


Prudhomme Dirx. N.Y.

Plate 9.

Fig.



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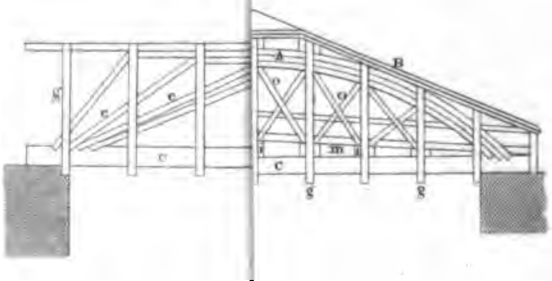
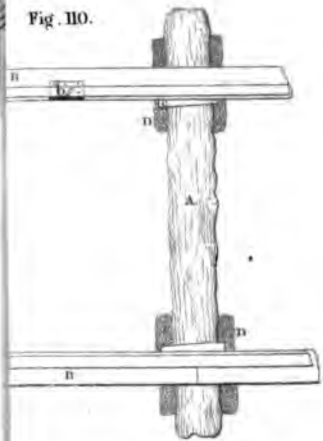
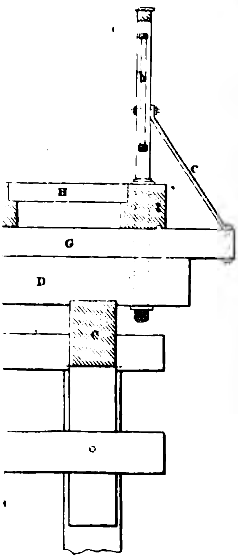
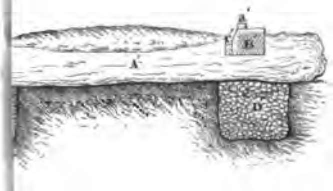


g. 107.

Fig. 110.







Prudhomme Direx NY.

Plate 10

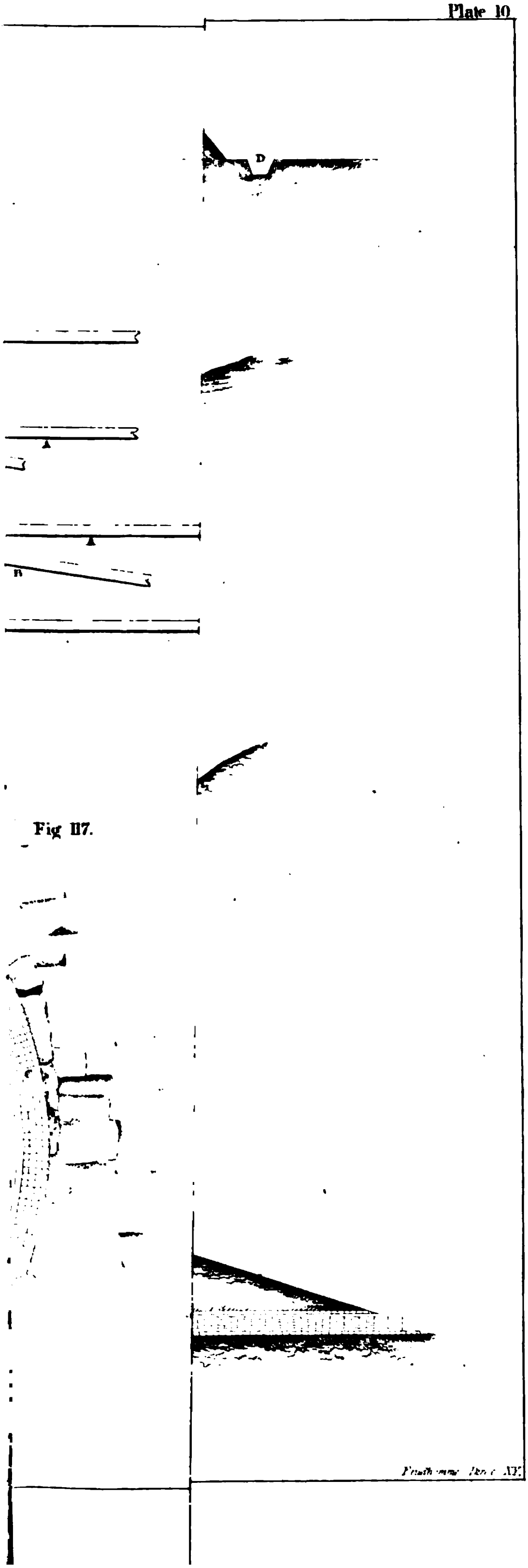
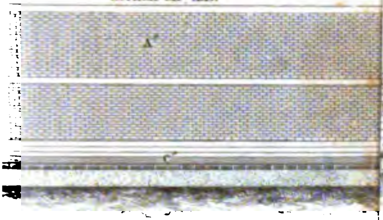


Fig. 12

Plate 11

Section on A.B.



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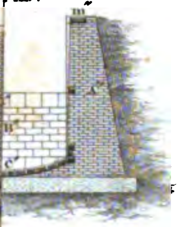
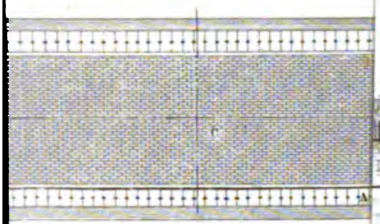
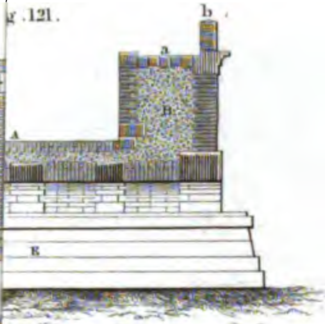


Fig. 118.

Plan



g. 121.



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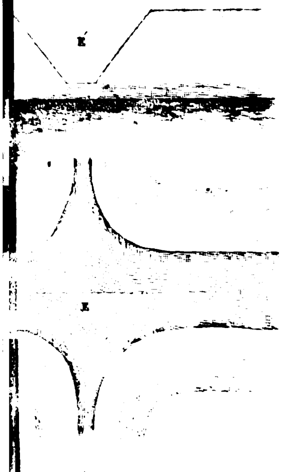

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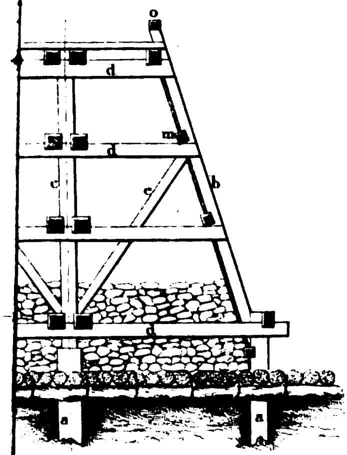

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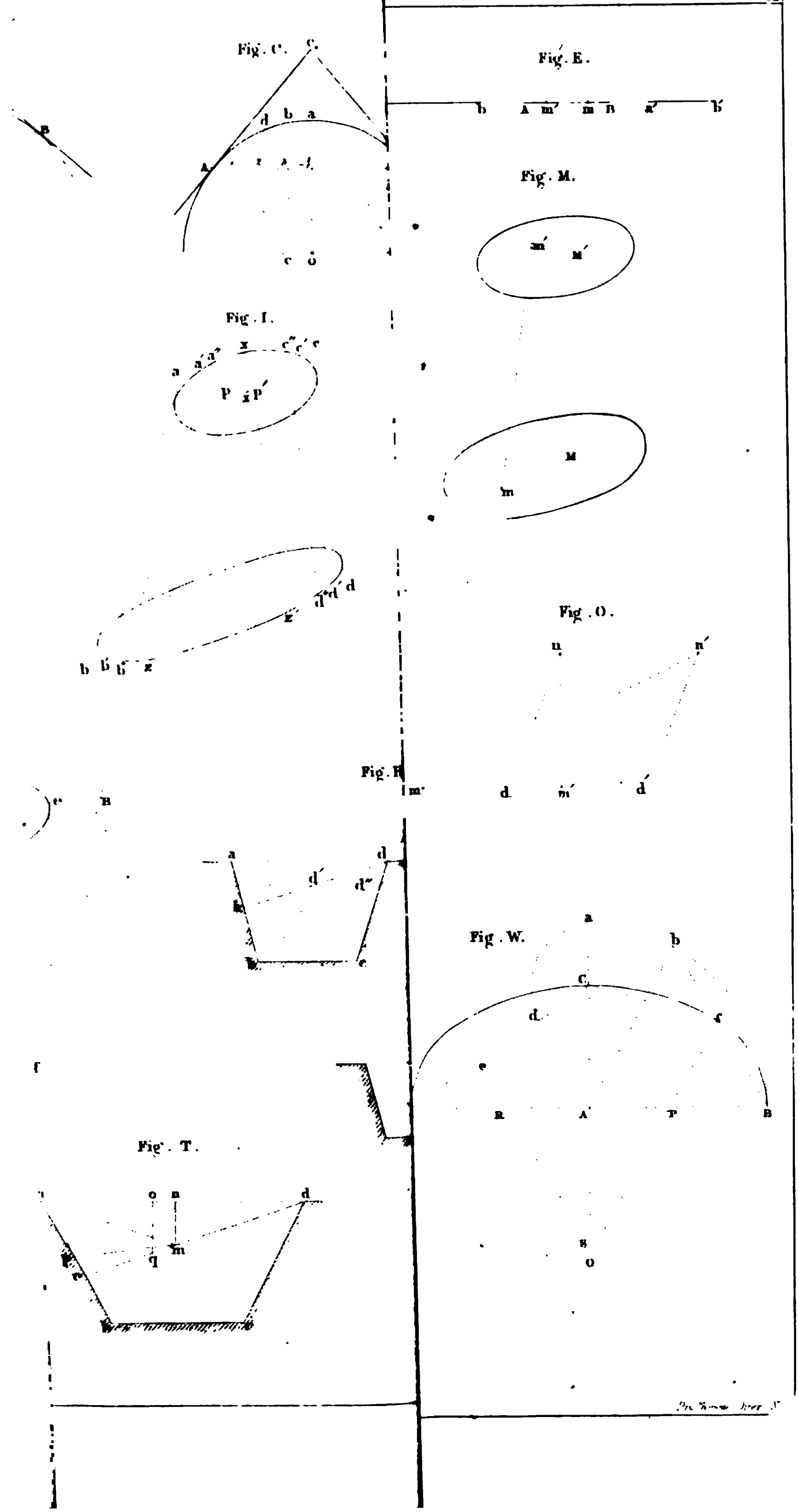
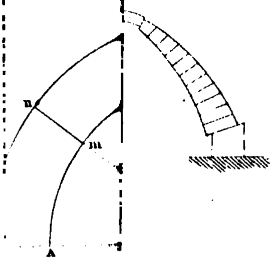


Plate 13

Fig. D.D.



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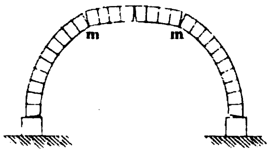

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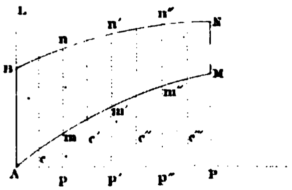

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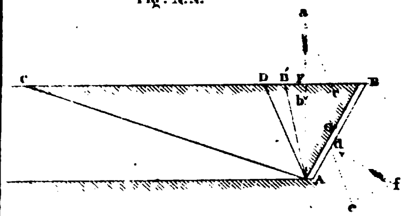

Fig. P.P.

Fig. T.T.

Fig. U.U.

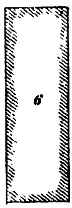

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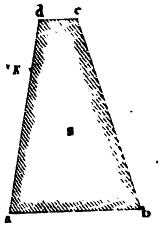
Fig. W.W.



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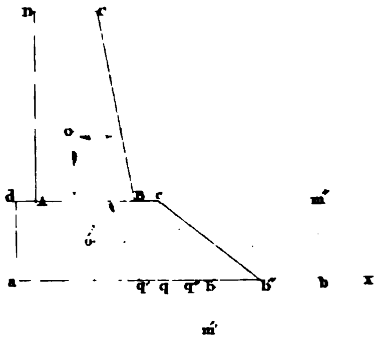

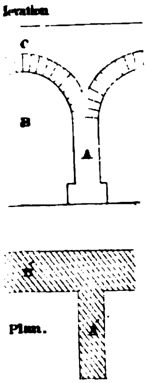

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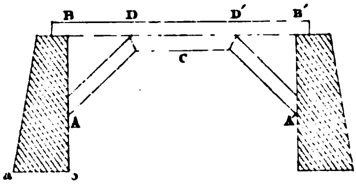

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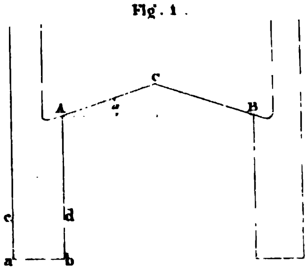

31

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